Global CKM Fits

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Chapter Outline

- Section: Introduction and goals 2p
- Section: Methodology
  - Subsection: CKMfitter 2p
  - Subsection: UTfit 2p
  - Subsection: Scanning method 2p
- Section: Experimental Inputs
  - Subsection: B-factories results: β, α (which decays to consider), γ, $2\beta+\gamma$, V_{ub}, V_{cb}, Δm_d, A^d_{SL}, $B(B\to\tau\nu)$,radiative penguins (how to use them) 4p
  - Subsection: Non-B-factories results (briefly on their threatment): ε_K, Δm_s, A^s_{SL}, TD $B_s \to J/\psi\phi$, $\Delta\Gamma_s$ (with order calculation). 2-3p
    - Rather than having subsubsections we indicate in the table which are inputs for the SM fit and inputs for the BSM fits
- Section Theoretical Inputs
  - Subsection Derivation of hadronic observables 2p
  - Subsection Lattice QCD inputs 4p
- Benchmark models 5p
- Section Results from the global fits
- Section Global fits beyond the Standard Model 4p
  - Subsection New-physics parameterizations 4p
  - Subsection Operator analysis 2p
- Section Conclusions 1-2p
- total: 36-38 pages

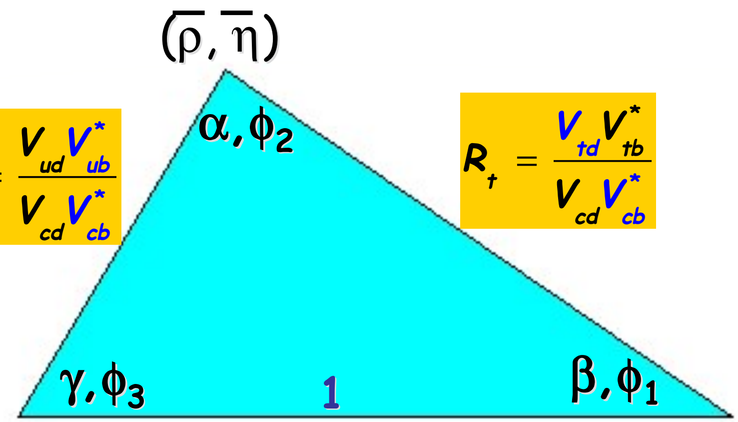
Motivation

- The CKM matrix is specified by 4 independent parameters,
  ➔ in the Wolfenstein approximation they are λ, A, $\bar{\rho}$, and $\bar{\eta}$

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+A^2\lambda^5(\frac{1}{2}-\rho-i\eta) & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4-\frac{1}{2}A^2\lambda^4 & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2+A\lambda^4(\frac{1}{2}-\rho-i\eta) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} + O(\lambda^6)$$

- Unitarity of the CKM matrix specifies relations among the parameters
  e.g. $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

- Combine measurements from the B and K systems to overconstrain the triangle
  ➔ test if phase of CKM matrix is only source of CP violation

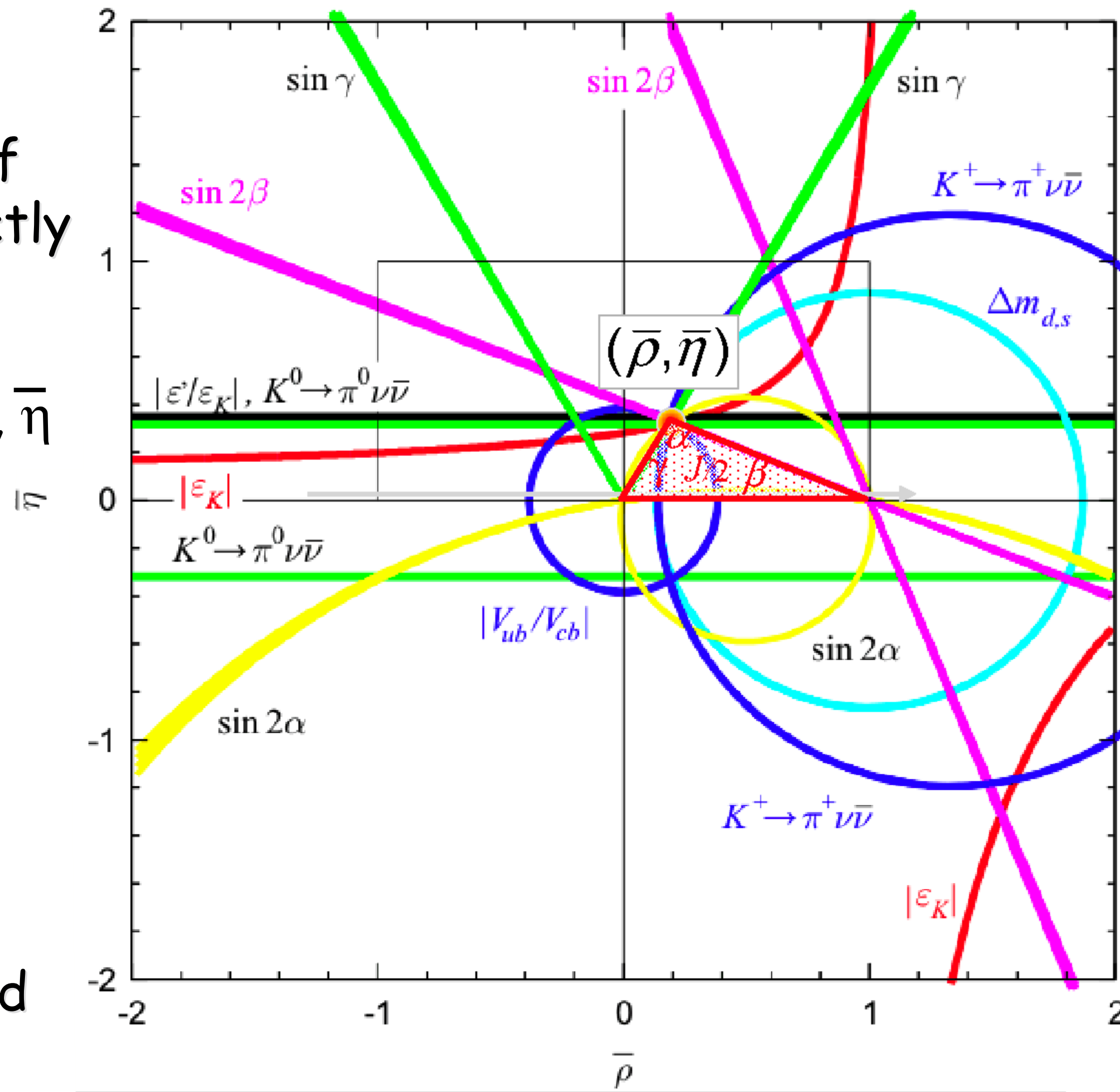


$$R_u = \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \qquad R_t = \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}$$

$$\alpha,\phi_2 = arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \qquad \beta,\phi_1 = arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \qquad \gamma,\phi_3 = arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

Motivation

- In the SM in the absence of errors all measurements of UT properties exactly meet in $\bar{\rho}$ and $\bar{\eta}$
- The extraction of $\bar{\rho}$, $\bar{\eta}$ depends on QCD parameters that have large theory uncertainties
- We use 3 different fit methods: CKMfitter, UTfit and Scanning method which differ in the treatment of QCD parameters



