Introduction to the Continuum Discretized Coupled Channels method (CDCC).

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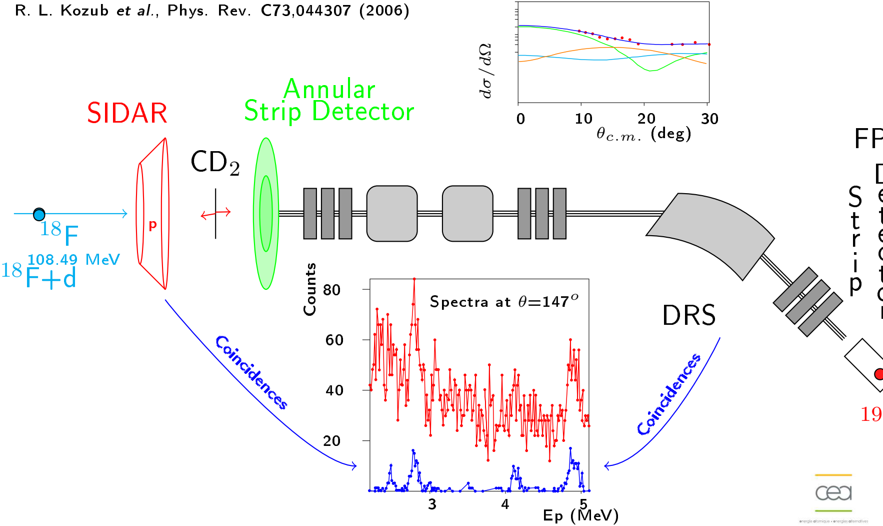
Contents

Introduction: an example of (d,p) reaction.

R. L. Kozub *et al.*, Phys. Rev. C73,044307 (2006)



Introduction: what can we learn from d induced reactions?

Nuclear spectroscopy from deuteron scattering.

1. From transfer reaction ${}^{A}_{Z}X(d,p){}^{A+1}_{Z}X$:
    1. The spectrum of the nucleus ${}^{A+1}_{Z}X$.
    2. The spin and parity of the levels of this nucleus.
2. From elastic and inelastic scattering ${}^{A}_{Z}X(d,d'){}^{A}_{Z}X^{*}$:
    1. The excitations energies of the nucleus ${}^{A}_{Z}X$.
    2. The deformation parameters of this nucleus.

The deuteron features in the cross section calculations.

Our goal is to include the deuteron properties in the cross section calculations. We wish to take into account that the deuteron

- is a composite system;
- and it is a weakly bound nucleus (low binding energy (2.2 MeV) and no excited states).

Some references.

CDCC formalism

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M. Yahiro and M. Kamimura, Prog. Theor. Phys. **65** (1981) 2046.

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M. Yahiro *et al.*, Prog. Theor. Phys. **67** (1982) 1467.

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I will mainly follow the notations of [8].

Discretization methods

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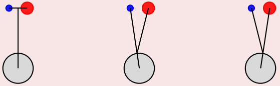
Introduction.

The CDCC approach is a quantum mechanical description of the scattering of a composite projectile by a nucleus which includes the structure of this projectile. But since this problem is very complex, some approximations and simplifications will be done.

First simplifications.



To describe the 3-body system, one should use these three partitions but within the CDCC approach, only the first partition is taken into account.

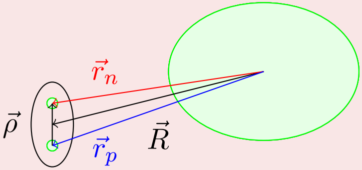
A quantum mechanical approach: $(\hat{H}_{\text{eff}} - E)\Psi^J_M = 0$.

"Quantum mechanics" means that:

1. the dynamics of the "deuteron+target" (d+A) system will be obtained by solving the Schrödinger equation for a given Hamiltonian, $\hat{H}_{\text{eff}}$.
2. the d+A system will be described by a wave function, Ψ^J_M.

The formalism: the Hamiltonian.

Some notations: the c.m. coordinates and the relative ones.



$$\begin{cases} \vec{R} = 1/2\,(\vec{r}_n + \vec{r}_p) \\ \vec{\rho} = (\vec{r}_n - \vec{r}_p) \end{cases}$$

$$\begin{cases} \vec{r}_n = \vec{R} + 1/2\,\vec{\rho} \\ \vec{r}_p = \vec{R} - 1/2\,\vec{\rho} \end{cases}$$

$$r_i = \sqrt{a_i^2 R^2 + b_i^2 \rho^2 + 2 a_i b_i R \rho \cos(\vec{R}, \vec{\rho})}$$

with $a_i = 1$ and $b_i = \pm 1/2$.

The effective 3-body hamiltonian: $(\hat{H}_{eff} - E)\Psi_M^J = 0$

$$\hat{H}_{\text{eff}} = \bar{p}\left[\hat{T}_{\vec{R}} + U_p(\vec{r}_p) + U_n(\vec{r}_n) + \underbrace{\hat{T}_{\vec{\rho}} + V_{pn}(\vec{\rho})}_{\hat{H}_{\text{pn}}} + V_p^{(Coul)}(R)\right]\bar{p}\,.$$

The formalism: the Hamiltonian.

An effective Hamiltonian: choice of the interactions.

CDCC is an effective formalism in which it is assumed that

1. The target remains inert.
2. The energy of each nucleon of the projectile is $E/2$.
3. The interaction between the nucleon and the target is given by a local optical potential

$$U_i(\vec{r}_i) = U_{\text{eff}}(\vec{r}_i, E/2).$$

4. The interaction between the proton and the neutron is ajusted to reproduce the binding energy, the phase shifts...

The formalism: the Hamiltonian in the spherical case.

Multipole expansion of the potentials: $U_i(\vec{r}_i) \equiv U_i(r_i)$ ($i = n$ or p).

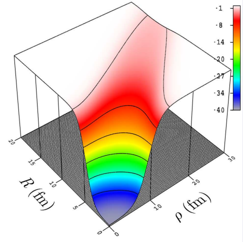
The optical potential depending on r_i will be written as function of R, ρ and the angle between $\vec{R}$ and $\vec{\rho}$:

$$\begin{aligned} U_i(r_i) &= U_i\left(\sqrt{a_i^2 R^2 + b_i^2 \rho^2 + 2a_i b_i R\rho \cos(\vec{R},\vec{\rho})}\right) = U_i(R,\rho,\gamma) \\ &= \sum_{\lambda=0}^{\infty} V_{i,\lambda}(R,\rho) P_\lambda(\gamma) \end{aligned}$$

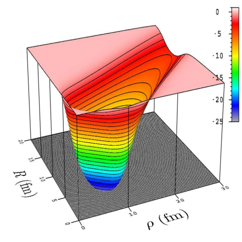
where $\gamma = \cos(\vec{R},\vec{\rho})$. Due to the orthogonality properties of the Legendre polynomials, these $V_{i,\lambda}(R,\rho)$ are given by:

$$V_{i,\lambda}(R,\rho) = \frac{2\lambda+1}{2} \int_{-1}^{+1} U_i\left(\sqrt{a_i^2 R^2 + b_i^2 \rho^2 + 2a_i b_i R\rho\gamma}\right) P_\lambda(\gamma)\, d\gamma\,.$$

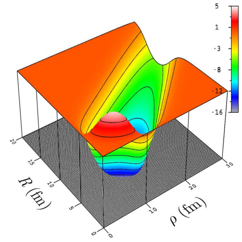
The formalism: the multipole expansion $V_{i,\lambda}(R,\rho)$.



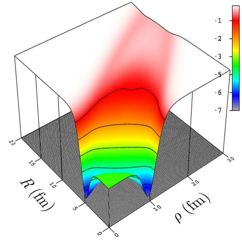
The real part of $V_{i,0}(R,\rho)$



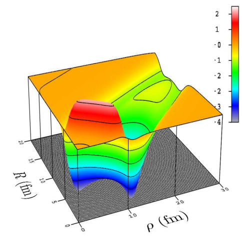
The real part of $V_{i,1}(R,\rho)$



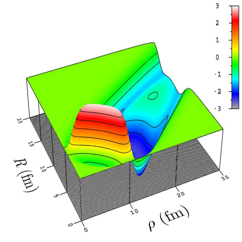
The real part of $V_{i,2}(R,\rho)$



The imaginary part of $V_{i,0}(R,\rho)$



The imaginary part of $V_{i,1}(R,\rho)$



The imaginary part of $V_{i,2}(R,\rho)$

The formalism: the Hamiltonian in the spherical case.

