# 13

# The CKM matrix and the Kobayashi Maskawa mechanism

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# Scope of the section

- Provide the basic notation to be used later
- Describe the CKM matrix and the KM mechanism in a nutshell
- Give a very brief historical review

# Status

- Historical bckgrnd is not yet written
- Basic formulae have been typeset
- Figures have been added

## The CKM matrix and the Kobayashi-Maskawa mechanism

Scope: *The KM mechanism as an extension to quark mixing; and resulting in the CKM matrix. Introduce approximations that will be encountered in the rest of the book.* The following journal papers should be cited

- KM: Kobayashi and Maskawa (1973)
- Nobel Release: [REF?]
- PDG CKM Review article via PDG: Amsler et al. (2008)
- Wolfenstien: Wolfenstein (1983)

The observed charged-current interactions can be written as

$$H_{\mathrm{cc}} = \frac{g}{\sqrt{2}} (\bar{u}_L \, \bar{c}_L \, \bar{t}_L) V_{\mathrm{CKM}} \gamma^{\mu} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_{\mu}^{+}, \tag{1}$$

where $V_{\mathrm{CKM}}$ is given by

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix}. \tag{2}$$

Here the $V_{ij}$ are couplings of quark mixing transitions from an up-type quark $i = u, c, t$ to a down-type quark $j = d, s, b$.

In the standard model the CKM matrix is unitary by construction. Using the freedom of phase redefinitions for the quark fields, the CKM matrix has $(n-1)^2$ physical parameters for the case of $n$ families. Out of these, $n(n-1)/2$ are (real) rotation angles, and $((n-3)n+2)/2$ are phases, which induce CP violation. For $n = 2$, no CP violation is possible, while for $n = 3$ a single phase appears.

The CKM matrix for 3 families may be represented three rotations and a matrix generating the phase:

$$U_{12} = \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad U_{13} = \begin{bmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{bmatrix} , \quad (3)$$

$$U_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} , \quad U_{\delta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta_{13}} \end{bmatrix} .$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and $\delta$ is the complex phase responsible for $CP$ violation, and by convention the mixing angles $\theta_{ij}$ are chosen to lie in the first quadrant so that the $s_{ij}$ and $c_{ij}$ are positive. A common way to write it is proposed by the PDG [REF]:

$$V_{\mathrm{CKM}} = U_{23} U_{\delta}^{\dagger} U_{13} U_{\delta} U_{12} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} . \quad (4)$$

The elements of the CKM matrix exhibit a pronounced hierarchy. While the diagonal elements are close to unity, the off-diagonal elements are small, such that e.g. $V_{ud} \gg V_{us} \gg V_{ub}$. In terms of the angles $\theta_{ij}$ we have $\theta_{12} \gg \theta_{23} \gg \theta_{13}$. This fact is usually expressed in terms of the Wolfenstein parametrization, which can be understood as an expansion in $\lambda = |V_{us}|$. It reads up to order $\lambda^3$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 - i\eta A^2\lambda^4 & A\lambda^2(1+i\eta\lambda^2) \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \quad (5)$$

The parameters $A$, $\rho$, and $\eta$ turn out to be of order one. When using the above Wolfenstein parametrization, one has to keep in mind that unitarity is satisfied only up to the order $\lambda^4$.

One can obtain an exact parameterisation of the CKM matrix in terms of $A$, $\lambda$, $\rho$, and $\eta$, for example, by following the convention of Buras, Lautenbacher, and Ostermaier (1994), where

$$\lambda = s_{12}, \quad (6)$$

$$A = s_{23}/\lambda^2, \quad (7)$$

$$A\lambda^3(\rho - i\eta) = s_{13}e^{-i\delta}, \quad (8)$$

on substituting Eqns. 6 through 8 into Eq. 4, and noting that $\sin^2\theta = 1 - \cos^2\theta$.

These relations can be depicted by triangles in the complex plane; inserting the Wolfenstein parametrization, both relations turn out to be identical, up to terms of order $\lambda^5$. Eq. (9) is referred to as the unitarity triangle, and is depicted in Figure 1. The apex of the unitarity triangle is given by the coordinate $(\overline{\rho}, \overline{\eta})$, where $\overline{\rho} = \rho[1 - \lambda^2/2 + \mathcal{O}(\lambda^4)]$, and $\overline{\eta} = \eta[1 - \lambda^2/2 + \mathcal{O}(\lambda^4)]$.

A non-trivial unitarity triangle indicates CP violation, which is proportional to the area of the triangle. Observation of CP violation has shown that the unitarity triangle is indeed non-trivial, and the angles of the unitarity triangle are defined as

$$\phi_1 = \beta \equiv \arg\left[-V_{\mathrm{cd}}V_{\mathrm{cb}}^*/V_{\mathrm{td}}V_{\mathrm{tb}}^*\right], \quad (11)$$

$$\phi_2 = \alpha \equiv \arg\left[-V_{\mathrm{td}}V_{\mathrm{tb}}^*/V_{\mathrm{ud}}V_{\mathrm{ub}}^*\right], \quad (12)$$