CKMfitter Methodology

- CKMfitter (Rfit) is a frequentist-based approach to the global fit of CKM matrix
- Likelihood function: $\mathcal{L}[y_{mod}] = \mathcal{L}_{exp}[x_{exp} - x_{th}(y_{mod})] \times \mathcal{L}_{th}[y_{QCD}]$

➔ First term measures agreement between data, x_{exp}, and prediction, x_{th}

➔ Second term expresses our present knowledge on QCD parameters

➔ y_{mod} are a set of fundamental and free parameters of theory (m_t, etc)

- Minimize $\chi^2(y_{mod}) \equiv -2\ln(\mathcal{L}[y_{mod}])$

  and determine $\Delta\chi^2(y_{mod}) = \chi^2(y_{mod}) - \chi^2_{min;y_{mod}}$

  where $\chi^2_{min;ymod}$ is the absolute minimum value of χ^2 function

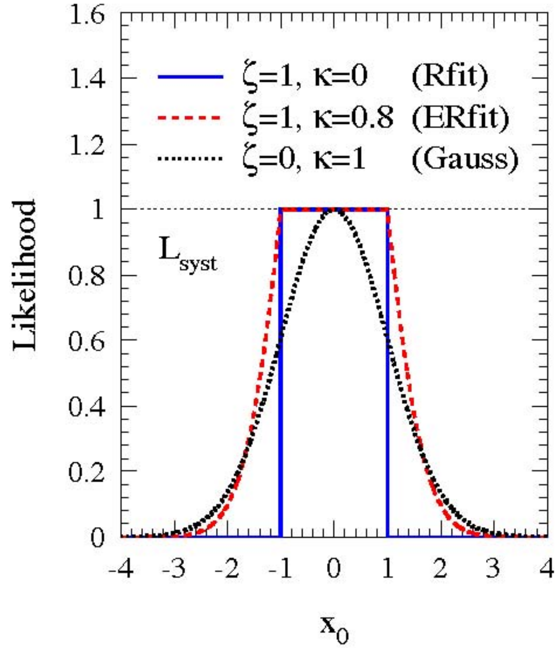
- Separate uncertainties of QCD parameters into statistical (σ) and non-statistical (theory) uncertainties (δ)

➔ statistical uncertainties are treated like experimental errors with a Gaussian likelihood

Rfit Methodology

- Treatment of theory uncertainties (δ):
  - ➔ If fitted parameter a lies within the predicted range $x_0 \pm \delta x_0$ contribution to χ^2 is zero
  - ➔ If fitted parameter a lies outside the predicted range $x_0 \pm \delta x_0$ the likelihood $\mathcal{L}_{th}[\,y_{QCD}]$ drops rapidly to zero, define:

$$-2\ln\mathcal{L}_{th}[x_0,\kappa,\zeta] = \begin{cases} 0, & \forall x_0 \in \left[\bar{x}_0 \pm \zeta\delta x_0\right] \\ \left(\dfrac{x_0-\bar{x}_0}{\kappa\delta x_0}\right)^2 - \left(\dfrac{\zeta}{\kappa}\right)^2, & \forall x_0 \notin \left[\bar{x}_0 \pm \zeta\delta x_0\right] \end{cases}$$



- 3 different analysis goals
  - Within SM achieve best estimate of y_{th}
  - Within SM set CL that quantifies agreement between data and theory
  - Within extended theory framework search for specific signs of new physics

UTfit Methodology

- UTfit is a Bayesian-based approach to the global fit of CKM matrix
- For M measurements c_j that depend on $\bar{\rho}$ and $\bar{\eta}$ plus other N parameters x_i the function $\mathbf{f(\bar{\rho}, \bar{\eta}, x_1, \ldots . x_N \mid c_1, \ldots . c_M)}$ needs to be evaluated by integrating over x_i and c_j
- Using Bayes theorem one finds

$$f(\bar{\rho}, \bar{\eta}, x_1, \ldots . x_N \mid c_1, \ldots . c_M) \propto \prod_{j=1,M} f_j(c_j \mid \bar{\rho}, \bar{\eta}, x_1, \ldots, x_N) \prod_{i=1,N} f_i(x_i) f_0(\bar{\rho}, \bar{\eta})$$

where $f_0(\bar{\rho}, \bar{\eta})$ is the a-priory probability for $\bar{\rho}$ and $\bar{\eta}$

- The output pdf for $\bar{\rho}$ and $\bar{\eta}$ is obtained by integrating over c_j and x_i

UTfit Methodology

- Measurement inputs and all theory parameters are described by pdfs
- Errors are typically treated with a Gaussian model, only for B_k, ξ and $f_B\sqrt{B_B}$ a flat distribution representing the theory uncertainty is convolved with a Gaussian representing the statistical uncertainty
- So if available, experimental inputs are represented by likelihoods
- The method does not make any distinction between measurement and theory parameters
- The allowed regions are well defined in terms of probability
  ➔ allowed regions at 95% probability means that you expect the "true" value in this range with 95% probability
- By changing the integration variables any pdf can be extracted
  ➔ this yields an indirect determination of any interesting quantity

The Scanning Method

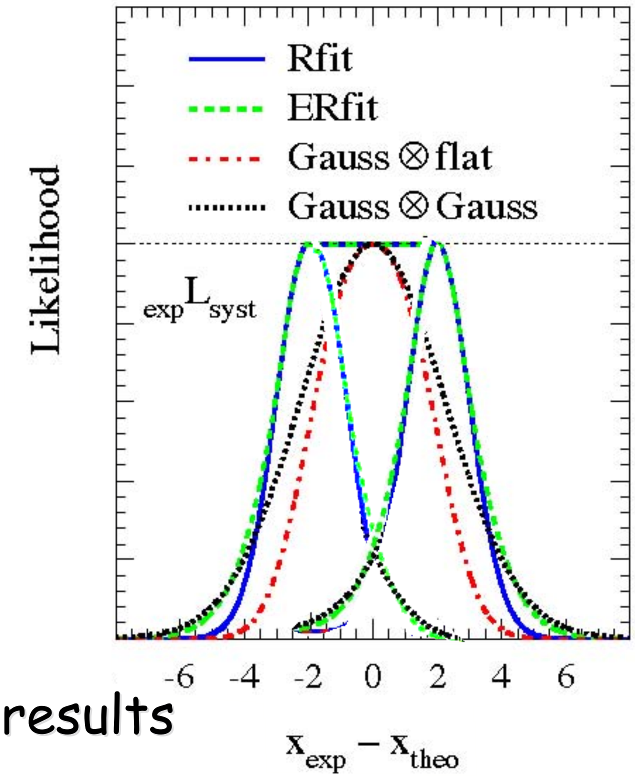
- The basis is the original approach by M.H. Schune & S. Plaszczynski used for the BABAR physics book
- The fit method was extended to include over 250 single measurements
- The four QCD parameters B_k, f_B, B_B, ξ and V_{ub}, V_{cb}, have significant theory uncertainties, thus they are scanned in the following way
  - We express each parameter in terms of $x_0 \pm \sigma_x \pm \delta_x$, where σ is a statistical uncertainty and δ_x is the theory uncertainty
  - We select a specific value $x^* \in [x_0 - \delta_x, x_0 + \delta_x]$ as a model
  - We consider all models inside the $[x_0 - \delta_x, x_0 + \delta_x]$ interval
  - In each model the uncertainty σ_q is treated in a statistical way
- The uncertainties in the QCD parameters η_{cc}, η_{ct}, η_{tt}, and η_B and the quark masses $m_c(\overline{m}_c)$ and $m_b(\overline{m}_b)$ are treated like statistical uncertainties, since these uncertainties are relatively small
  ➔ however, if necessary, we can scan over any of these parameters