Multipole expansion of the potentials: $U_i(\vec{r}_i) \equiv U_i(r_i)$ ($i = n$ or p)

The multipole expansion of the optical potential is given by:

$$U_i(r_i) \;=\; \sum_{\lambda=0}^{\infty} V_{i,\lambda}(R,\rho) P_\lambda(\gamma)\,.$$

Using the addition theorem (Messiah p. 422)

$$\frac{2\lambda+1}{4\pi} P_\lambda(\gamma) \;=\; \sum_{\mu=-\lambda}^{\lambda} Y_\lambda^{\mu *}(\hat{R}) Y_\lambda^{\mu}(\hat{\rho})\,,$$

the optical potential reads:

$$U_i(r_i) \;=\; 4\pi \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \frac{V_{i,\lambda}(R,\rho)}{2\lambda+1} Y_\lambda^{\mu *}(\hat{R}) Y_\lambda^{\mu}(\hat{\rho})\,.$$

The formalism: the Hamiltonian in the spherical case.

Multipole expansion of the potentials: $U_i(\vec{r}_i) \equiv U_i(r_i)$ ($i = n$ or p)

$$U_i(r_i) = 4\pi \sum_{\lambda=0}^{\infty} \frac{V_{i,\lambda}(R,\rho)}{\sqrt{2\lambda+1}} (-1)^{\lambda} \left[Y_\lambda(\hat{R}) \otimes Y_\lambda(\hat{\rho}) \right]_0^{(0)}$$

$$U_i(r_i) = 4\pi \sum_{\lambda=0}^{\infty} \frac{V_{i,\lambda}(R,\rho)}{\sqrt{2\lambda+1}} (-1)^{\lambda} \left[Y_\lambda(\hat{R}) \otimes Y_\lambda(\hat{\rho}) \right]_0^{(0)}$$

$$U_i(r_i) = 4\pi \sum_{\lambda=0}^{\infty} \frac{V_{i,\lambda}(R,\rho)}{\sqrt{2\lambda+1}} (-1)^{\lambda} \left[Y_\lambda(\hat{R}) \otimes Y_\lambda(\hat{\rho}) \right]_0^{(0)}$$

The formalism: the wave function.

The wave function for the d+target system.

The wave function $\Psi^J_M(\vec{R},\vec{\rho})$ of the 3-body system reads as a superposition of states:

$$\Psi^J_M = \sum_{L=|J-1|}^{J+1} \left[\Phi_0(\vec{\rho}) \otimes \chi_0(L, J\,;\, P_0, \vec{R})\right]^J_M +$$

$$\sum_{l=0}^{\infty} \sum_{I=|l-S|}^{l+S} \sum_{L=|J-I|}^{J+I} \underbrace{\int_0^{\infty} \left[\Phi({}^{2S+1}l_I\,;\, k, \vec{\rho}) \otimes \chi({}^{2S+1}l_I, L, J\,;\, P_k, \vec{R})\right]^J_M dk}_{BU_{lSILJ}} .$$

1. The Φ_i are known wave functions of the p-n hamiltonian: $\hat{H}_{\mathsf{pn}}\Phi_0 = \varepsilon_0\Phi_0$ and $\hat{H}_{\mathsf{pn}}\Phi = \varepsilon_k\Phi$ with $\varepsilon_0 < 0$ and $\varepsilon_k > 0$.
2. The first term describes the motion of the deuteron.
3. The BU_{lSILJ} term describes the motion of a broken pair with relative momentum $\vec{k}$.

The formalism: the wave function.

Total energy conservation : relation between $E, P_0, P_k, \varepsilon_0, \varepsilon_k$.

$$E = \hbar^2 P_0^2/2\mu_R + \varepsilon_0 \text{ where } \varepsilon_0 \text{ is the deuteron ground state energy.}$$
$$E = \hbar^2 P_k^2/2\mu_R + \varepsilon_k \text{ with } \varepsilon_k = \hbar^2 k^2/2\mu_\rho \text{ and } \hat{H}_{pn}\Phi = \varepsilon_k \Phi \,.$$

How to handle states into the continuum?

Since the wave function $\Psi^J_M(R,\rho)$ of the 3-body system reads as:

$$\Psi^J_M = \sum_{L=|J-1|}^{J+1} \left[\Phi_0(\vec{\rho}) \otimes \chi_0(L, J\,;\, P_0, \vec{R})\right]^J_M +$$
$$\sum_{l=0}^{\infty} \sum_{I=|l-S|}^{l+S} \sum_{L=|J-I|}^{J+I} \underbrace{\int_0^\infty \left[\Phi(^{2S+1}l_I\,;\, k, \vec{\rho}) \otimes \chi(^{2S+1}l_I, L, J\,;\, P_k, \vec{R})\right]^J_M dk}_{BU_{lSILJ}}$$

The continuum discretization.

We discretize the continuum into bins and we assume that, for each bin $[k_i, k_{i+1}]$:

$$\chi(^{2S+1}l_I, L, J\,;\,P_k, \vec{R}) \sim \chi(^{2S+1}l_I, L, J\,;\,\hat{P}_i, \vec{R})\,, \forall k \in [k_i, k_{i+1}]\,.$$

Discretization & Truncation: $i \leq N$ et $l \leq l_{\text{max}}$.

$$BU_{lSILJ} \sim \sum_{i=1}^{N}\left[\tilde{\Phi}_i(^{2S+1}l_I\,;\,\vec{\rho}) \otimes \tilde{\chi}_i(^{2S+1}l_I, L, J\,;\,\vec{R})\right]_M^{(J)}\,.$$

The CDCC wave function.

$$\Psi_M^J \sim \sum_{L=|J-1|}^{J+1}\left[\Phi_0(\vec{\rho}) \otimes \chi_0(L, J\,;\,P_0, \vec{R})\right]_M^J +$$

$$\sum_{l=0}^{l_{\text{max}}}\sum_{I=|l-S|}^{l+S}\sum_{L=|J-I|}^{J+I}\sum_{i=1}^{N}\left[\tilde{\Phi}_i(^{2S+1}l_I\,;\,\vec{\rho}) \otimes \tilde{\chi}_i(^{2S+1}l_I, L, J\,;\,\vec{R})\right]_M^J\,.$$

The formalism: the CDCC wave function.

The wave function: how to derive the $\tilde{\Phi}_j({}^{2S+1}l_I\,;\,\vec{\rho})$?

Each $\tilde{\Phi}_j({}^{2S+1}l_I\,;\,\vec{\rho})$ of the wave function can be written as:

$$\tilde{\Phi}_j({}^{2S+1}l_I\,;\,\vec{\rho}) = \left[i^l \frac{\tilde{\phi}_{jl}(\rho)}{\rho} Y_l(\hat{\rho}) \otimes \eta_S \right]^I .$$

To derive the $\tilde{\phi}_{jl}(\rho)$, ie the radial parts of $\tilde{\Phi}_j$, we first solve the following equation:

$$-\frac{\hbar^2}{2\mu_\rho}\frac{d^2\phi_{kl}}{d\rho^2} + \frac{\hbar^2 l(l+1)}{2\mu_\rho \rho^2}\phi_{kl} + V_{pn}\phi_{kl} = \varepsilon_k \phi_{kl}$$

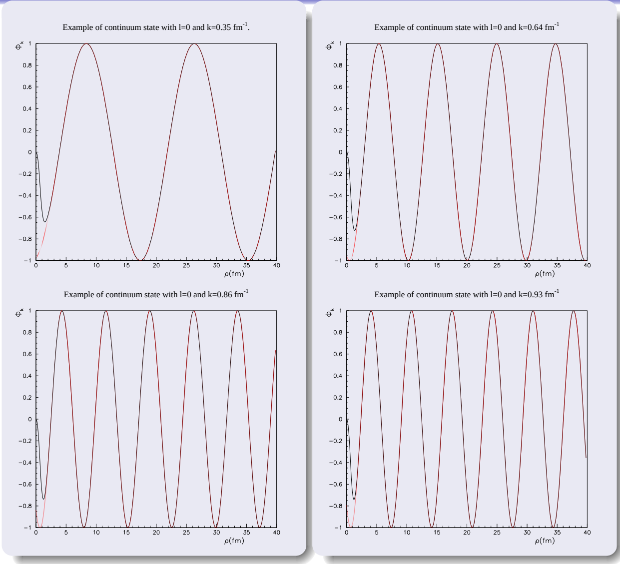
with $\varepsilon_k = \hbar^2 k^2/2\mu_\rho$ and with the asymptotic form:

$$\phi_k \sim \sin(k\rho - l\pi/2 - \delta(l,k)) .$$

The formalism: the CDCC wave function.

