Methods to describe direct reactions.

II. Transfer reactions. Distorted wave approximations and beyond.

N.K. Timofeyuk

University of Surrey, UK

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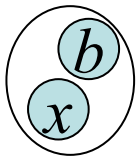
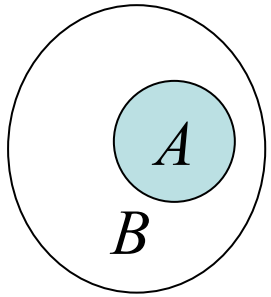
Wave function of the system



$$A + a \rightarrow B + b \quad (\alpha \rightarrow \beta)$$



The total wave function Ψ of the system satisfies the Schrodinger equation

$$(H - E)\,\Psi = 0$$

and contains outgoing spherical waves in all channels and incoming waves in the α channel. Let us denote it $\Psi_\alpha^{(+)}$

The amplitude of the reaction will be determined by the projection of $\Psi_\alpha^{(+)}$ into $\psi_b\,\psi_B$:

$$\chi_\beta(\boldsymbol{r}_\beta) = \left(\psi_b \psi_B \middle| \Psi_\alpha^{(+)}\right)$$

Exact expression for reaction amplitude

The Schrodinger equation can be rewritten as

$$\left(E - H_b - H_B - \frac{\boldsymbol{p}_\beta^2}{2\mu_\beta}\right)\Psi_\alpha^{(+)} = V_\beta \Psi_\alpha^{(+)}, \qquad V_\beta = \sum_{i\in b, j\in B} V_{ij}$$

Multiplying from the left by $\psi_b{}^*\,\psi_B{}^*$ and integrating over ξ_b and ξ_B we get

$$\left(E_\beta - \frac{\boldsymbol{p}_\beta^2}{2\mu_\beta}\right)\chi_\beta(\boldsymbol{r}_\beta) = \int d\xi_b d\xi_B \; \psi_b^*(\xi_b)\psi_B^*(\xi_B) V_\beta(\boldsymbol{r}_\beta, \xi_b, \xi_B)\Psi_\alpha^{(+)}$$

$$= \left(\psi_b \psi_B \middle| V_\beta \middle| \Psi_\alpha^{(+)}\right)$$

The solution of this equation is

Green's function

$$\chi_\beta(\boldsymbol{r}_\beta) = e^{i\boldsymbol{k}_\alpha \boldsymbol{r}_\alpha}\delta_{\alpha\beta} + \int d\boldsymbol{r'}_\beta \; G_\beta^{(+)}(\boldsymbol{r}_\beta, \boldsymbol{r'}_\beta)\left(\psi_b \psi_B \middle| V_\beta \middle| \Psi_\alpha^{(+)}\right)$$

Reaction amplitude and cross section

To get amplitude for the $A + a \rightarrow B + b$ ($\alpha \rightarrow \beta$) reaction, the total wave function $\Psi_\alpha^{(+)}$ should be projected into $\psi_\beta = \psi_b \psi_B$

$$\chi_\beta(\boldsymbol{r}_\beta) = \left(\psi_\beta \middle| \Psi_\alpha^{(+)}\right) = \left(\psi_b \psi_B \middle| \Psi_\alpha^{(+)}\right) = \int d\xi_b d\xi_B \; \psi_b^*(\xi_b)\psi_B^*(\xi_B)\Psi_\alpha^{(+)}$$

Asymptotically,

$$\chi_\beta(\boldsymbol{r}_\beta)\Big|_{r\to\infty} \rightarrow e^{i\boldsymbol{k}_\alpha \boldsymbol{r}_\alpha}\delta_{\alpha\beta} + f_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta)\frac{e^{ik_\beta r_\beta}}{r_\beta}$$

Reaction amplitude

Cross section: $$\frac{d\sigma}{d\Omega} = \frac{v_\beta}{v_\alpha}\left|f_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta)\right|^2$$

$$\chi_\beta^{(+)}(\boldsymbol{r}_\beta) = e^{i\boldsymbol{k}_\alpha \boldsymbol{r}_\alpha}\delta_{\alpha\beta} + \int d\boldsymbol{r}'_\beta\, G_\beta^{(+)}(\boldsymbol{r}_\beta, \boldsymbol{r}'_\beta)\left(\psi_b\psi_B\middle|V_\beta\middle|\Psi_\alpha^{(+)}\right)$$

Green's function

$$G_\beta^{(+)}(\boldsymbol{r}_\beta, \boldsymbol{r}'_\beta) \equiv \left(E_\beta - \frac{\boldsymbol{p}_\beta^2}{2\mu_\beta}\right)^{-1} = -\frac{\mu_\beta}{2\pi\hbar^2}\frac{e^{ik_\beta|\boldsymbol{r}_\beta - \boldsymbol{r}'_\beta|}}{|\boldsymbol{r}_\beta - \boldsymbol{r}'_\beta|}$$

To get reaction amplitude, the limit $\boldsymbol{r}_\beta \to \infty$ should be considered.