The Scanning Method

- We perform maximum likelihood fits using a frequentist approach
- A model is considered consistent with data if $P(\chi^2_M)_{min} > 5\%$
- For consistent models we determine the best estimate and plot a 95% CL $(\bar{\rho}, \bar{\eta})$ contour ➔ we overlay contours of consistent models
  ➔ however, though only one of the contours is the correct one, we do not know which and thus show a representative numebr of them
- For accepted fits we also study the correlations among the theoretical parameters extending their range far beyond the range specified by the theorists
- We can input α, ϕ_2 and γ, ϕ_3 via a likelihood function or directly using individual $B\to\pi\pi$, $\rho\pi$, $\rho\rho$, $a_1\pi$, $b_1\pi$ measurements and GLW, ADS and Dalitz plot measurements in $B\to D^{(*)}K^{(*)}$ & $\sin(2\beta+\gamma)$, respectively
- We can further determine PP, PV VV amplitudes and strong phases using Gronau and Rosner parameterizations in powers of λ
- Work is in progress to include $\cos 2\beta$, β_s, A^q_{SL}, $\Delta\Gamma_s$ and τ_s add contours of $\sin 2\alpha$, γ and $\sin(2\beta+\gamma)$ and improve on display

Differences among the 3 Methods

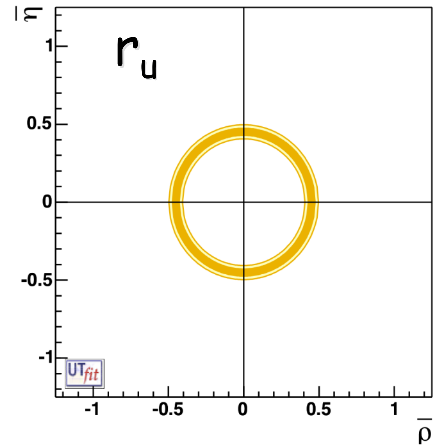
- Fit methodologies differ: 2 frequentist approaches vs 1 Bayesian approach
- Theory uncertainties are treated differently in the global fits
- Presently, measurement input values differ plus some assumptions differ
- For V_{ub} and V_{cb} there is an issue how to combine inclusive and exclusive results
  - Inclusive/exclusive averages ↔ individual results
  - Resulting errors

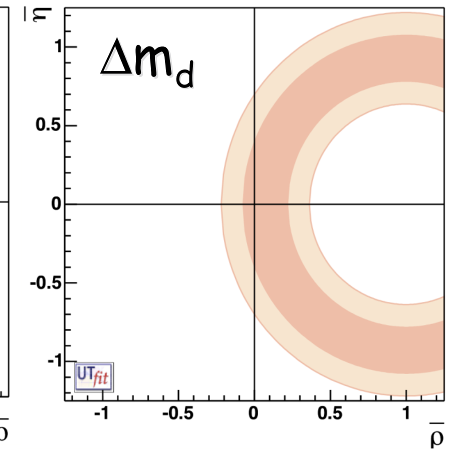


- QCD parameters inputs differ, eg f_{Bs}, B_{Bs}, f_{Bs}/f_{Bd}, B_{Bs}/B_{Bd} ↔ f_{Bd}, B_{Bd}, ξ ↔ $f_{Bd}\sqrt{B_{Bd}}$, ξ
- ➔ We need to standardize measurement inputs, QCD parameters (at least numerical values should agree) and assumptions

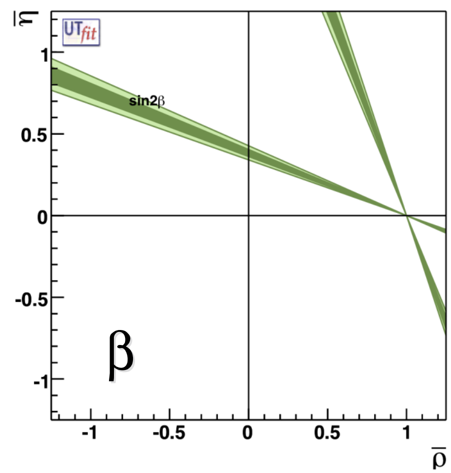
Input Measurements from B Factories

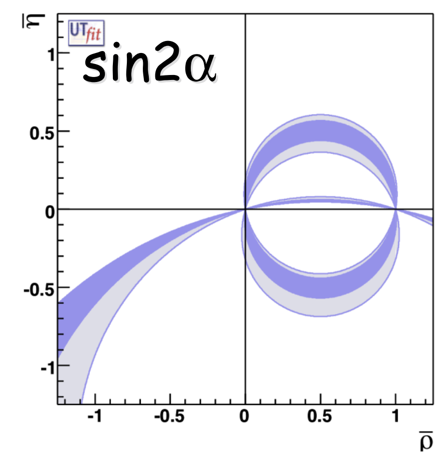
- V_{ub} and V_{cb} measured in exclusive and inclusive semileptonic B decays

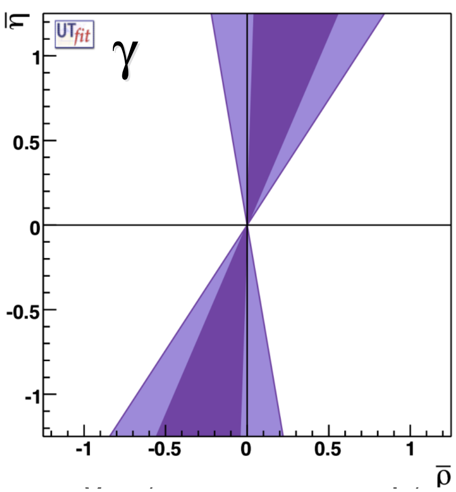




- Δm_d from $B_d \bar{B}_d$ oscillations
- CP asymmetries $a_{cp}(\psi K_S)$ from $B \to c\bar{c}K_s$ decays
  ➔ angle β
- α from $B \to \pi\pi$, $B \to \rho\rho$, & $B \to \rho\pi$ CP measurements, add $B \to a_1\pi$, $B \to b_1\pi$