Derivation of the $\tilde{\Phi}_j(^{2S+1}l_I\ ;\ \vec{\rho})$: calculation of states into the continuum.

Four examples of continuum wave functions (black lines) and their asymptotic forms (red dashed lines) are plotted for $k = 0.35\ \text{fm}^{-1}$, $k = 0.64\ \text{fm}^{-1}$, $k = 0.86\ \text{fm}^{-1}$ and $k = 0.93\ \text{fm}^{-1}$.



The formalism: the CDCC wave function.

Derivation of the $\tilde{\Phi}_j({}^{2S+1}l_I\,;\,\vec{\rho})$: discretization by integrating over k (the average method).

Once we get the continuum states, the discretization can be performed by integrating over k within each bin $[k_i, k_{i+1}]$.
If it is assumed that the phase shifts $\delta(l,k)$ remain constant and equal to δ then an approximation of the discretized states is given by:

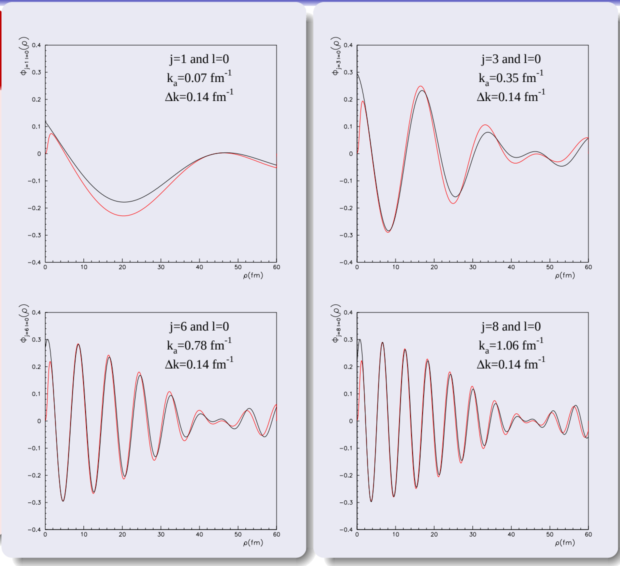
$$\begin{aligned}\int_{k_i}^{k_f} \sin(k\rho+\delta)dk &= -\frac{1}{\rho}\left[cos(k\rho+\delta)\right]_{k_i}^{k_f} \\ &= \frac{2}{\rho}\sin(\Delta k\rho)\sin(k_a\rho+\delta)\end{aligned}$$

where $\Delta k = (k_f - k_i)/2$ and $k_a = (k_f + k_i)/2$. Thus the discretized states should behave as $1/\rho\sin(\Delta k\rho)\sin(k_a\rho+\delta)$.

The formalism: the CDCC wave function.

Example of the $\tilde{\Phi}_j(^{2S+1}l_I\ ;\ \vec{\rho})$.

Four examples of discretized wave functions (red lines) and their asymptotic forms (black lines) are plotted assuming that $k_{\text{max}} = 1.4\ \text{fm}^{-1}$ and using 10 bins to discretize the continuum. The asymptotic form is defined by $1/\rho \sin(\Delta k \rho) \sin(k_a \rho + \delta)$.



The formalism: the CDCC wave function.

Other mehods to discretize the continuum.

Some other methods have been proposed to discretize the continuum:

- The mid-point method in which the continuum states are the scattering states for given values of scattering energies.
- Some authors [9, 11, 12] have also developed approaches based upon pseudo-states (PS): the projectile wave function are eigenstate of the Hamiltonian in a truncated basis of square-integrable functions.

Some studies have been performed to compare these different methods of discretizations [10, 13] and the pseudo-state method has been improved by introducing some transformed harmonic oscillator basis in order to overcome some issue streming from the gaussian asymptotic decay of the HO basis.

The formalism: the CDCC equations.

Derivation of the CDCC equations.

To obtain the CDCC equations, just follow the recipes:

- Introduce the CDCC wave function into the Schrödinger equation.
- Left-multiply the Sch. equation by $\left[\tilde{\Phi}_i({}^{2S+1}l_I, L, J\,; \vec{\rho}) \otimes Y_L(\hat{R})\right]_M^{(J)}$.
- Integrate over $\vec{\rho}$ and the angular variables $\hat{R}$.
- Solve the following set of coupled differential equations for the radial parts $u_c^J(R)$ of the wave functions :

$$\left(-\frac{\hbar^2}{2\mu_R}\frac{d^2}{dR^2} + \frac{\hbar^2 L(L+1)}{2\mu_R R^2} + V_p^{(Coul)} - E_i\right) u_c^J(R) = -\sum_{c'} F_{cc'}^J(R) u_{c'}^J(R)$$

$$\text{with } u_c^J(R) \;\rightarrow\; \delta_{c_0 c}\, U^{(-)}(\tilde{P}_i R) - \sqrt{\tilde{P}_i/\tilde{P}_0}\, \hat{S}_{c_0 c}^{(J)}\, U^{(+)}(\tilde{P}_i R)\,,$$

where $\hat{S}_{c_0 c}^{(J)}$ are the CDCC S-matrix elements, c_0 denotes the elastic channel and $U^{(+)}$ and $U^{(-)}$ are the outgoing and incoming Coulomb wave functions.

The formalism: the CDCC equations.

Definition of the form factors.

The form factors are thus defined by

$$F^J_{cc'}(R) = \langle \left[\tilde{\Phi}_i \otimes Y_L(\hat{R})\right]^{(J)}_M |U_{pA} + U_{nA}| \left[\tilde{\Phi}_{i'} \otimes Y_{L'}(\hat{R})\right]^{(J)}_M \rangle_{\hat{R},\hat{\rho},\rho} .$$

The brackets $\langle\,\rangle_{\hat{R},\hat{\rho},\rho}$ denote the integration over $\hat{R}$ and $\vec{\rho}$.

Derivation of the form factors by using the multipole expansion.

The easiest way to compute these form factors is to use the previous multipole expansion of the potentials:

$$U_i(r_i) = 4\pi \sum_{\lambda=0}^{\infty} \frac{V_{i,\lambda}(R,\rho)}{\sqrt{2\lambda+1}} (-1)^\lambda \left[Y_\lambda(\hat{R}) \otimes Y_\lambda(\hat{\rho})\right]^0_0 ,$$

where i denotes proton or neutron.

The formalism: the CDCC equations.

Derivation of the form factors.

The integration over the angular variables can be performed by using the Wigner-Eckart theorem :

$$\frac{4\pi}{\hat{\lambda}}(-1)^{\lambda}\langle\left[\tilde{\Phi}_i\otimes Y_L(\hat{R})\right]^{(J)}_M\left|\left[Y_\lambda\left(\hat{\rho}\right)\otimes Y_\lambda\left(\hat{R}\right)\right]^{(0)}_0\right|\left[\tilde{\Phi}_{i'}\otimes Y_{L'}(\hat{R})\right]^{(J)}_M\rangle_{\hat{R},\hat{\rho}}$$
$$=\frac{4\pi}{\hat{\lambda}\hat{J}}(-1)^{\lambda}\langle\left[\tilde{\Phi}_i\otimes Y_L(\hat{R})\right]^{(J)}\left\|\left[Y_\lambda\left(\hat{\rho}\right)\otimes Y_\lambda\left(\hat{R}\right)\right]^{(0)}_0\right\|\left[\tilde{\Phi}_{i'}\otimes Y_{L'}(\hat{R})\right]^{(J)}\rangle_{\hat{R},\hat{\rho}}$$

This reduced matrix element is transformed as follows

$$\langle\left[\tilde{\Phi}_i\otimes Y_L(\hat{R})\right]^{(J)}\left\|\left[Y_\lambda\left(\hat{\rho}\right)\otimes Y_\lambda\left(\hat{R}\right)\right]^{(0)}_0\right\|\left[\tilde{\Phi}_{i'}\otimes Y_{L'}(\hat{R})\right]^{(J)}\rangle_{\hat{R},\hat{\rho}}$$
$$=\quad \hat{J}^2\left\{\begin{array}{ccc} I' & L' & J \\ \lambda & \lambda & 0 \\ I & L & J \end{array}\right\}\underbrace{\langle Y_L(\hat{R})\left\|Y_\lambda(\hat{R})\right\|Y_{L'}(\hat{R})\rangle}_{=A}\underbrace{\langle\tilde{\Phi}_i\left\|Y_\lambda(\hat{\rho})\right\|\tilde{\Phi}_{i'}\rangle}_{=B}.$$

The formalism: the CDCC equations.

Derivation of the form factors: $9j$ coefficient.