For $\boldsymbol{r}_\beta >> \boldsymbol{r}'_\beta$, $\quad |\boldsymbol{r}_\beta - \boldsymbol{r}'_\beta| \approx r_\beta - \hat{\boldsymbol{r}}_\beta \cdot \boldsymbol{r}'_\beta = r_\beta - \hat{\boldsymbol{k}}_\beta \cdot \boldsymbol{r}'_\beta$

$$\chi_\beta^{(+)}(\boldsymbol{r}_\beta) = e^{i\boldsymbol{k}_\alpha \boldsymbol{r}_\alpha}\delta_{\alpha\beta} - \frac{\mu_\beta}{2\pi\hbar^2}\frac{e^{ik_\beta r_\beta}}{r_\beta}\int d\boldsymbol{r}'_\beta\, e^{-i\boldsymbol{k}_\beta \boldsymbol{r}'_\beta}\left(\psi_b\psi_B\middle|V_\beta\middle|\Psi_\alpha^{(+)}\right)$$

$$f_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = -\frac{\mu_\beta}{2\pi\hbar^2}\left\langle e^{i\boldsymbol{k}_\beta \boldsymbol{r}'_\beta}\psi_b\psi_B\middle|V_\beta\middle|\Psi_\alpha^{(+)}\right\rangle$$

Distorted waves

Let us introduce arbitrary potential $U_\beta(\boldsymbol{r}_\beta)$

$$\left(E - H_b - H_B - \frac{\boldsymbol{p}_\beta^2}{2\mu_\beta} - U_\beta(\boldsymbol{r}_\beta) \right) \Psi_\alpha^{(+)} = (V_\beta - U_\beta(\boldsymbol{r}_\beta))\Psi_\alpha^{(+)},$$

Multiplying from the left by $\psi_b{}^* \psi_B{}^*$ and integrating over ξ_b and ξ_B we get

$$\tilde{\chi}_\beta^{(+)}(\boldsymbol{r}_\beta) = \chi_\beta^{(+)}\delta_{\alpha\beta} + \int d\boldsymbol{r}'_\beta \; \tilde{G}_\beta^{(+)}(\boldsymbol{r}_\beta, \boldsymbol{r}'_\beta)\left(\psi_b\psi_B \left| V_\beta - U_\beta(\boldsymbol{r}_\beta) \right| \Psi_\alpha^{(+)}\right)$$

The reaction amplitude is then

$$f_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = -\frac{\mu_\beta}{2\pi\hbar^2} \left\langle \chi_\beta^{(-)}\psi_b\psi_B \left| V_\beta - U_\beta(\boldsymbol{r}_\beta) \right| \Psi_\alpha^{(+)} \right\rangle$$

Exact amplitude contain exact wave function $\Psi_\alpha^{(+)}$

$$f_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta,\boldsymbol{k}_\beta)=-\frac{\mu_\beta}{2\pi\hbar^2}T_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta,\boldsymbol{k}_\beta)=-\frac{\mu_\beta}{2\pi\hbar^2}\left\langle\chi_\beta^{(-)}\psi_b\psi_B\middle|V_\beta-U_\beta(\boldsymbol{r}_\beta)\middle|\Psi_\alpha^{(+)}\right\rangle$$

This wave function satisfies the Schrodinger equation

$$\left(E-H_a-H_A-\frac{\boldsymbol{p}_\alpha^2}{2\mu_\alpha}-U_\alpha(\boldsymbol{r}_\alpha)\right)\Psi_\alpha^{(+)}=(V_\alpha-U_\alpha(\boldsymbol{r}_\alpha))\Psi_\alpha^{(+)}$$

The formal solution can be written as

$$\Psi_\alpha^{(+)}=\chi_\alpha^{(+)}\psi_a\psi_A+\underbrace{\left(E-H_a-H_A-\frac{\boldsymbol{p}_\alpha^2}{2\mu_\alpha}-U_\alpha(\boldsymbol{r}_\alpha)\right)^{-1}}_{G_\alpha^{(+)}}(V_\alpha-U_\alpha(\boldsymbol{r}_\alpha))\Psi_\alpha^{(+)}$$

Then the amplitude can be rewritten as

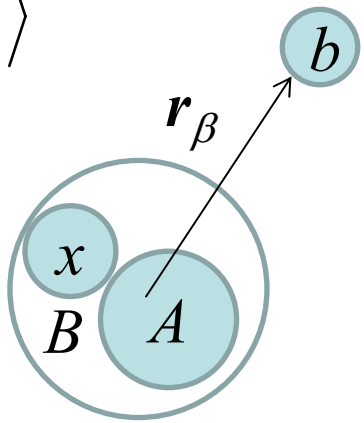
Born approximation

$$T_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta,\boldsymbol{k}_\beta)=\left\langle\chi_\beta^{(-)}\psi_b\psi_B\middle|V_\beta-U_\beta(\boldsymbol{r}_\beta)\middle|\chi_\alpha^{(+)}\psi_a\psi_A\right\rangle$$

$$=\left\langle\chi_\beta^{(-)}\psi_b\psi_B\middle|(V_\beta-U_\beta(\boldsymbol{r}_\beta))G_\alpha^{(+)}(V_\alpha-U_\alpha(\boldsymbol{r}_\alpha))\middle|\Psi_\alpha^{(+)}\right\rangle$$

Distorted wave Born approximation (DWBA)

$$T_{\beta\alpha}^{DWBA}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = \left\langle \chi_\beta^{(-)} \psi_b \psi_B \middle| V_\beta - U_\beta(\boldsymbol{r}_\beta) \middle| \chi_\alpha^{(+)} \psi_a \psi_A \right\rangle$$



χ_α is obtained from optical model in channel α
χ_β is obtained from optical model in channel β

$$V_\beta = V_{bB} = \sum_{i\in b}\sum_{j\in B} v_{ij} = \sum_{i\in b}\left\{\sum_{j\in x} + \sum_{j\in A}\right\} v_{ij} = V_{bx} + V_{bA}.$$

The assumption $V_{bA} - U_\beta(\boldsymbol{r}_\beta) \approx 0$ is often made. This assumption may look reasonable if x is a nucleon and A is large. Then

$$T_{\beta\alpha}^{DWBA}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = \iint d\boldsymbol{r}_\alpha d\boldsymbol{r}_\beta \chi_\beta^{(-)}\left(\boldsymbol{k}_\beta, \boldsymbol{r}_\beta\right)^* I_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{r}_\beta) \chi_\alpha^{(+)}\left(\boldsymbol{k}_\alpha, \boldsymbol{r}_\alpha\right)$$

$$I_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{r}_\beta) = \left\langle \psi_b \psi_B \middle| V_{bx} \middle| \psi_a \psi_A \right\rangle = \left\langle \psi_b \middle| V_{bx} \middle| \psi_a \right\rangle \left\langle \psi_B \middle| \psi_A \right\rangle$$