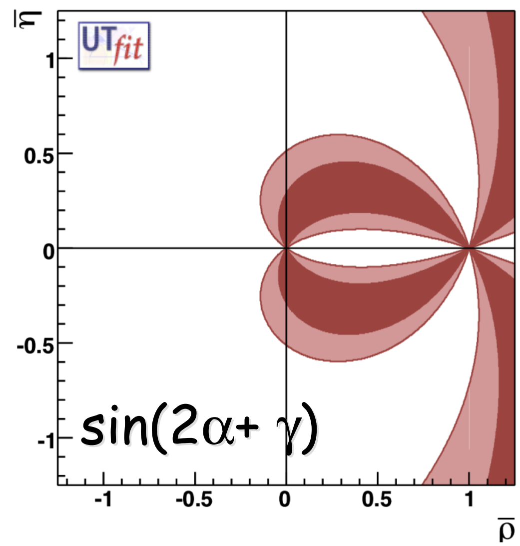


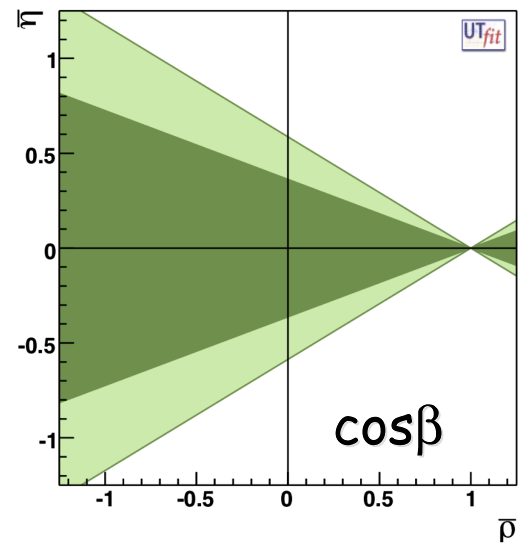


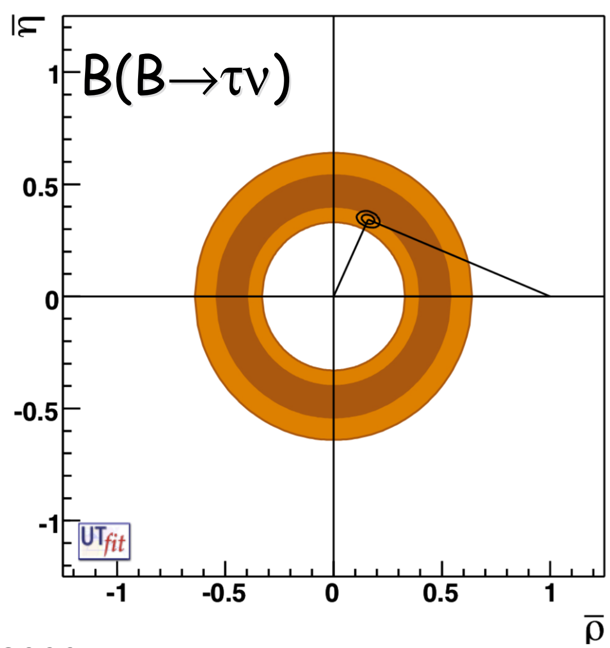
- GLW, ADS and GGSZ analyses in $B \to D^{(*)}K^{(*)}$

Input Measurements from B Factories

- $\sin(2\beta+\gamma)$ measurement from $B \to D^{(*)}\pi(\rho)$
- $\cos 2\beta$ from $B \to J/\psi K^*$ and $B \to D^0\pi^0$
- $B \to \tau\nu$ branching fraction

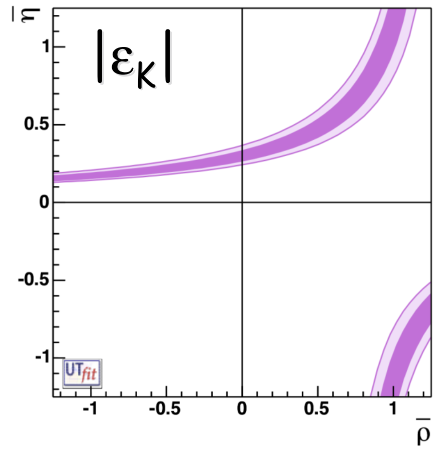


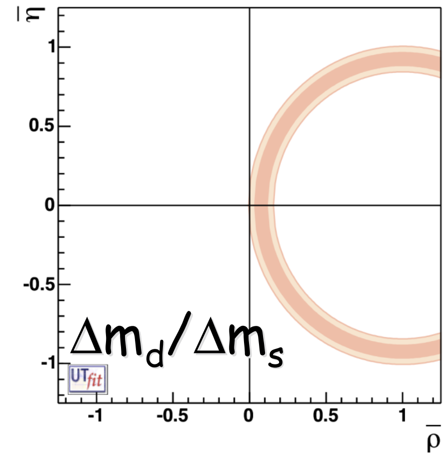


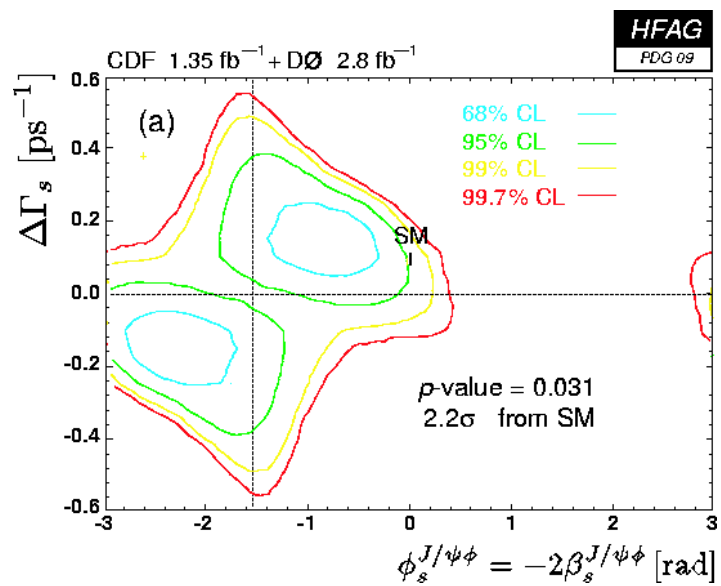


Other Input Measurements

- $|\varepsilon_K|$ from CP violation in K decays
- $\Delta m_d/\Delta m_s$ from $B_d\bar{B}_d$ and $B_s\bar{B}_s$ oscillations
- β_s - $\Delta\Gamma_s$ from B_s measurements at the Tevatron
- CKM elements V_{ud}, V_{us}, V_{cd}, V_{cs}, V_{tb}







Measurement Inputs

Observable	CKMfitter	UTfit	Scanning M
$\|V_{us}\|$	0.2246±0.0012	0.2259±0.0009	0.2258±0.0021
$\|V_{ub}\|$ [10^{-3}]	3.79±0.09±0.41*	$4.11^{+0.27}_{-0.28}$ (inc)	
		3.38±0.36 (exc)	3.84±0.16±0.29 (ex
$\|V_{cb}\|$ [10^{-3}]	40.59±0.37±0.58*	41.54±0.73 (inc)	
		38.6±1.1 (exc)	40.9±1.0±1.6 (exc)
$B(B\to\tau\nu)$ [10^{-4}]	1.73±0.35	1.51±0.33	1.79±0.72
Δm_{Bd} [ps^{-1}]	0.507±0.005	0.507±0.005	0.508±0.005
Δm_{Bs} [ps^{-1}]	17.77±0.12	17.77±0.12	17.77±0.12
$\|\varepsilon_K\|$ [10^{-3}]	2.229±0.010	2.229±0.010	2.232±0.007
sin 2β	0.671±0.023	0.671±0.023	0.68±0.025
α[$\pi\pi$, $\rho\pi$, $\rho\rho$]	1-CL(α)	$-2\Delta\ln(\mathcal{L})$	B, S, C for $\pi\pi$ & $\rho\rho$
γ [GGSZ, GLW, ADS]	1-CL(γ)	$-2\Delta\ln(\mathcal{L})$	GGSZ, GLW, ADS
cos 2β	$J/\psi K^*$	$J/\psi K^*$, $D^0\pi^0$	To be done
$2\beta+\gamma$	$D^{(*)}\pi(\rho)$	$-2\Delta\ln(\mathcal{L})$	$D^{(*)}\pi(\rho)$