Firstly the $9j$ coefficient can be simplified:

$$\left\{\begin{array}{ccc} I' & L' & J \\ \lambda & \lambda & 0 \\ I & L & J \end{array}\right\} = (-1)^R \left\{\begin{array}{ccc} I' & L' & J \\ I & L & J \\ \lambda & \lambda & 0 \end{array}\right\} = \left\{\begin{array}{ccc} I & L & J \\ I' & L' & J \\ \lambda & \lambda & 0 \end{array}\right\}$$

$$= \left\{\begin{array}{ccc} I & I' & \lambda \\ L & L' & \lambda \\ J & J & 0 \end{array}\right\} = \frac{(-1)^{J+\lambda+L+I'}}{\hat{J}\,\hat{\lambda}} \left\{\begin{array}{ccc} I & I' & \lambda \\ L' & L & J \end{array}\right\}$$

$$= \frac{(-1)^{J+\lambda+L+I'}}{\hat{J}\,\hat{\lambda}} \left\{\begin{array}{ccc} L' & L & \lambda \\ I & I' & J \end{array}\right\} = \frac{(-1)^{J+\lambda+L+I'}}{\hat{J}\,\hat{\lambda}} \left\{\begin{array}{ccc} L & L' & \lambda \\ I' & I & J \end{array}\right\}$$

$$= \frac{(-1)^{J+\lambda+L+I'}}{\hat{J}\,\hat{\lambda}} (-1)^{I+I'+L+L'} W\left(LL'II';\lambda J\right) .$$

The formalism: the CDCC equations.

Derivation of the form factors: $A = \langle Y_L(\hat{R}) \left\| Y_\lambda(\hat{R}) \right\| Y_{L'}(\hat{R}) \rangle$.

The calculation of A is straightforward:

$$A = \langle Y_L(\hat{R}) \left\| Y_\lambda(\hat{R}) \right\| Y_{L'}(\hat{R}) \rangle = \frac{\hat{L}'\hat{L}}{\sqrt{4\pi}}(-1)^{L-\lambda}\langle L0L'|\lambda 0\rangle \,.$$

Derivation of the form factors: $B = \langle \tilde{\Phi}_i \left\| Y_\lambda(\hat{\rho}) \right\| \tilde{\Phi}_{i'} \rangle$.

$\hat{B}$, the angular part of B, can also be transformed:

$$\begin{aligned}
\hat{B} &= (-1)^{I'+l+S+\lambda}\delta_{SS'}\langle Y_l \left\| Y_\lambda \right\| Y_{l'}\rangle \hat{I}'\hat{I} \begin{Bmatrix} l & \lambda & l' \\ I' & S & I \end{Bmatrix} \\
&= (-1)^{I'+l+S+\lambda}\delta_{SS'}\langle Y_l \left\| Y_\lambda \right\| Y_{l'}\rangle \hat{I}'\hat{I} \begin{Bmatrix} l & l' & \lambda \\ I' & I & S \end{Bmatrix} \\
&= (-1)^{I'+l+S+\lambda}\delta_{SS'}\langle Y_l \left\| Y_\lambda \right\| Y_{l'}\rangle \hat{I}'\hat{I}\,(-1)^{l+l'+I+I'} W\left(l\,l'\,I\,I';\,\lambda\,S\right) \\
&= (-1)^{I'+S}\delta_{SS'}\frac{ll'}{\sqrt{4\pi}}\langle l0l'0|\lambda 0\rangle \hat{I}'\hat{I}\,(-1)^{l+l'+I+I'} W\left(l\,l'\,I\,I';\,\lambda\,S\right) \,.
\end{aligned}$$

The formalism: the CDCC equations.

Expression of the form factors.

Therefore the form factors can be written as:

$$F^J_{cc'}(R) = \sum_\lambda Z(c, c'; \lambda, J) f(c, c'; \lambda)$$

where

$$\begin{aligned} Z(c, c'; \lambda, J) &= i^{L-L'l-l'}(-1)^{J-S+\lambda}\delta_{SS'}\frac{\hat{I}\hat{I}'\hat{L}\hat{L}'\hat{l}\hat{l}'}{\hat{\lambda}^2}\langle l0l'0|\lambda 0\rangle\langle L0L'0|\lambda 0\rangle \\ & W\left(LL'II'; \lambda J\right) W\left(l\,l'\,I\,I';\,\lambda\,S\right) \end{aligned}$$

and

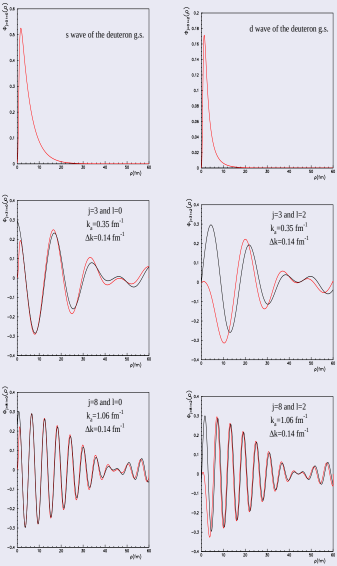
$$f(c, c'; \lambda) = \int_0^\infty d\rho\, \tilde{\varphi}_c(\rho) \left(U^{(\lambda)}_{nA}(\rho, R) + U^{(\lambda)}_{pA}(\rho, R)\right) \tilde{\varphi}'_c(\rho)\,.$$

This expansion is very usefull since one has only had to compute a one-dimensional integral which is independent of J.

Summary: the main CDCC ingredients

s waves

d waves



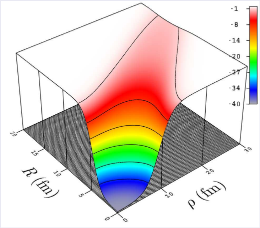
Folding

Coupl. chan. eq.

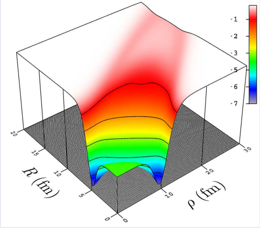
S-matrix

Cross Sections

Nucleon-nucleus optical potentials



Real part of $V_0(R, \rho)$.



Imaginary part of $V_0(R, \rho)$.

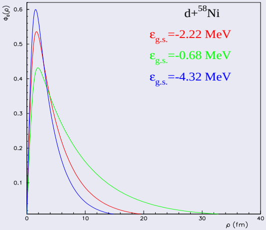
Ground state and binding energy.

We investigate the effect of the shape of the ground state wave functions and of the binding energy on the cross section calculations. These calculations are performed assuming that:

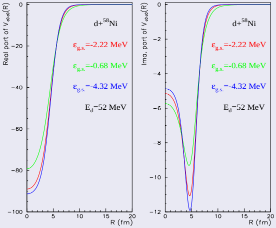
- The target is the ^{58}Ni.
- The nucleon-target optical potentials are those proposed by A. Koning and J.-P. Delaroche.
- The deuteron ground state is calculated with the V_{pn} interaction with a Gaussian shape for 3 sets of parameters: three different wave functions have been obtained with
  - $\varepsilon_{\mathrm{g.s.}} = -0.68$ MeV,
  - $\varepsilon_{\mathrm{g.s.}} = -2.22$ MeV
  - and $\varepsilon_{\mathrm{g.s.}} = -4.32$ MeV.
- The incident energy ranges between 5 MeV and 80 MeV.

Calculations with different g.s. wave functions.

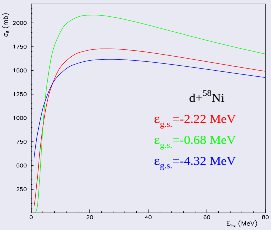
The radial part of the g.s. w.f.



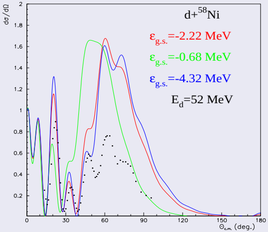
Form factor obtained by folding.



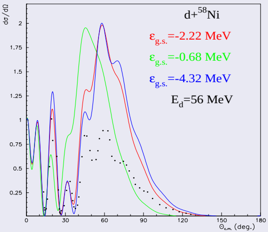
Reaction cross sections.



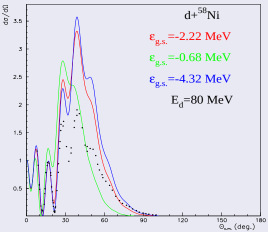
Differential cross section.



Differential cross section.



Differential cross section.



Effect of the projectile wave function.

From these figures, we can draw the following conclusions:

1. The depth and the width of folding potentials are modified.
2. The threshold and the amplitude of the reaction cross sections depend strongly on this w.f.
3. The oscillary patterns of the differential cross sections also depend on the projectile w.f.