Overlap integrals

Overlap integrals $\langle \psi_B | \psi_A \rangle$ carry information about nuclear structure. They are solutions of an integral equation.

$$(T_A + V_A - E_A)\psi_A = 0, \qquad (T_B + V_B - E_B)\psi_B = 0$$

$$\psi_A (T_B + V_B - E_B)\, \psi_B = 0$$

$$\langle \psi_A | T_A + (T_B - T_A) + V_A + (V_B - V_A) - E_A + (E_A - E_B) | \psi_B \rangle = 0$$

$$\langle \psi_A | (T_B - T_A) + (E_A - E_B) | \psi_B \rangle = \langle \psi_A | (V_A - V_B) | \psi_B \rangle$$

$$(T_x + \varepsilon)\langle \psi_A | \psi_B \rangle = -\langle \psi_A | V_{Ax} | \psi_B \rangle$$

Partial wave expansion of the overlap integral

$$I_{AB}(\boldsymbol{r}) \equiv \left\langle \psi_B \middle| \psi_A \right\rangle = \sum_{M_A M_B m \sigma} (lm\, \tfrac{1}{2}\sigma \mid jm_j)(jm_j J_A M_A \mid J_B M_B) I_{lj}(r) Y_{lm}(\hat{r}) \chi_{1/2\sigma} \chi_{1/2\tau}$$

Properties of the overlap integrals

I) Asymptotic behaviour

At large r the overlap integral satisfies the equation

$$(T_x + \varepsilon) I_{AB}(\boldsymbol{r}) = -\langle \psi_A | V_{Ax} | \psi_B \rangle \approx 0 \qquad (\textit{for neutral particle } x)$$

$$(T_x + V_{coul}(r) + \varepsilon) I_{AB}(\boldsymbol{r}) = -\langle \psi_A | V_{Ax} - V_{Coul}(r) | \psi_B \rangle \approx 0 \qquad (\textit{for charged particle } x)$$

The asymptotic part of the overlap functions $I_{lj}(r)$ is given by

$$I_{lj}(r) \approx C_{lj}\, W_{-\eta, l+\frac{1}{2}}(2\kappa r)/r$$

C_{lj} is the **<u>asymptotic normalization coefficient</u> (*ANC*),**
W is the Whittaker function,
$\kappa = (2\mu\varepsilon)^{1/2}$, ε is the nucleon separation energy

Example: for *B=A+neutron* and *l=0:* $I_{lj}(r) \approx C_{lj}\, exp(-\kappa r)/r$

II) Normalization

Definition: the norm of $I_{lj}(r)$ is called the **spectroscopic factor.**

$$S_{lj} = \int_0^\infty dr\, r^2 I_{lj}^2(r) \quad (\times B)$$

The meaning of the spectroscopic factor from the shell model point of view.
The shell model wave function is a linear combination of the Slater determinants

$$\psi_A = \sum_{\alpha_A} C_{A,\alpha_A} D_{A,\alpha_A} \qquad \alpha_A = \{n_1 l_1 j_1 m_1 \tau_1, ..., n_A l_A j_A m_A \tau_A\}$$

$$\psi_B = \sum_{\alpha_B} C_{B,\alpha_B} D_{B,\alpha_B}$$

$$\langle \psi_A | \psi_B \rangle = \sum_{\alpha_A \alpha_B} C_{A,\alpha_A} C_{B,\alpha_B} \varphi_{\gamma(\alpha_A \alpha_B)}$$

The spectroscopic factor is expressed only via coefficients C_{A,α_A} and C_{B,α_B} which are probability amplitudes of a particular shell occupation scheme.

Modelling the overlap functions:

$$(T_x + \varepsilon)\langle \psi_A |\, \psi_B \rangle = -\langle \psi_A \,|V_{Ax}|\psi_B \rangle \approx -V_{Ax}\langle \psi_A \,|\psi_B \rangle$$