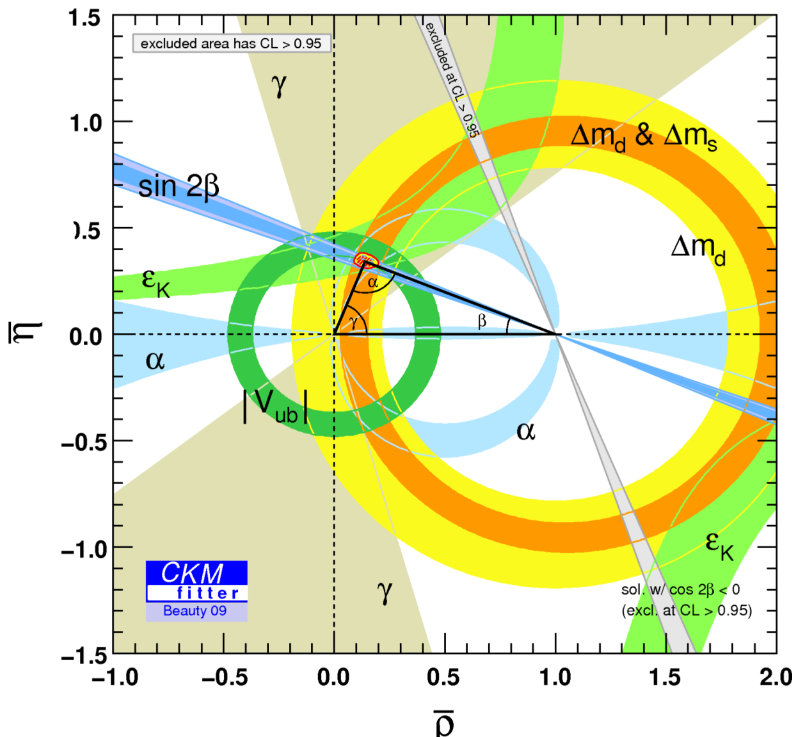
* use average values

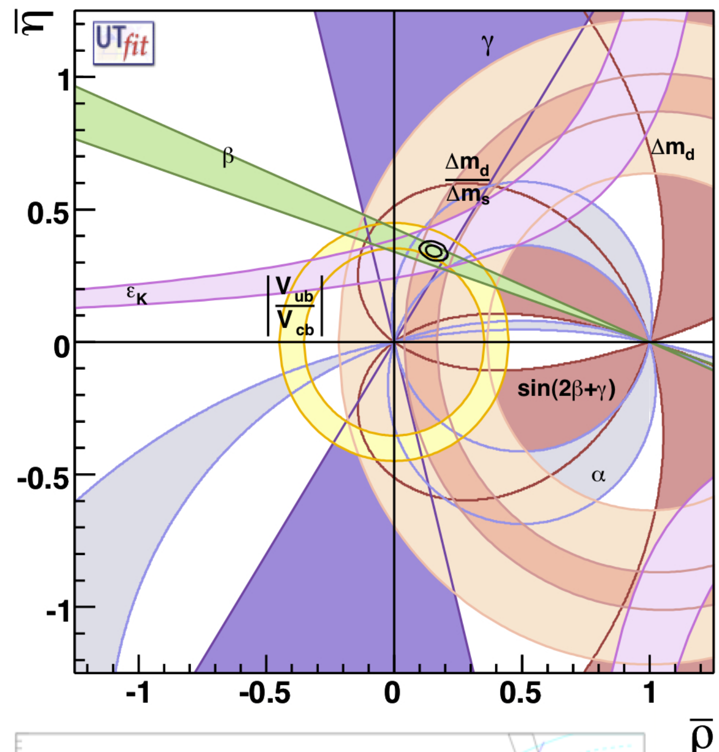
Lattice QCD Inputs

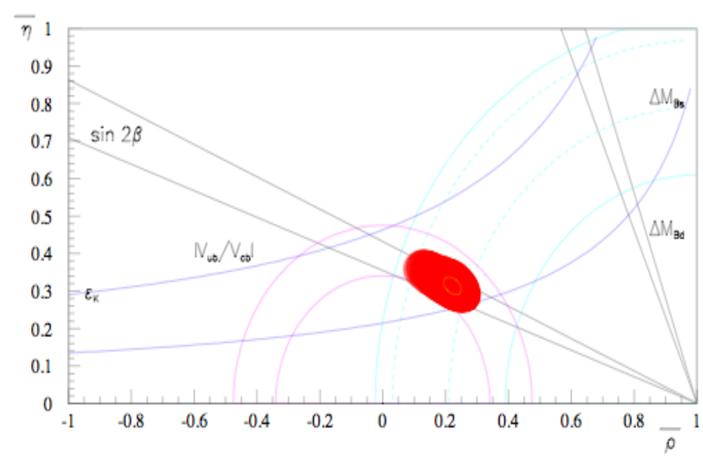
Parameter	CKMfitter			UTfit		Scanning		
	Mean	σ_{stat}	δ_{theo}	Mean	σ	Mean	σ_{stat}	δ_{theo}
f_{Bs} [f_{Bd}]	228	±3	±17	245	±25	[216	±10	± 20]
f_{Bs}/f_{Bd}	1.199	±0.008	±0.023	1.21	±0.04			
B_{Bs} [B_{Bd}]	1.23	±0.03	±0.05	1.22	±0.12	[1.29	±.05	±.08]
B_{Bs}/B_{Bd} [ξ]	1.05	±0.02	±0.05	1.00	±0.03	[1.2	±.028	±.05]
B_K [2 GeV]	0.525	±0.0036	±0.052					
B_K	0.721	±0.005	±0.040	0.75	±0.07	0.79	±0.04	±0.09
$m_c(\overline{m}_c)$ [GeV]	1.286	±0.013	±0.040	1.3	±0.1	1.27±0.11		
$m_t(\overline{m_t})$ [GeV]	165.02	±1.16	±0.11	161.2	±1.7	163.3	±2.1	
η_{cc}	Calculated from $m_c(m_c)$ & α_s			1.38	±0.53	1.46±0.22		
η_{ct}	0.47±0.04			0.47	±0.04	0.47±0.04		
η_{tt}	0.5765±0.0065			0.574	±0.004	0.5765±0.0065		
$\eta_B(\overline{MS})$	0.551±0.007			0.55	±0.01	0.551±0.007		
α_s	0.1176±0.0020			0.119	±0.03	0.118		

Global Fit Results

- Inputs: $|V_{ud}|$, $|V_{us}|$
  - $|V_{cb}|$, $|V_{ub}|$
  - $\mathcal{B}(B\to\tau\nu)$
  - $|\varepsilon_K|$
  - Δm_{Bd}, Δm_{Bs}
  - $\sin 2\beta$, 1-CL(α), 1-CL(γ)