An accurate measurement of the cross sections can thus provide a precise insight about the projectile features.

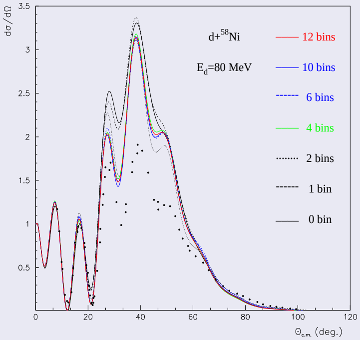
Continuum effect.

We want to check that the CDCC approach converges while increasing the number of states (i.e. the number of bins) used to discretize the continuum. We also wish to compare the calculated cross section with the experimental one. The following calculations are performed assuming that:

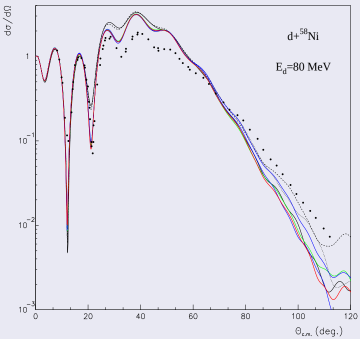
- The target is the ^{58}Ni.
- The nucleon-target optical potentials are those proposed by A. Koning and J.-P. Delaroche.
- The s and d waves of the deuteron g.s. and p-n continuum states are obtained by using the Reid93 potential.
- The deuteron is incident at 80 MeV on the target.
- The number of bins increases from 0 to 12.
- k_{max}=1.5 fm^{-1}.

Convergence of the CDCC calculations with the bin number.

Differential cross section.



Differential cross section (log scale).



Continuum effect.

We can conclude that:

1. The calculations converge while increasing the bin number.
2. For this incident energy, the elastic cross section calculation has converged by using 4 bins to describe the continuum .
3. A better agreement with the experimental data is obtained by including the continuum states.

As expected for weakly bound projectile, the coupling to continuum states plays a crucial role onto the elastic cross sections and it clearly improves the agreement with the experimental data even though there are still some discrepancies between the calculated cross section and the experimental one meaning that other channels (inelastic, transfer ?) should be included.

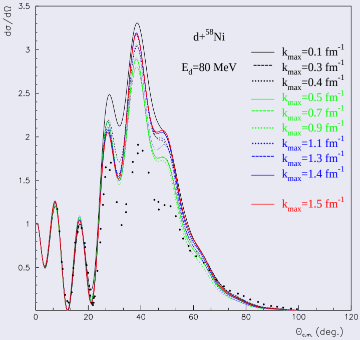
$k_{\max}$ effect.

We have checked that the CDCC approach converges while increasing the number of states (i.e. the number of bins) used to discretize the continuum. We also compare the calculated cross section with the experimental one. These calculations are performed assuming that:

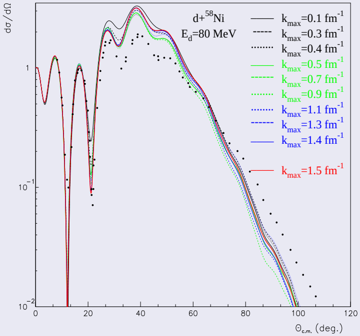
- The target is the ^{58}Ni.
- The nucleon-target optical potentials are those proposed by A. Koning and J.-P. Delaroche.
- The s and d waves of the deuteron g.s. and p-n continuum states are obtained by using the Reid93 potential.
- The deuteron is incident at 80 MeV on the target.
- The number of bins is set to 4.
- $k_{\max}$ belongs to $[0.1, 1.5]$ fm^{-1} .

Convergence of the CDCC calculations with k_{max}.

Differential cross section.



Differential cross section (log scale).

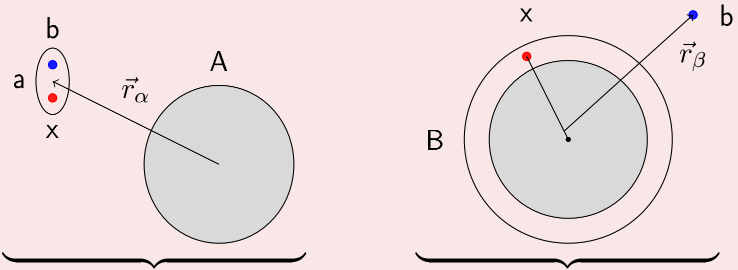


These calculations show that:

1. The calculated cross sections depend on the shape of the projectile wave function.
2. The CDCC cross sections converge while increasing the bin number.
3. The coupling between the elastic channel and the breakup ones has to be included to improve the agreement between calculations and experimental data.
4. It seems important to include all the open channels to describe the continuum.
5. It seems also that it is necessary to go beyond CDCC calculations and to include other reaction channels :
    - the inelastic channels to take into account the target excitations (CDCC*) ?
    - the channels describing the projectile excitations (XCDCC) ?
    - the transfer channels ?

Transfer reactions.

The partitions for the reaction $a(=b+x)+A \rightarrow b+B(=x+A)$.



α partition

$\alpha = a(=b+x)+A$

β partition

$\beta = b+B(=x+A)$

The model wave funtion reads:

$$\Psi_{\text{model}} = u_\alpha(\vec{r}_\alpha)\psi_\alpha(\xi_\alpha) + u_\beta(\vec{r}_\beta)\psi_\beta(\xi_\beta)$$

with $\xi_\alpha = \vec{xb}$ and spin variables and $\xi_\beta = \vec{xA}$ and spin variables.

Transfer reactions.

The system is thus described by:

$$\Psi_{\text{model}} = u_\alpha(\vec{r}_\alpha)\psi_\alpha(\xi_\alpha) + u_\beta(\vec{r}_\beta)\psi_\beta(\xi_\beta)$$

where α and β denote two partitions of the system : $\alpha = A + a$, $\beta = B + b$ and the $u_\alpha(\vec{r}_\alpha)$, $u_\beta(\vec{r}_\beta)$ are unknown functions. For each partition, one can define a basis:

$$\Psi_\alpha = \delta_\alpha(\vec{r} - \vec{r}_\alpha)\psi_\alpha(\xi_\alpha) \text{ and } \Psi_\beta = \delta_\beta(\vec{r} - \vec{r}_\beta)\psi_\beta(\xi_\beta)\,.$$

The Schrödinger Equation reads:

$$\hat{H}\,\Psi_{\text{model}} = E\,\Psi_{\text{model}}\,.$$

Thus one can get:

$$\langle\Psi_\alpha|\left(E - \hat{H}\right)|\Psi_{\text{model}}\rangle = 0 \text{ and } \langle\Psi_\beta|\left(E - \hat{H}\right)|\Psi_{\text{model}}\rangle = 0$$

with the two equivalent forms of H: $\hat{H} = H_\alpha + K_\alpha + V_\alpha = H_\beta + K_\beta + V_\beta$.

Transfer reactions.

Derivation of the equations.

Thus we get:

$$\begin{cases} \langle \Psi_\alpha | \left(E - \hat{H} \right) | \Psi_{\text{model}} \rangle = 0 \\ \langle \Psi_\beta | \left(E - \hat{H} \right) | \Psi_{\text{model}} \rangle = 0 \end{cases}$$

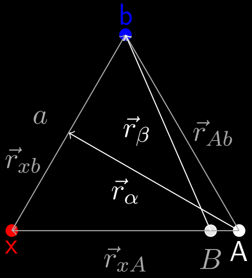
$$\begin{cases} \langle \Psi_\alpha | \left(E - \hat{H} \right) | u_\alpha(\vec{r}_\alpha) \psi_\alpha \rangle + \langle \Psi_\alpha | \left(E - \hat{H} \right) | u_\beta(\vec{r}_\beta) \psi_\beta \rangle = 0 \\ \langle \Psi_\beta | \left(E - \hat{H} \right) | u_\alpha(\vec{r}_\alpha) \psi_\alpha \rangle + \langle \Psi_\beta | \left(E - \hat{H} \right) | u_\beta(\vec{r}_\beta) \psi_\beta \rangle = 0 \end{cases}$$

$$\begin{cases} \langle \Psi_\alpha | \left(E - H_\alpha - K_\alpha - V_\alpha \right) u_\alpha(\vec{r}_\alpha) \psi_\alpha \rangle + \langle \Psi_\alpha | \left(E - \hat{H} \right) | u_\beta(\vec{r}_\beta) \psi_\beta \rangle = 0 \\ \langle \Psi_\beta | \left(E - \hat{H} \right) | u_\alpha(\vec{r}_\alpha) \psi_\alpha \rangle + \langle \Psi_\beta | \left(E - H_\beta - K_\beta - V_\beta \right) u_\beta(\vec{r}_\beta) \psi_\beta \rangle = 0 \end{cases}$$

$$\begin{cases} \left[\left(E - \varepsilon_\alpha \right) - K_\alpha - \langle \psi_\alpha | V_\alpha | \psi_\alpha \rangle \right] u_\alpha(\vec{r}_\alpha) = \langle \psi_\alpha | \left(E - \hat{H} \right) | u_\beta(\vec{r}_\beta) \psi_\beta \rangle \\ \left[\left(E - \varepsilon_\beta \right) - K_\beta - \langle \psi_\beta | V_\beta | \psi_\beta \rangle \right] u_\beta(\vec{r}_\beta) = \langle \psi_\beta | \left(E - \hat{H} \right) | u_\alpha(\vec{r}_\alpha) \psi_\alpha \rangle \end{cases}$$

Transfer reactions.