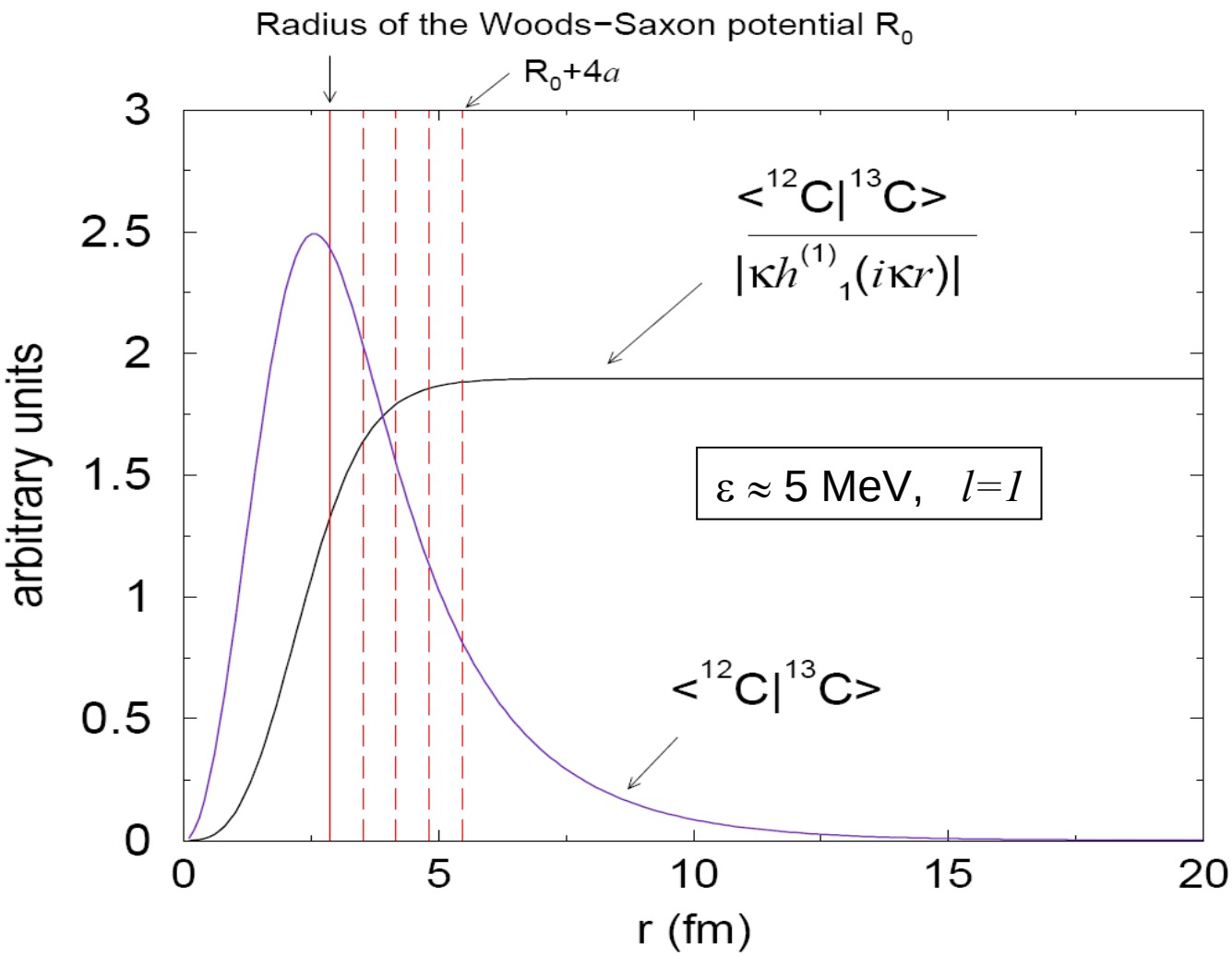
or $$(T_x + V_{Ax}(r) + \varepsilon)\, I_{lj}(r) = 0$$

$$I_{lj}(r) = S^{1/2}\, \varphi_{lj}(r), \qquad \int_0^\infty dr\, r^2 \varphi_{lj}^2(r) = 1$$

$\varphi_{lj}(r)$ is the normalized solution of the two-body equation and the spectroscopic factor S is thought to be determined from experiment.

Often, a standard Wood-Saxon potential with $r_0 \approx 1.25$ fm, $a \approx 0.65$ fm is used to determine $\varphi_{lj}(r)$ while the depth V_0 is fitted to reproduce ε.

Typical example of the overlap functions for stable nuclei



Peripheral transfer reactions

The reaction amplitude can be rewritten as follows:

$$T_{\beta\alpha}^{DWBA}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = \iint d\boldsymbol{r}_\alpha d\boldsymbol{r}_\beta \chi_\beta^{(-)}\left(\boldsymbol{k}_\beta, \boldsymbol{r}_\beta\right)^* I_{\alpha\beta}(\boldsymbol{r}_\alpha, \boldsymbol{r}_\beta)\chi_\alpha^{(+)}\left(\boldsymbol{k}_\alpha, \boldsymbol{r}_\alpha\right)$$

$$= \int_0^{R_{cut}} dr_{xA} .... + \int_{R_{cut}}^{\infty} dr_{xA} .... = T_{in} + T_{ext}$$

T_{int} probes the overlap integral $I_{lj}(r_{xA})$ in the nuclear interior.

T_{ext} probes the tail of the overlap integral $I_{lj}(r_{xA})$, the magnitude of which is given by the ANC.

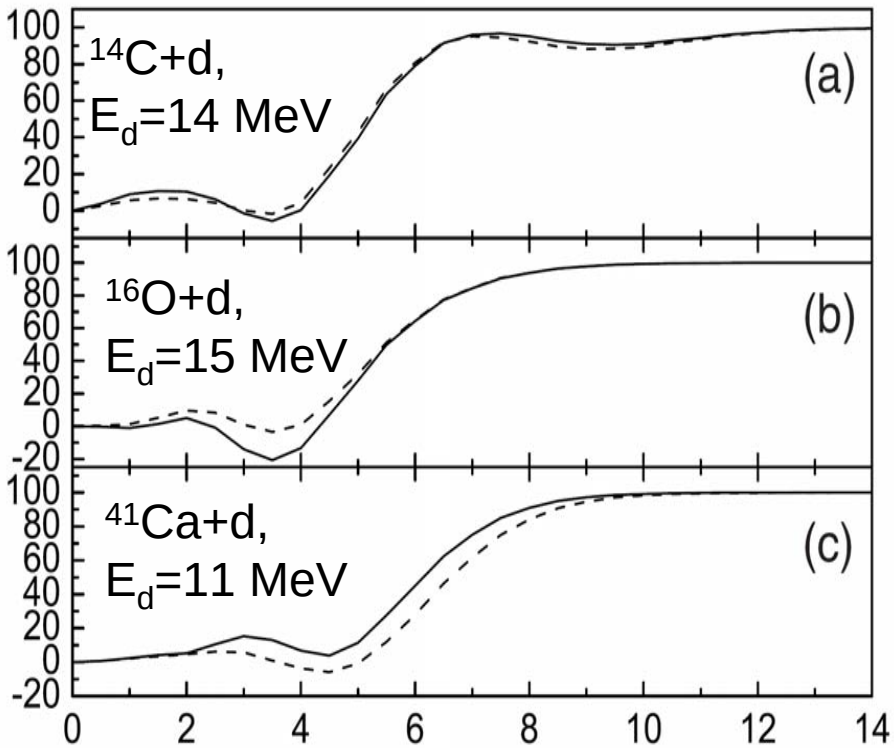
$$I_{lj}(r_{xA}) = S^{1/2}\ \varphi_{lj}(r_{xA}) = S^{1/2}\ b_{lj}\ W_{-\eta,l+1/2}\ (2\kappa r_{xA})/r_{xA}$$