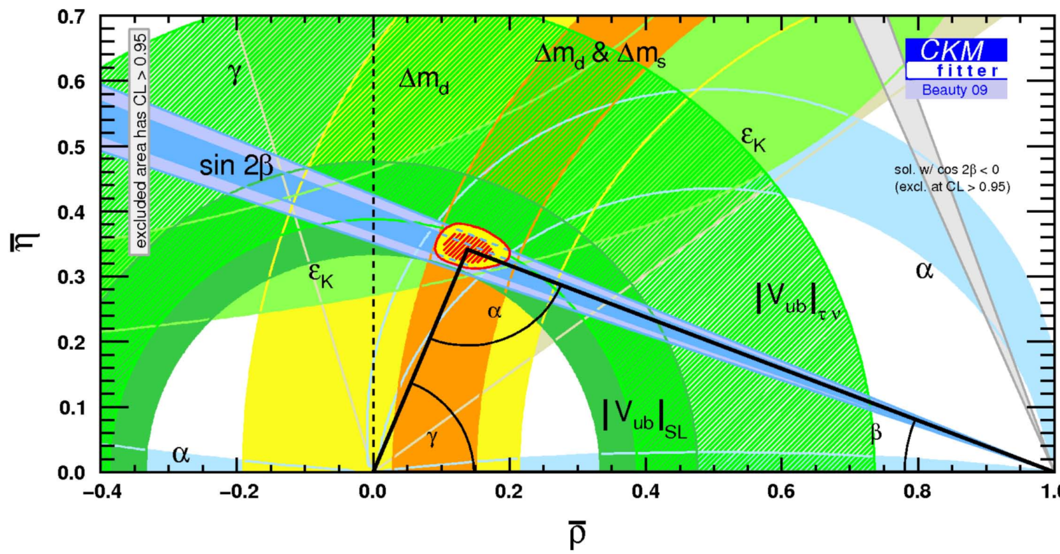


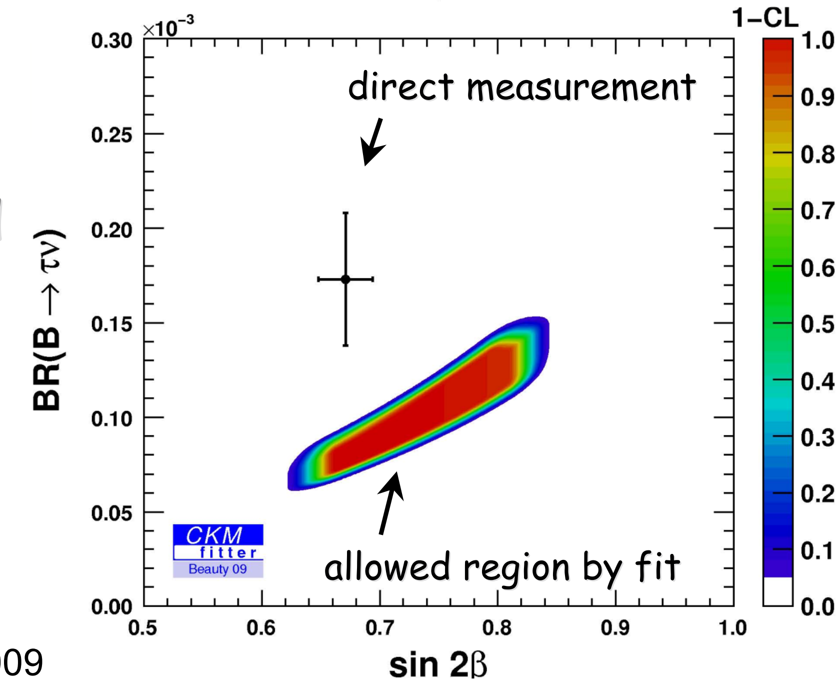
2008 inputs

Note scales are not the same!

Tension from $\mathcal{B}(B\to\tau\nu)$

- $\mathcal{B}(B\to\tau\nu)$ is proportional to $|V_{ub}|^2$ and f^2_{Bd}
- from global fit $\mathcal{B}(B\to\tau\nu)=(0.79^{+0.016}_{-0.010})\times10^{-4}$
- WA: $\mathcal{B}(B\to\tau\nu)=(1.73\pm0.35)\times10^{-4}$
- 2.4σ discrepancy
- If $\mathcal{B}(B\to\tau\nu)$ or $\sin 2\beta$ are removed χ^2_{min} in global fit drops by 2.4 $|V_{ub}|$, $|V_{cb}|$ remain unaffected



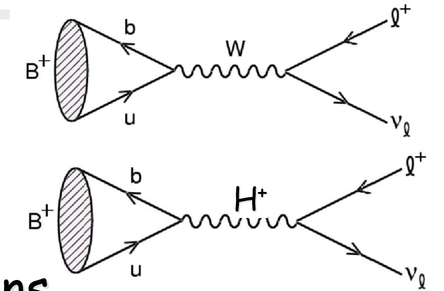


Constraints in the m_H-tanβ Plane

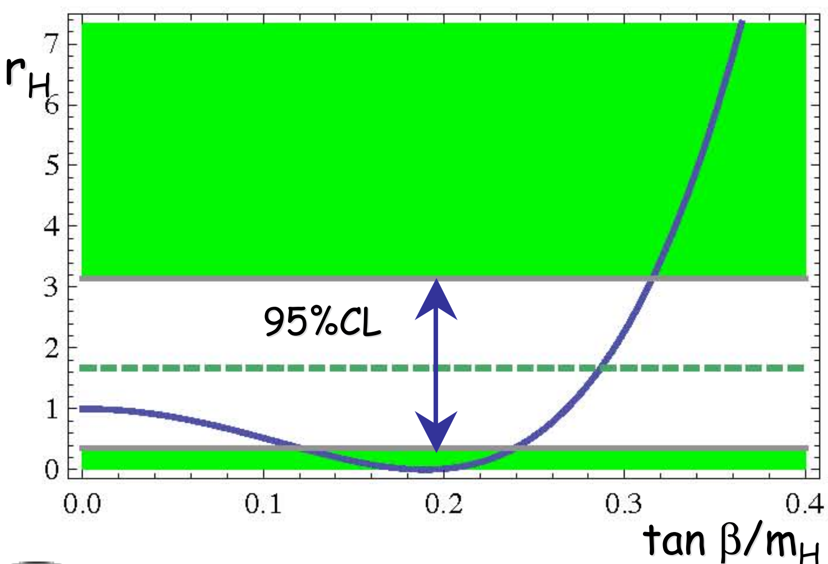
- From BABAR/Belle average we extract

$$r_H = 1.67 \pm 0.34_{exp} \pm 0.36_{f_B, V_{ub}}$$

$$r_H = \left(1 - \frac{m_B^2}{m_{H^+}^2 (1 + \varepsilon_0 \cdot \tan\beta)} \tan^2\beta\right)^2$$