Example of coordinate transformation.



a center of mass of $\{(x, m_x); (b, m_b)\}$:

$$\vec{xa} = \frac{m_b}{m_b + m_x}\vec{xb}\,.$$

B center of mass of $\{(x, m_x); (A, m_A)\}$:

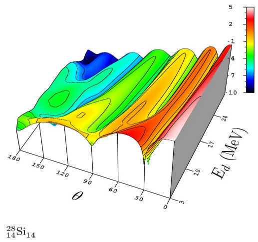
$$\vec{xB} = \frac{m_A}{m_A + m_x}\vec{xA}\,.$$

$$\begin{aligned}
\vec{xA} &= \vec{xa} + \vec{aA} = \frac{m_b}{m_b + m_x}\vec{xb} + \vec{aA} = \frac{m_b\left(\vec{xB} + \vec{Bb}\right)}{m_b + m_x} + \vec{aA} \\
&= \frac{m_b}{m_b + m_x}\left(\frac{m_A}{m_A + m_x}\vec{xA} + \vec{Bb}\right) + \vec{aA}\,. \\
\vec{xA} &= \frac{(m_b + m_x)(m_A + m_x)}{m_x(m_b + m_x + m_A)}\left(\frac{m_b}{m_b + m_x}\vec{r}_\beta - \vec{r}_\alpha\right)\,.
\end{aligned}$$

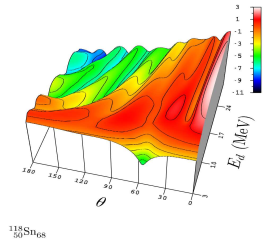
Evolution of the transfer cross sections with the E_d and Z.

^{A}X(d,p)$^{A+1}$Y differential cross sections for $3 \leq E_d \leq 30$ (MeV).

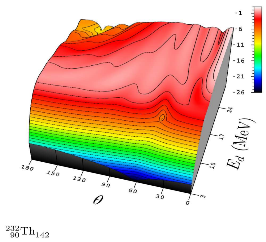
$^{28}_{14}$Si(d,p)$^{29}_{14}$Si cross sections.



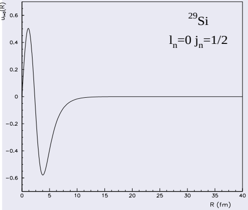
$^{118}_{50}$Sn(d,p)$^{119}_{50}$Sn cross sections.



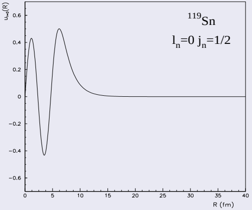
$^{232}_{90}$Th(d,p)$^{233}_{90}$Th cross sections.



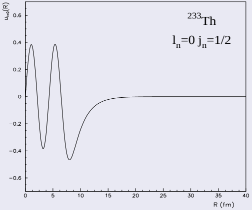
Neutron w. f.: $2s1/2$.



Neutron w. f.: $3s1/2$.



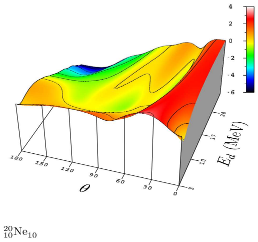
Neutron w. f.: $4s1/2$.



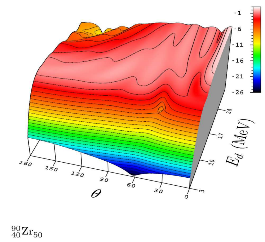
Evolution of the transfer cross sections with the E_d and Z.

^{A}X(d,p)$^{A+1}$Y differential cross sections for $3 \leq E_d \leq 30$ (MeV).

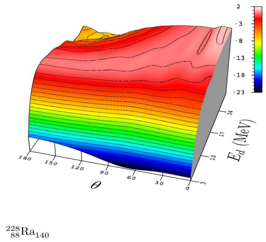
$^{20}_{10}$Ne(d,p)$^{21}_{10}$Ne cross sections.



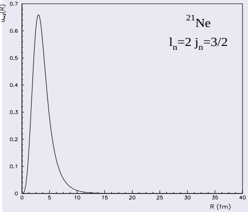
$^{90}_{50}$Zr(d,p)$^{91}_{50}$Zr cross sections.



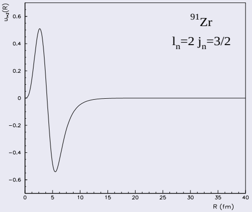
$^{228}_{88}$Ra(d,p)$^{229}_{88}$Ra cross sections.



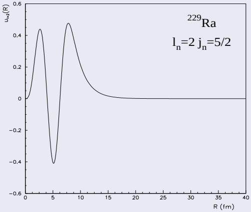
Neutron w. f.: $1d3/2$.



Neutron w. f.: $2d5/2$.



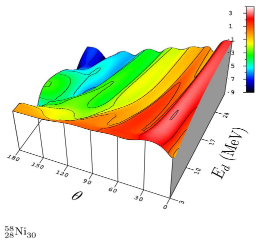
Neutron w. f.: $3d5/2$.



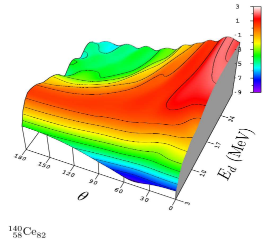
Evolution of the transfer cross sections with the E_d and Z.

^{A}X(d,p)${}^{A+1}$Y differential cross sections for $3 \leq E_d \leq 30$ (MeV).

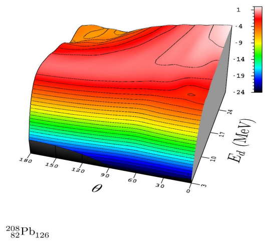
${}^{58}_{28}$Ni(d,p)${}^{59}_{28}$Ni cross sections.



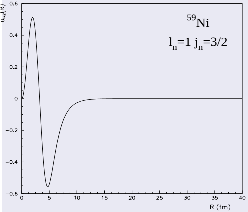
${}^{140}_{58}$Ce(d,p)${}^{141}_{58}$Ce cross sections.



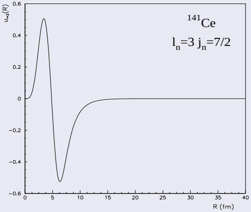
${}^{208}_{82}$Pb(d,p)${}^{209}_{82}$Pb cross sections.



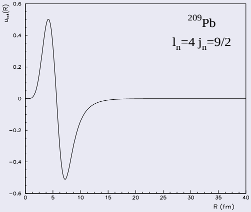
Neutron w. f.: $2p3/2$.



Neutron w. f.: $2f7/2$.



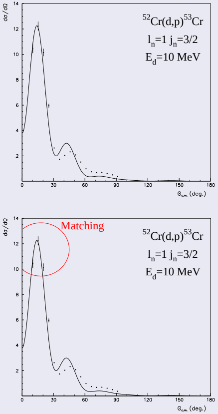
Neutron w. f.: $2g9/2$.



Effect of the orbital angular momentum of the neutron w.f..

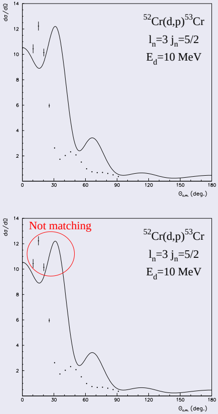
Determination of the l of the g.s. from the ^{52}Cr(d,p)^{53}Cr cross section.

Differential cross section.



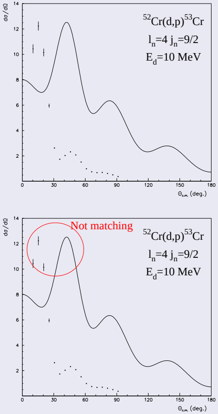
Neutron wave function : $l = 1$

Differential cross section.