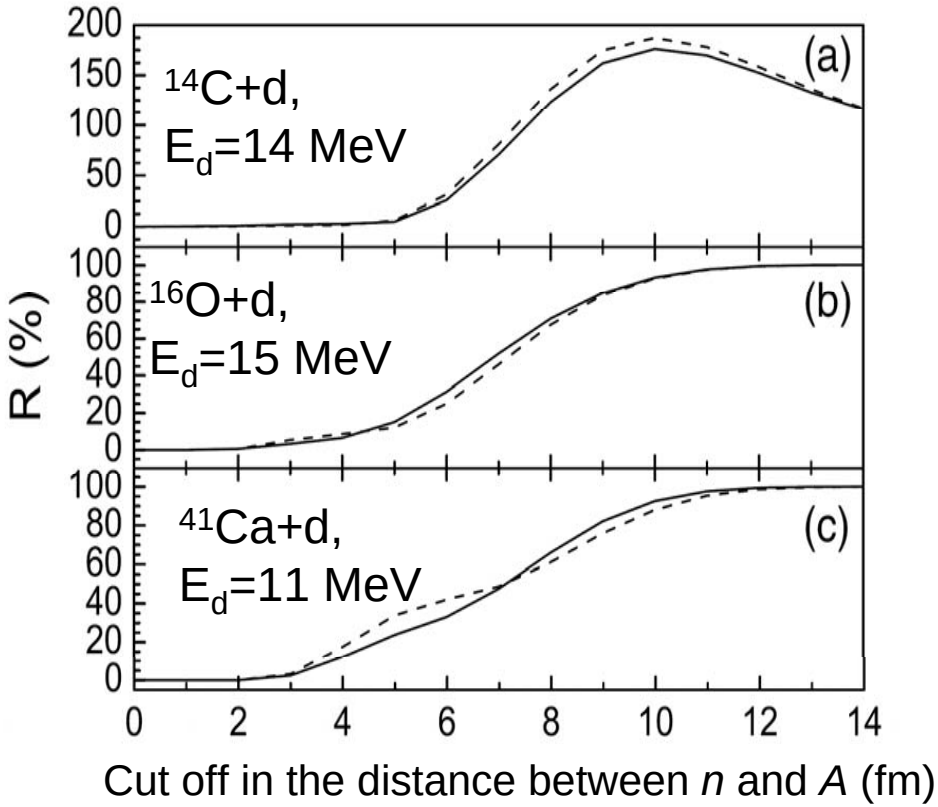
$$C_{lj} = S^{1/2} b_{lj}$$

b_{lj} is the single-particle ANC

Contribution to the A(d,p)B reaction amplitude

D.Y.Pang, F.M.Nunes, A.M.Mukhamedzhanov, Phys. Rev. **C 75**, 024601 (2007)





The contribution of the asymptotic region into (*d,p*) reaction amplitude dominates

Cross sections of peripheral reactions are factorized via ANCs:

$$\sigma(\theta) \sim \left|T_{\text{int}} + T_{ext}\right|^2 \approx \left|T_{ext}\right|^2 = \left|\sqrt{S} b \tilde{T}_{ext}\right|^2 = C^2 \left|\tilde{T}_{ext}\right|^2$$

does not depens on b

The ANC determined from experiment as

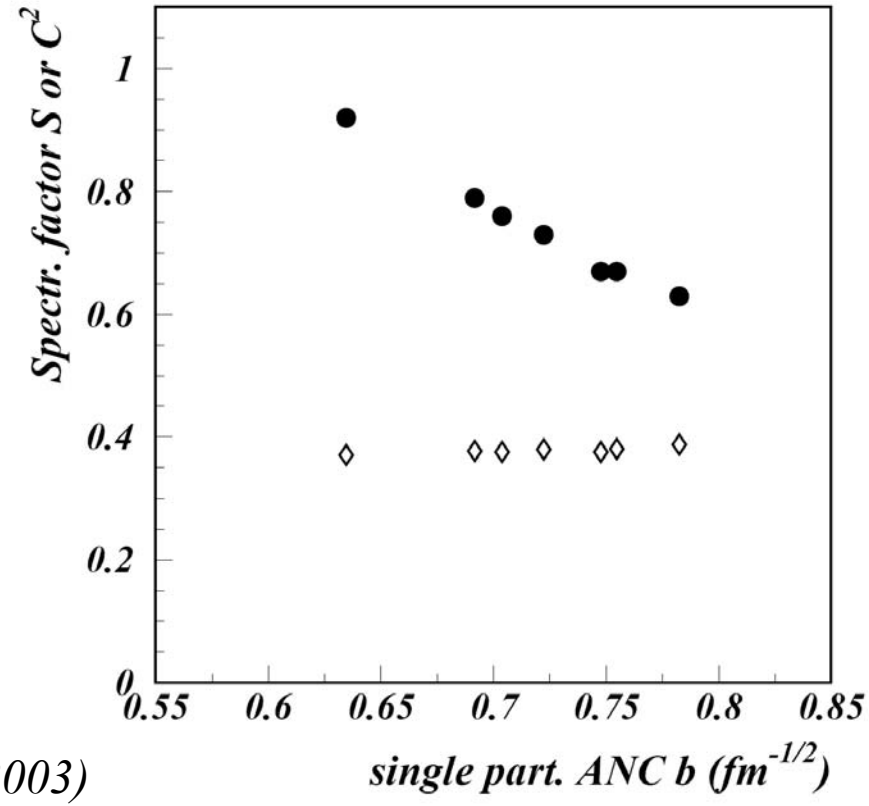
$$C^2 \sim \sigma_{\text{exp}}(\theta) / \left|\tilde{T}_{ext}\right|^2$$

does not depend on b.