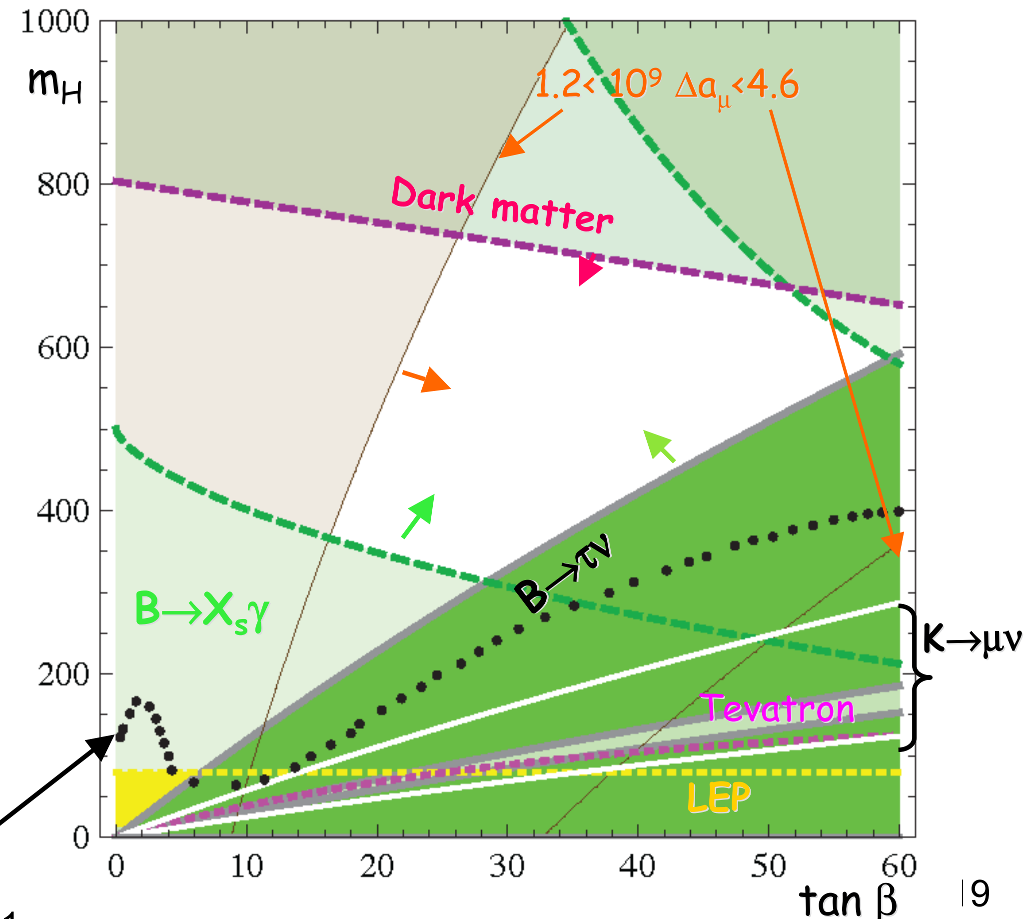


- We can use the 95% CL to present exclusions at 95% CL in the m_{H^+}-tanβ plane



ATLAS 5σ discovery curve

95% C.L. exclusions



Model-Independent Analysis of UT

- Assume that new physics only affects short-distance part of $\Delta B=2$
- We use model-independent parameterization for B_d and B_s

$$\frac{\langle B_q^0 | H_{\Delta B=2}^{full} | \bar{B}_q^0 \rangle}{\langle B_q^0 | H_{\Delta B=2}^{SM} | \bar{B}_q^0 \rangle} = \Delta_q^{NP} = |\Delta_q^{NP}| \exp\{2i\phi_q^{NP}\}$$

where $H^{full} = H^{SM} + H^{NP}$

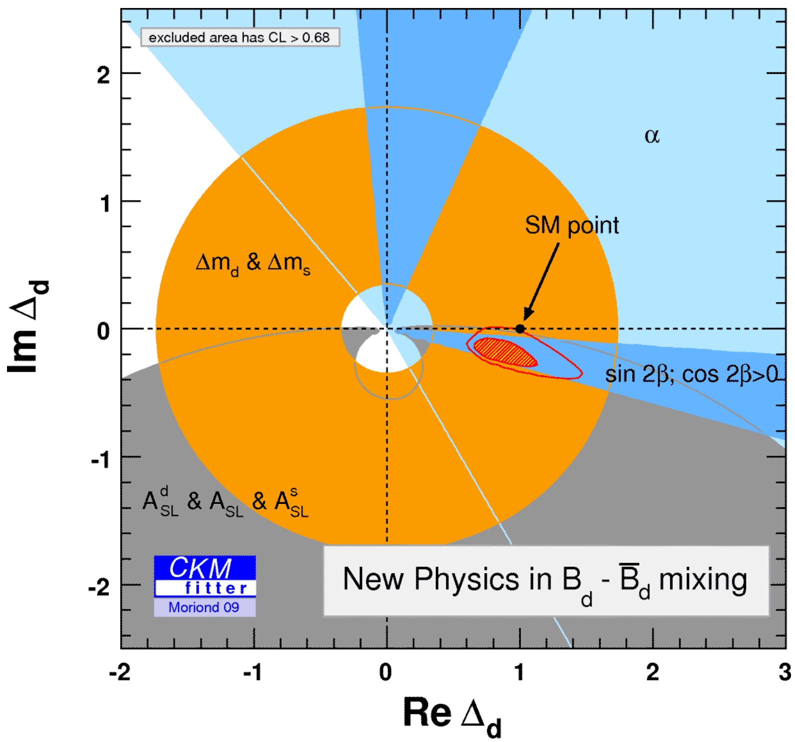
- Several observables are modified by the magnitude or phase of Δ_q^{NP}
- In B_d system we compare R_u & γ with sin 2β, sin2α and Δm_d that may be modified by NP parameters (Δ_d, ϕ_d)

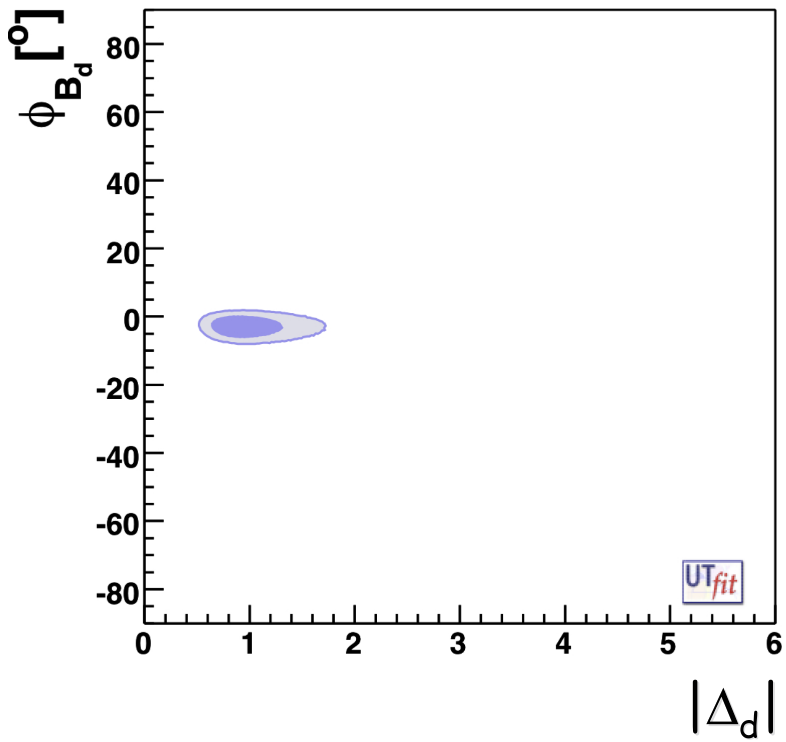
	parameter	prediction in the presence of NP
	Δm_q	$\lvert\Delta_q^{\rm NP}\rvert \times \Delta m_q^{\rm SM}$
phases	2β	$2\beta^{\rm SM} + \Phi_d^{\rm NP}$
phases	$2\beta_s$	$2\beta_s^{\rm SM} - \Phi_s^{\rm NP}$
phases	2α	$2(\pi - \beta^{\rm SM} - \gamma) - \Phi_d^{\rm NP}$
phases	$\Phi_{12,q} = {\rm Arg}[-\frac{M_{12,q}}{\Gamma_{12,q}}]$	$\Phi_{12,q}^{\rm SM} + \Phi_q^{\rm NP}$
	A_{SL}^q	$\frac{\Gamma_{12,q}}{M_{12,q}^{\rm SM}} \times \frac{\sin(\Phi_{12,q}^{\rm SM}+\Phi_q^{\rm NP})}{\lvert\Delta_q^{\rm NP}\rvert}$
	$\Delta\Gamma_q$	$2\lvert\Gamma_{12,q}\rvert \times \cos(\Phi_{12,q}^{\rm SM} + \Phi_q^{\rm NP})$