Neutron wave function : $l = 3$

Differential cross section.

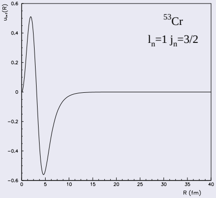


Neutron wave function : $l = 4$

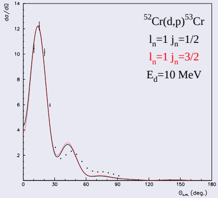
Effect of the spin of the neutron w.f. on the cross section.

^{52}Cr(d,p)^{53}Cr cross section and vector analyzing power.

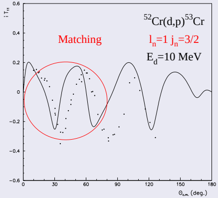
Neutron wave function:
$l = 1$ and $j = 3/2$.



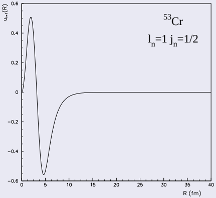
Differential cross section:
$l = 1$ and $j = 3/2$.



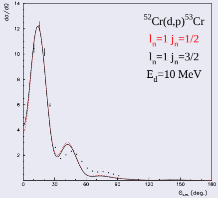
Vector analyzing power:
$l = 1$ and $j = 3/2$.



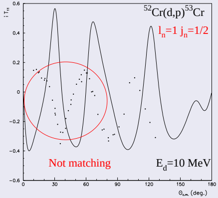
Neutron wave function:
$l = 1$ and $j = 1/2$.



Differential cross section:
$l = 1$ and $j = 1/2$.



Vector analyzing power:
$l = 1$ and $j = 1/2$.



Core excitations: XCDCC.

Introduction.

An extension to the CDCC approach has been proposed by N.C Summers, F. Nunes and I. Thompson to include the core excitations [1-2].

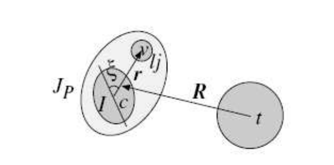


Figure 1: Coordinates for the three-body problem.

$$H = T_R + H_{proj} + V_{ct} + V_{vt} \,.$$

$$H_{proj} = T_r + V_{vc}(r,\xi) + h_{core}(\xi) \,.$$

The wave function of the system reads:

$$\Phi^{M_T}_{J_T}(R,r,\xi) = \sum_{\alpha} \Phi^{J_T}_{\alpha}(R) \left[\left[Y_L(\hat{R}) \otimes \Phi^{in}_{J_P}(r,\xi) \right] \otimes \Phi_{J_t}(\xi_t) \right]^{M_T}_{J_T} \,.$$

[1] N.C. Summers *et al.*, Phys. Rev. C73, 0631603(R) (2006).
[2] N.C. Summers *et al.*, Phys. Rev. C74, 014606 (2006).

Core excitations: XCDCC.

The XCDCC projectile wave function.

The projectile ground state includes contributions of several core states which are coupled:

$$\Phi_0(r,\xi) = \sum_{\alpha} \Phi_\alpha(r) \left[[Y_l(\hat{r}) \otimes \chi_s] \otimes \phi_I(\xi)\right]_{J_P}$$

By denoting

$$\langle \hat{R}, r, \xi | \alpha; J_T \rangle = \left[\left[Y_L(\hat{R}) \otimes \Phi^{in}_{J_P}(r,\xi)\right] \otimes \Phi_{J_t}(\xi_t)\right]_{J_T} ,$$

the form factors to be computed are given by

$$U^{J_T}_{\alpha\alpha'}(R) = \langle \alpha; J_T | V_{ct}(R,r,\xi) + V_{vt}(R,r,\xi) | \alpha'; J_T \rangle \,.$$

Core excitations: XCDCC.

The XCDCC form factors.

The form factors have been derived by N.C Summers *et al.* by using a multipole expansion:

$$U^{J_T}_{\alpha\alpha'}(R) = \hat{L}\hat{L}'\hat{J}_P\hat{J}'_P(-1)^{J_P+J}\sum_{\Lambda}(-1)^{\Lambda}\hat{\Lambda}^2\begin{pmatrix}\Lambda & L & L' \\ 0 & 0 & 0\end{pmatrix}$$
$$\begin{pmatrix}J_P & J'_P & \Lambda \\ L' & L & J\end{pmatrix}F^{\Lambda}_{J_P in:J'_P i'n'}(R)$$

where

$$F^{\Lambda}_{J_P in:J'_P i'n'}(R) = \sum_{KQ\lambda a:a'} R^{KQ\lambda}_{ain:a'i'n'}(R)P^{KQ\lambda:\Lambda}_{a:a'}.$$

Core excitations: XCDCC.

The radial part and the angular part.

The radial part and angular one are respectively given by

$$R^{KQ\lambda}_{ain:a'i'n'}(R) = \hat{K}\int_0^{R_m} u^{i}_{a:n}{}^{*}(r)\, V^{QK}_{ct}(r,R)\, R^{\lambda}(-\gamma r)^{Q-\lambda}\, u^{i'}_{a':n'}(r)\, dr\,,$$

$$\begin{aligned} P^{KQ\lambda:\Lambda}_{a:a'} = {} & (-1)^{j'+l+l'+s+Q}\hat{Q}\hat{K}'\hat{j}\hat{j}'\hat{l}\hat{l}'\sqrt{\frac{(2Q)!}{(2\lambda)![2(Q-\lambda)]!}}\langle I \parallel C_Q(\xi) \parallel I'\rangle \\ & \begin{pmatrix} K & \lambda & \Lambda \\ 0 & 0 & 0 \end{pmatrix} \sum_{\Lambda'} \hat{\Lambda}'^2 \begin{pmatrix} K & Q-\lambda & \Lambda' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Lambda' & l & l' \\ 0 & 0 & 0 \end{pmatrix} \\ & \begin{Bmatrix} \Lambda' & \Lambda & Q \\ \lambda & Q-\lambda & K \end{Bmatrix} \begin{Bmatrix} j & j' & \Lambda' \\ l' & l & s \end{Bmatrix} \begin{Bmatrix} J_P & J'_P & \Lambda \\ j & j' & \Lambda' \\ I & I & Q \end{Bmatrix}. \end{aligned}$$

^{9}Be(^{11}Be,^{10}Be+n) 60 MeV/nucl.

The model space used by N.C. Summers *et al.*

The ^{11}Be(^{10}Be+n) ground state is a $J_P = 0^+$ with two components: a neutron s wave coupled to a 0^+ core (^{10}Be) state and a neutron d wave coupled to a 2^+ core state.
They have compared the calculations including the core excitations with those obtained without these excitation (Single-Particle Incoherent Sum). The comparaison is summarized in the table below:

Model	σ_{0^+} (mb)	σ_{2^+} (mb)	σ (mb)
SPIS	109	1	110
XCDCC	109	8	115

They conclude that the σ_{2^+} is strongly underestimated by the SPIS model.

Target excitations: CDCC*.

The 3-body wave function.

$$|\Psi_{J_T M_T}(\vec{R},\vec{\rho})\rangle = \sum_{i\,l\,S\,I_p\,L\,J\,I_t} |(i\,l\,S)\,I_p\,L\,J I_t\,;J_T\,M_T\rangle$$

where $\vec{R}$ denotes the deuteron center of mass coordinates and $\vec{\rho}$ the proton-neutron relative coordinates. The channels of the system for a given J_T are characterized by the following quantum numbers:

- The bin number i to discretize the continuum;
- The deuteron spin $S = 1$;
- The relative orbital angular momentum l associated to $\vec{\rho}$;
- The angular momentum J;
- The orbital angular momentum L associated to $\vec{R}$;
- The spin of the target I_t.

with $\vec{S} + \vec{l} = \vec{I_p}$, $\vec{L} + \vec{I_p} = \vec{J}$ and $\vec{I_t} + \vec{J} = \vec{J_T}$.

Target excitations: CDCC*.

The new Schrödinger equation.