The spectroscopic factor determined from experiment as

$$S = \sigma_{\text{exp}}(\theta) / \left|b\tilde{T}_{ext}\right|^2$$

depends on b.



Example: $^{12}C(^{8}Li,^{7}Li)^{13}C$

L.Trache et al, Phys.Rev. C 67, 062801 (2003)

Can b_{lj} be determined from experimental data in a mode-independent way?

Original idea (*S.A.Goncharov, et al, Yad.Fiz. 35, 662 (1982)*):

$$\sigma(\theta) \propto \left|T_{\text{int}} + T_{ext}\right|^2 = \left|\sqrt{S}\tilde{T}_{\text{int}}(b) + \sqrt{S}b\tilde{T}_{ext}\right|^2$$

does not depends on *S*

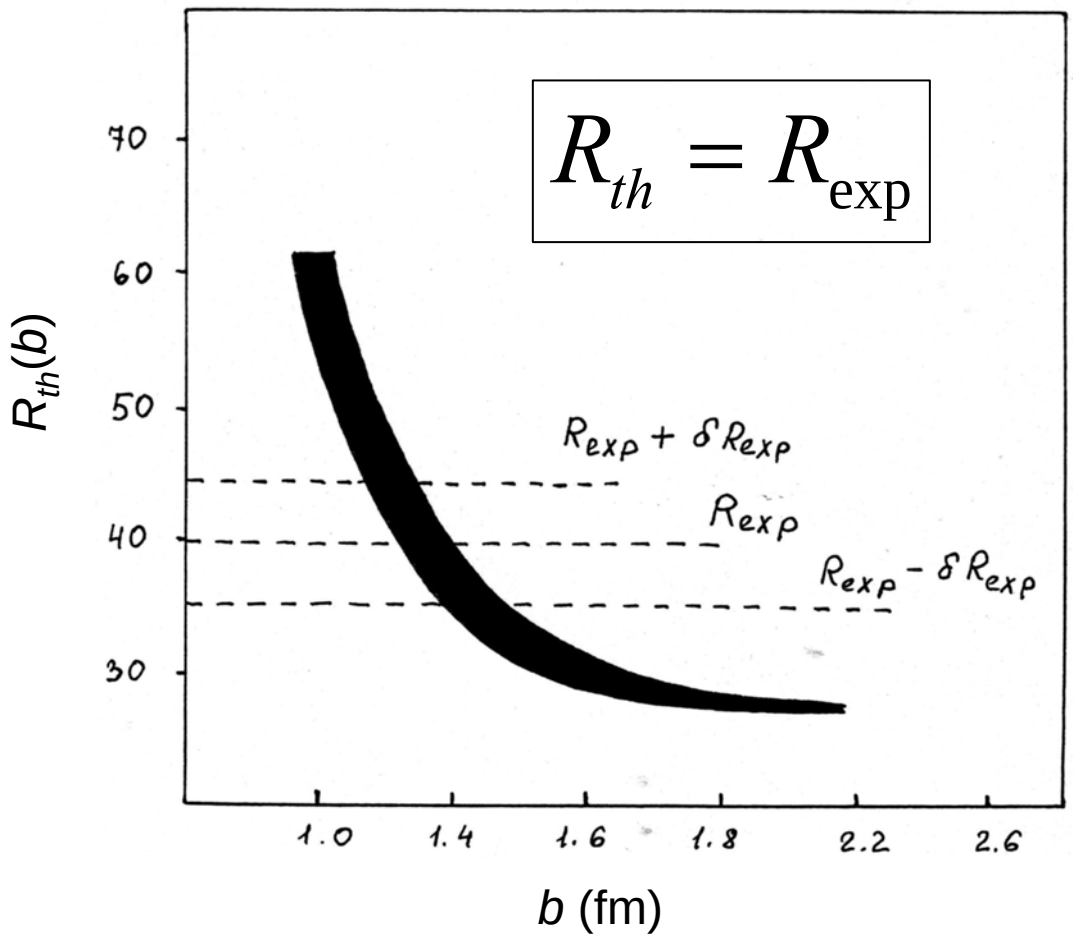
T_{ext} can be fixed using ANCs measured from peripheral reactions.
Then T_{int} can be determined.

$$\sigma(\theta) \propto Sb^2\left|\frac{\tilde{T}_{\text{int}}(b)}{b} + \tilde{T}_{ext}\right|^2 \quad \Rightarrow \quad \frac{\sigma(\theta)}{Sb^2} \propto \left|\frac{\tilde{T}_{\text{int}}(b)}{b} + \tilde{T}_{ext}\right|^2$$

$$\boxed{R_{th}(b) \equiv \frac{\sigma_{th}(\theta)}{S_{th}b_{th}^2} = \frac{\sigma_{\text{exp}}(\theta)}{C_{\text{exp}}^2} \equiv R_{\text{exp}}}$$

$^{13}C(p,d)^{12}C$ E_p = 18.6 MeV

S.A.Goncharov et al, Yad.Fiz. 44, 303 (1986)