## The Unitarity Triangle

The unitarity relations $V_{\mathrm{CKM}} \cdot V_{\mathrm{CKM}}^{\dagger} = 1$ and $V_{\mathrm{CKM}}^{\dagger} \cdot V_{\mathrm{CKM}} = 1$ yield six independent relations corresponding to the off-diagonal zeros in the unit matrix. However, most of these triangles are "squashed", i.e. they have disparate sides. Only two triangles have sides of comparable length, which means that they are of same order in the Wolfenstein parameter $\lambda$. The two relations are

$$V_{\mathrm{ud}}V_{\mathrm{ub}}^{*} + V_{\mathrm{cd}}V_{\mathrm{cb}}^{*} + V_{\mathrm{td}}V_{\mathrm{tb}}^{*} = 0, \tag{9}$$

$$V_{\mathrm{ud}}V_{\mathrm{td}}^{*} + V_{\mathrm{us}}V_{\mathrm{ts}}^{*} + V_{\mathrm{ub}}V_{\mathrm{tb}}^{*} = 0. \tag{10}$$

$$\phi_3 = \gamma \equiv \arg\left[-V_{\mathrm{ud}}V_{\mathrm{ub}}^{*}/V_{\mathrm{cd}}V_{\mathrm{cb}}^{*}\right], \tag{13}$$

where this definition is independent of the specific phase choice expressed in Eq. (4).

Conventionally different notations have been used in the literature for the angles of the Unitarity Triangle. In particular the *BABAR* experiment has used $\alpha$, $\beta$, and $\gamma$ to denote the angles, whereas the Belle experiment has reported results in terms of $\phi_1$, $\phi_2$, and $\phi_3$. For clarity on presenting and discussing results in later sections, after reminding the reader of the different notational choices, we proceed to discuss results in terms of the $\phi_1$, $\phi_2$, $\phi_3$ basis.

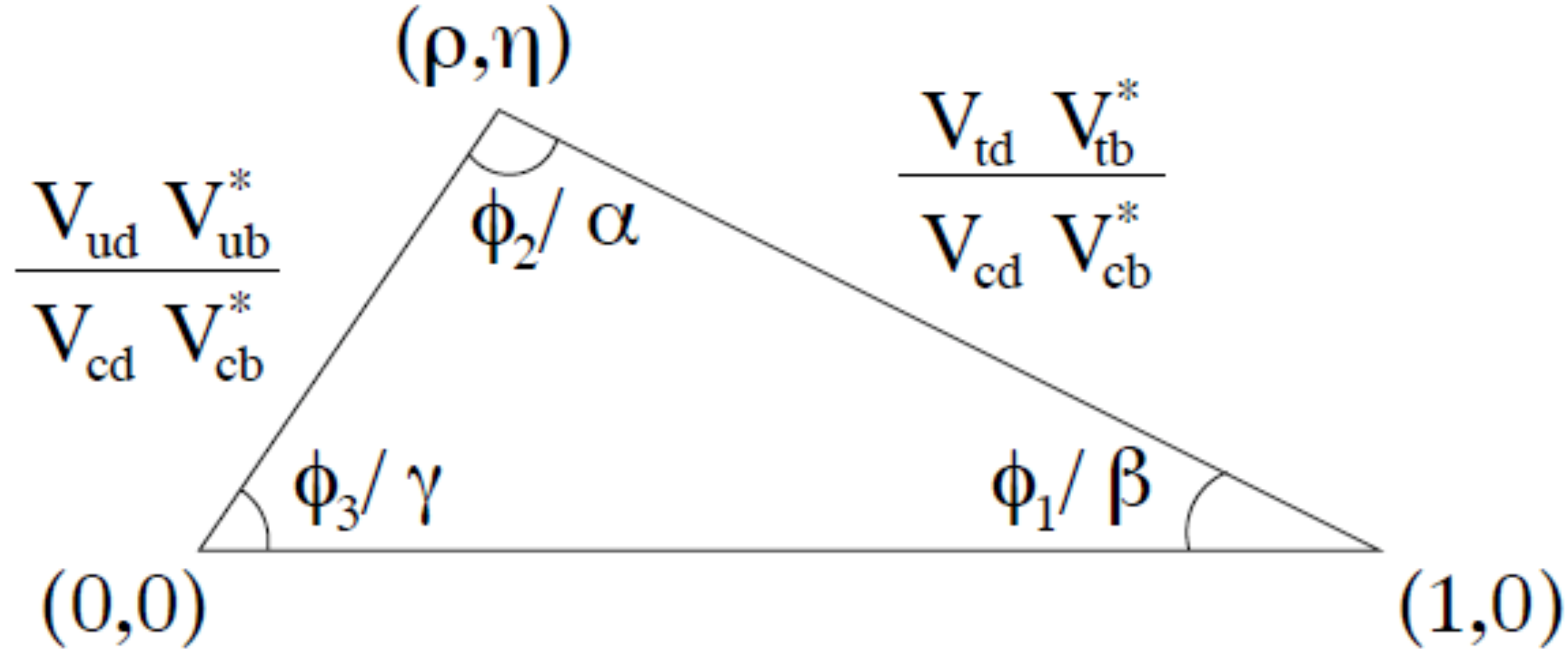


**Fig. 1.** The Unitarity Triangle.

Chapters 14.6, 14.7, and 14.8 discuss measurements of the angles $\phi_1$, $\phi_2$, and $\phi_3$, respectively. Measurements of the magnitues of CKM matrix elements $V_{\rm ub}$, $V_{\rm cb}$, $V_{\rm td}$, and $V_{\rm ts}$ can be found in Chapters 14.1 and 14.2. It is possible to perform *global fits* using data from many decay processes and theoretical input from Lattice QCD calcuations to over-constrain our knowledge of the CKM mechanism in the determination of the apex of the triangle. These global fits, both in the context of the SM and allowing for physics beyond the SM are discussed in Chapter 22.

# Outlook

- Can be finished very soon
- Still need to provide a balanced view on the history
- ... which should be sufficiently compact.