The previous wave function satisfies the following Schrödinger equation:

$$\hat{H}|\Psi_{J_T M_T}(\vec{R},\vec{\rho})\rangle = E|\Psi_{J_T M_T}(\vec{R},\vec{\rho})\rangle$$

where

$$\hat{H} = \hat{T}_R + V_{pA}(\vec{r}_p,\xi_t) + V_{nA}(\vec{r}_n,\,\xi_t) + V_{Coul} + \hat{H}_{pn} + \hat{H}_A$$

with

$$\hat{H}_{pn}\,\varphi_{il}(\vec{\rho}) = \varepsilon_i\,\varphi_{il}(\vec{\rho}),\ \hat{H}_A\,\psi_{I_t}(\xi_t) = \hat{\varepsilon}_{I_t}\,\psi_{I_t}(\xi_t)$$

and

$$V(\vec{r}_i,\,\xi_t) = V(r_i,\theta_i,\phi_i,\,\xi_t) = \sum_\lambda v^{(\lambda)}(r_i)P_\lambda(\cos(\theta_i'))\,.$$

Due to the deformation, for each nucleon, the optical potential depends on $\vec{r}_i$ ($i = n$ or p).

Target excitations: CDCC*.

The starting point: T. Tamura's work [3].

The optical potential between the target and a nucleon was derived by T. Tamura and is given by

$$V_{coupl}(\vec{r}_i) = \sum_{\lambda \neq 0,\, \mu} \hat{\lambda}(-1)^{\lambda+1} v_{\lambda}^{(rot)}(r_i) D_{\mu 0}^{\lambda} Y_{\mu}^{\lambda}(\hat{r}_i)\,.$$

The solid spherical harmonics addition theorem for $\vec{r}_i = x_i\vec{R} + y_i\vec{\rho}$

$$r_i^{\lambda} Y_{\mu}^{\lambda}(\hat{r}_i) = \sum_{0 \leq p \leq \lambda} \frac{\sqrt{4\pi(2\lambda+1)!}\, x_i^p R^p y_i^p \rho^{\lambda-p}}{\sqrt{(2p+1)!(2(\lambda-p)+1)!}} \left[Y^p(\hat{R}) \otimes Y^{\lambda-p}(\hat{\rho}) \right]_{\mu}^{\lambda}\,.$$

[3] T. Tamura, Rev. Mod. Phys. **37** (1965) 679.

Target excitations: CDCC*.

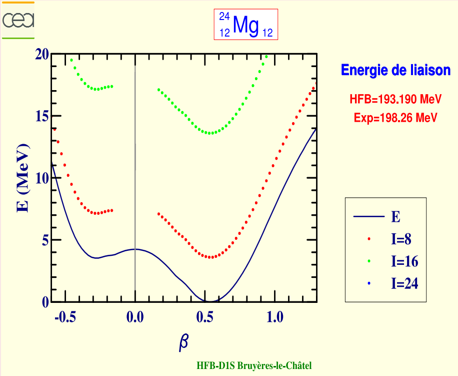
The new form factors $V_{c\,c'} = \langle c|V(\vec{r_i})|c'\rangle$.

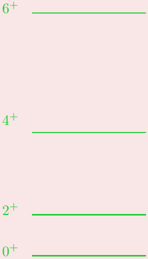
For a rotating nucleus, one gets:

$$
\begin{aligned}
V_{c\,c'}(R) \quad = & \sum_{\lambda\,p\,\sigma\,L''\,l''} \left(\frac{1}{4\pi}\right)^2 (-1)^{p+\sigma+\lambda-p}(-1)^{L'+l'}(-1)^{I'_t-\lambda} \\
& \hat{l}\hat{I}_p\hat{L}\hat{J}\hat{I}_t \quad \hat{l}'\hat{I}'_p\hat{L}'\hat{J}'\hat{I}'_t \quad \hat{\sigma}\hat{L''}^2\hat{l''}^2\widehat{(\lambda-p)}\hat{p}\,(-1)^{J_T+M_T} \begin{Bmatrix} L & I_p & J \\ L'' & l'' & \lambda \\ L' & I'_p & J' \end{Bmatrix} \\
& \delta_{s\,s'} \begin{pmatrix} \sigma & p & L'' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sigma & (\lambda-p) & l'' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & L'' & L' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & l'' & l' \\ 0 & 0 & 0 \end{pmatrix} \\
& \begin{Bmatrix} \lambda & l'' & L'' \\ \sigma & p & (\lambda-p) \end{Bmatrix} \begin{Bmatrix} I'_p & l'' & I_p \\ l & s & l' \end{Bmatrix} \begin{Bmatrix} I_t & I'_t & \lambda \\ J' & J & J_T \end{Bmatrix} \begin{pmatrix} I'_t & \lambda & I_t \\ 0 & 0 & 0 \end{pmatrix} \\
& \int \frac{4\pi}{\hat{\sigma}} u_{l'}(\rho)u_l(\rho)v_p^{(\lambda)\sigma}(R,\rho)d\rho\,.
\end{aligned}
$$

Target excitations: CDCC*.

Elastic and inelastic cross sections for a magnesium target.





^{24}Mg spectrum

$\beta_2 = 0.4$

Target excitations: CDCC*.

Convergence test: elastic and inel. cross sections for d+^{24}Mg and $E_d = 70.0$ MeV

0 bin: deuteron g.s.

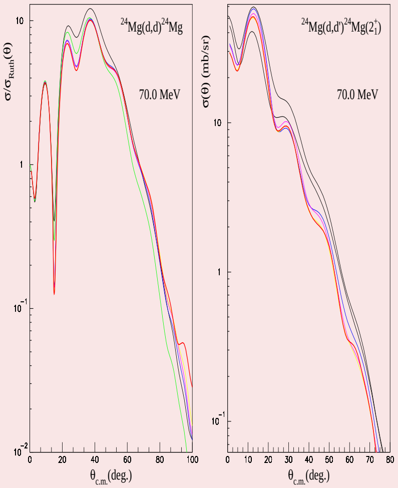
1 bin to discretize the cont.

3 bins to discretize the cont.

4 bins to discretize the cont.

8 bins to discretize the cont.

10 bins to discretize the cont.



Target excitations: CDCC*.

Elastic and inelastic cross sections (2_1^+ state) for a ^{24}Mg target and for $72.0 \leq E_d \leq 90.0$ MeV.

^{24}Mg(d,d)^{24}Mg at 90.0 MeV

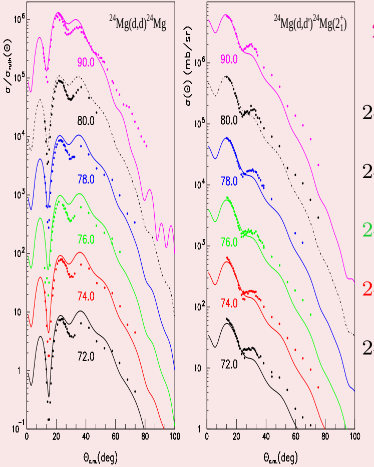
^{24}Mg(d,d)^{24}Mg at 80.0 MeV

^{24}Mg(d,d)^{24}Mg at 78.0 MeV

^{24}Mg(d,d)^{24}Mg at 76.0 MeV

^{24}Mg(d,d)^{24}Mg at 74.0 MeV

^{24}Mg(d,d)^{24}Mg at 72.0 MeV



^{24}Mg(d,d')^{24}Mg*

^{24}Mg(d,d')^{24}Mg*

^{24}Mg(d,d')^{24}Mg*

^{24}Mg(d,d')^{24}Mg*

^{24}Mg(d,d')^{24}Mg*

^{24}Mg(d,d')^{24}Mg*

Conclusion.

Summary.

Within this lecture, we have tried to present

1. The main ideas of the CDCC approach:
   1. How to choose the interactions?
   2. How to discretize the continuum?
   3. How to compute the equations?
2. Some applications (the effect of the continuum on the elastic cross section...).

We would like to emphasize that CDCC is an effective approach and that the convergence must be tested by increazing the bin numbers, the angular momentum truncation....

Conclusion.

Extensions.

The CDCC method has been applied to analyse reaction involving other weakly bound projectiles such as ^{6}Li, ^{6}He, ^{7}Li...

The CDCC formalism has been extended to describe other reaction mechanisms:

1. Core excitations have also been included.
   N.C. Summers *et al.*, Phys. Rev. **C73**, 0631603(R) (2006).
   N.C. Summers *et al.*, Phys. Rev. **C74**, 014606 (2006).
   N.C. Summers *et al.*, Phys. Rev. **C76**, 014611 (2007).
2. The formalism has also been extended to include the target excitations and to calculate the inelastic cross sections (for rotationnal and vibrationnal nuclei).
3. 4-body approaches have been developped.
   T. Matsumoto *et al.*, Phys. Rev. **C70**, 061601 (2004).
   M. Rodríguez-Gallardo *et al*, Phys. Rev. **C80**, 051601 (2009).
   P.N. de Faria *et al.*, Phys. Rev. **C81**, 044605 (2010).

Some other formalisms have been developped to include the breakup channels (e.g. O. A. Rubtsova *et al.*, Phys. Rev. C**78**, 034603 (2008)).

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