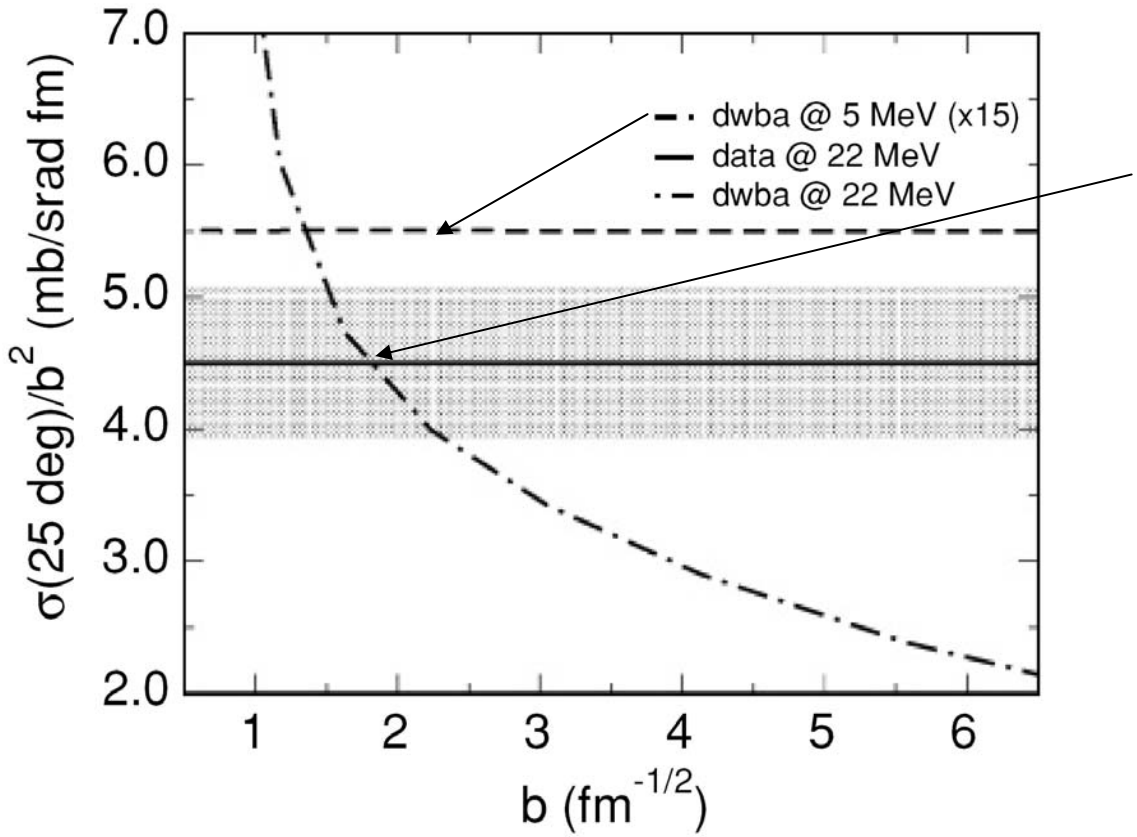
$b_{exp}=1.3\ ^{-0.17}_{+0.26}\ fm^{-1/2}$

$S_{exp}=1.19^{+0.08}_{-0.21}$

r_0 = 1.0 fm, a = 0.49 fm

^{208}Pb(*d*,*p*)^{209}Pb

A.M. Mukhamedzhanov and F.Nunes, Phys. Rev. C ***72****, 017602 (2005)*



$b = 1.82\ fm^{-1/2}$

$S = 0.74$

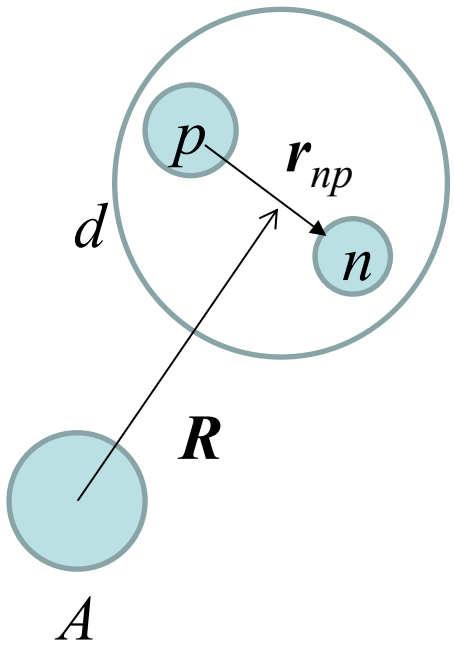
A(d,p)B reactions. Beyond the Born approximation.

Exact w.f.

$$T_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = \left\langle \chi_\beta^{(-)} \psi_b \psi_B \middle| V_\beta - U_\beta(\boldsymbol{r}_\beta) \middle| \Psi_\alpha^{(+)} \right\rangle$$

Born approximation:

$$\Psi_\alpha^{(+)} \approx \chi_d^{(+)}(\boldsymbol{r}_d)\psi_d(\boldsymbol{r}_{np})\psi_A$$



Beyond the Born approximation:
taking deuteron breakup into account.

$$\Psi_\alpha^{(+)} \approx \psi_{Anp}^{(+)}(\boldsymbol{R}, \boldsymbol{r}_{np})\, \psi_A$$

$$\left(T_R + T_{np} + V_{np} + V_{nA} + V_{pA} - E\right)\psi_{Anp}^{(+)}(\boldsymbol{R}, \boldsymbol{r}_{np}) = 0$$

Solving 3-body Schrödinger equation in the adiabatic approximation. Johnson-Soper model.

R.C. Johnson and P.J.R. Soper, Phys. Rev. C1,976 (1970)

Adiabatic assumption: $\left(T_{np}+V_{np}+\varepsilon_d\right)\psi^{(+)}_{Anp}(\boldsymbol{R},\boldsymbol{r}_{np})=0$

Then the three-body equation becomes

$$\left(T_R+V_{nA}+V_{pA}-E_d\right)\psi^{(+)}_{Anp}(\boldsymbol{R},\boldsymbol{r}_{np})=0, \qquad E_d=E-\varepsilon_d$$

To calculate the reaction amplitude

$$T_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta,\boldsymbol{k}_\beta)=\left\langle \chi^{(-)}_\beta \psi_b \psi_B \middle| V_{np}(\boldsymbol{r}_{np}) \middle| \psi^{(+)}_{Anp}(\boldsymbol{R},\boldsymbol{r}_{np}) \right\rangle$$