- In B_s system we compare R_u & γ with Δm_s, β_s, and $\Delta\Gamma_s$ that may be modified by NP parameters (Δ_d, ϕ_d)

Model-Independent Analysis of UT for $|\Delta_d|$- ϕ_d

- Inputs: Δm_d, Δm_s, sin 2β, α, $\Delta\Gamma_d$, A_{SL}^{Bd}, A_{SL}^{Bs} , w/o $\mathcal{B}(B\to\tau\nu)$

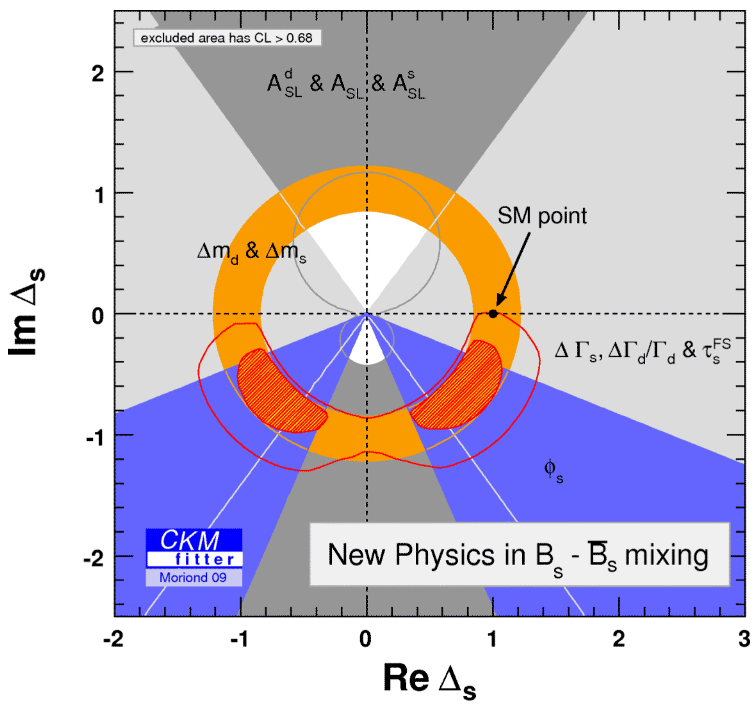


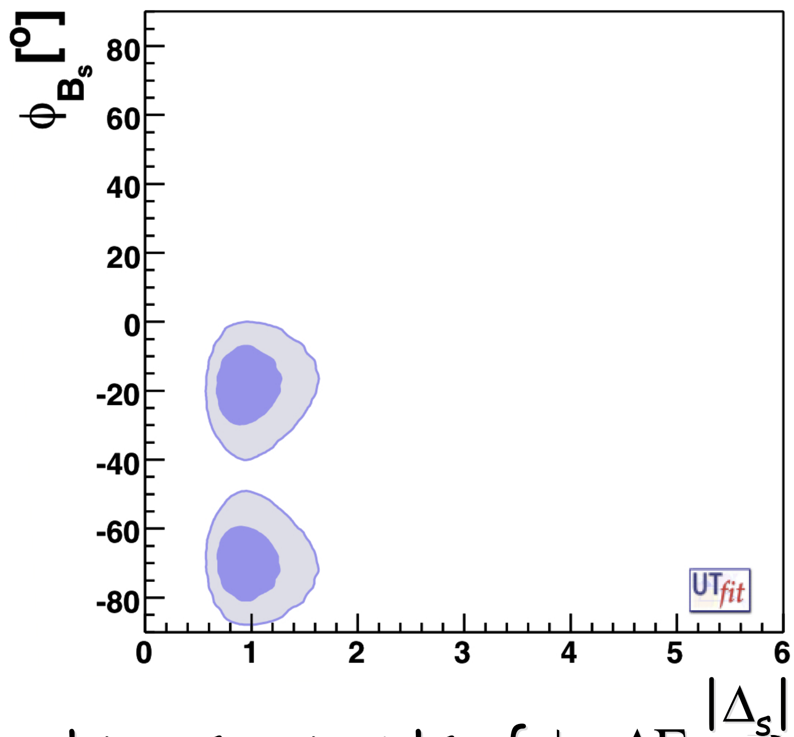


- Dominant constraints come from β and Δm_d
- Semileptonic asymmetries A_{SL} exclude symmetric solution with η<0
- Δ_d=1 (SM) is disfavored by 2.1σ (discrepancy $\mathcal{B}(B\to\tau\nu)$ and sin 2β)
➔ $\phi_d^{NP}=(-12^{+9}_{-6})^0$ @ 95% CL (➔discrepancy is 0.6σ w/o $\mathcal{B}(B\to\tau\nu)$)

Model-Independent Analysis of UT for $|\Delta_s|$- ϕ_s

- Inputs: ϕ_s, Δm_d, Δm_s, A_{SL}^{Bd}, A_{SL}^{Bs}, $\Delta\Gamma_s$, τ_s, $\mathcal{B}(B\to\tau\nu)$





- Dominant constraints come from direct measurements of ϕ_s, $\Delta\Gamma_s$ in $B\to J/\psi\phi$ and Δm_s from the Tevatron
- ϕ_s is 2.2σ away from the SM prediction
- Δ_s =1 is disfavored at 1.9σ level independent of $\mathcal{B}(B\to\tau\nu)$

Final Remarks

- Among the 3 global CKM fitting methods we need to standardize
  - All measurement inputs
  - What QCD parameters to use, their central values, their statistical errors and their theory errors
  - The notation for quantities in the text, on plots and in equations

- We need to specify which input parameters are used in the fits and standardize on the assumptions
  ➔ This is important for comparing results

- We will present results in form of plots with values listed in tables
  ➔ we accompany the results with a few remarks
  ➔in particular in cases of discrepancies we need to discuss them

- Most of the writing probably has to be done by the co-conveners

More Recent Publications

- CKMfitter publications
  - J. Charles et al., Eur. Phys. J. C41, 1-135, 2005.

- UTfit publications:
  - M. Bona et al., JHEP 0610:081, 2006.
  - M. Bona et al., Phys.Rev.D76:014015, 2007.
  - M. Bona et al., Phys.Rev.Lett.97:151803, 2006.
  - M Bona et al., Phys.Lett.B687:61-69, 2010.

- Scanning method
  - G. Eigen et al., Eur.Phys.J.C33:S644-S646,2004.
  - G.P. Dubois-Felsmann et al., hep-ph/0308262.