Only those part of the wave function, where $r_{np}\approx 0$, are needed:

$$\left(T_R+V_{nA}(\boldsymbol{R})+V_{pA}(\boldsymbol{R})-E_d\right)\psi^{(+)}_{Anp}(\boldsymbol{R},0)=0$$

Johnson-Soper model

• The zero-range (d,p) reaction amplitude formally looks exactly the same as the zero-range DWBA amplitude

$$T_{\beta\alpha}^{ZR} = \left\langle \chi_{\beta}^{(-)}(\tfrac{A}{A+x}\boldsymbol{R}) I_{AB}(\boldsymbol{R}) \,|\, \psi_{Anp}^{(+)}(\boldsymbol{R},0) \right\rangle$$

• The three-body wave function is calculated from the two-body Schrödinger equation

$$\left(T_R + V_{nA}(\boldsymbol{R}) + V_{pA}(\boldsymbol{R}) - E_d\right)\psi_{Anp}^{(+)}(\boldsymbol{R},0) = 0$$

• The adiabatic interaction potential is a sum of the proton and neutron potentials taken at half deuteron energy

• The model takes the deuteron breakup into account as $\psi_d(\boldsymbol{R},0)$ includes all deuteron continuum states

Solving 3-body Schrödinger equation using Weinberg state expansion. Johnson-Tandy model.

R.C. Johnson and P.C. Tandy, Nucl. Phys. A235, 56 (1974)

$$\psi^{(+)}_{Anp}(\boldsymbol{R},\boldsymbol{r}_{np}) = \sum_{i=1}^{\infty} \varphi_i(\boldsymbol{r}_{np})\,\chi_i(\boldsymbol{R}) \qquad\qquad \chi_i = -\left\langle \varphi_i \middle| V_{np} \middle| \psi^{(+)}_{Anp} \right\rangle$$

Weinberg basis:

$$\left(T_{np} + \alpha_i V_{np} + \varepsilon_d\right)\varphi_i(\boldsymbol{r}_{np}) = 0, \qquad \alpha_1 = 1,\ \varphi_1 = \varphi_d,\ \left\langle \varphi_i \middle| V_{np} \middle| \varphi_j \right\rangle = -\delta_{ij}$$

It is assumed that only first term of the expansion is important. It can be found if all the coupling to the other channels are neglected

$$\left(T_R + V(\boldsymbol{R}) - E_d\right)\chi^{(+)}_1(\boldsymbol{R}) = 0$$

$$V(\boldsymbol{R}) = \int d\boldsymbol{r}_{np}\,\left|\varphi_1(\boldsymbol{r}_{np})\right|^2 V_{np}(\boldsymbol{r}_{np})\left(V_{nA}(\boldsymbol{R} + \frac{\boldsymbol{r}_{np}}{2}) + V_{pA}(\boldsymbol{R} - \frac{\boldsymbol{r}_{np}}{2}) \right)$$

Johnson-Tandy model

• The (d,p) reaction amplitude formally looks exactly the same as the finite-range DWBA amplitude

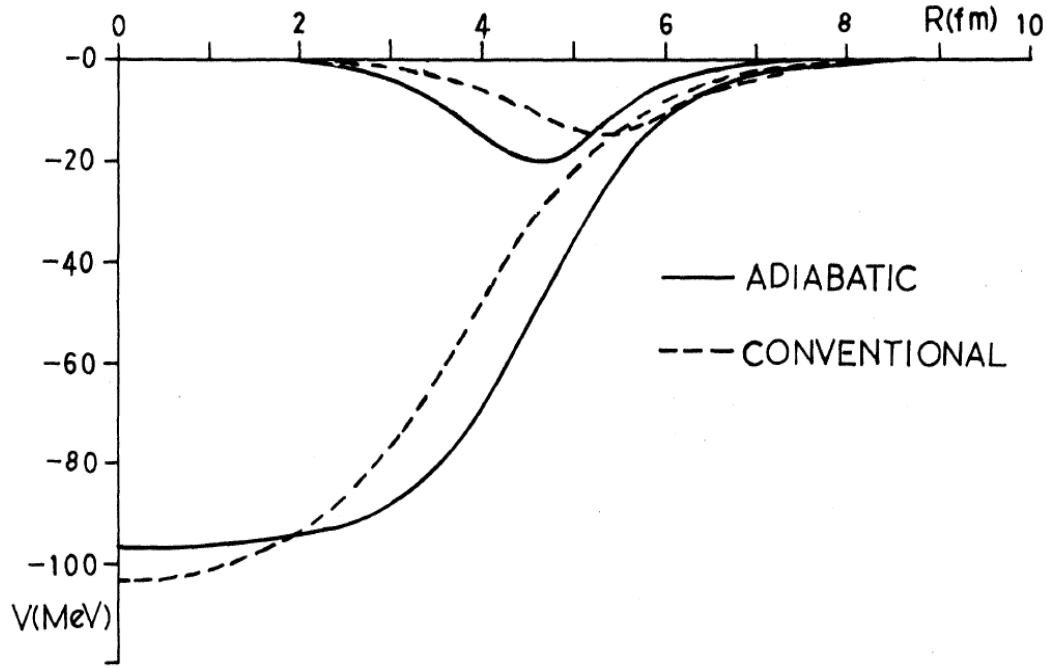
• The three-body wave function $\psi^{(+)}_{Anp}(\boldsymbol{R}, \boldsymbol{r}_{np})$ is calculated from the two-body Schrödinger equation

• The Johnson-Tandy two-body "deuteron" potential is calculated in a folding procedure involving a sum of the proton and neutron potentials taken at half deuteron energy

• The model takes the deuteron breakup into account as the projection of $V_{np}(r_{np})\varphi_1(r_{np})\ \chi_1(\boldsymbol{R})$ onto any deuteron continuum state is not zero

Deuteron potential: adiabatic vs conventional

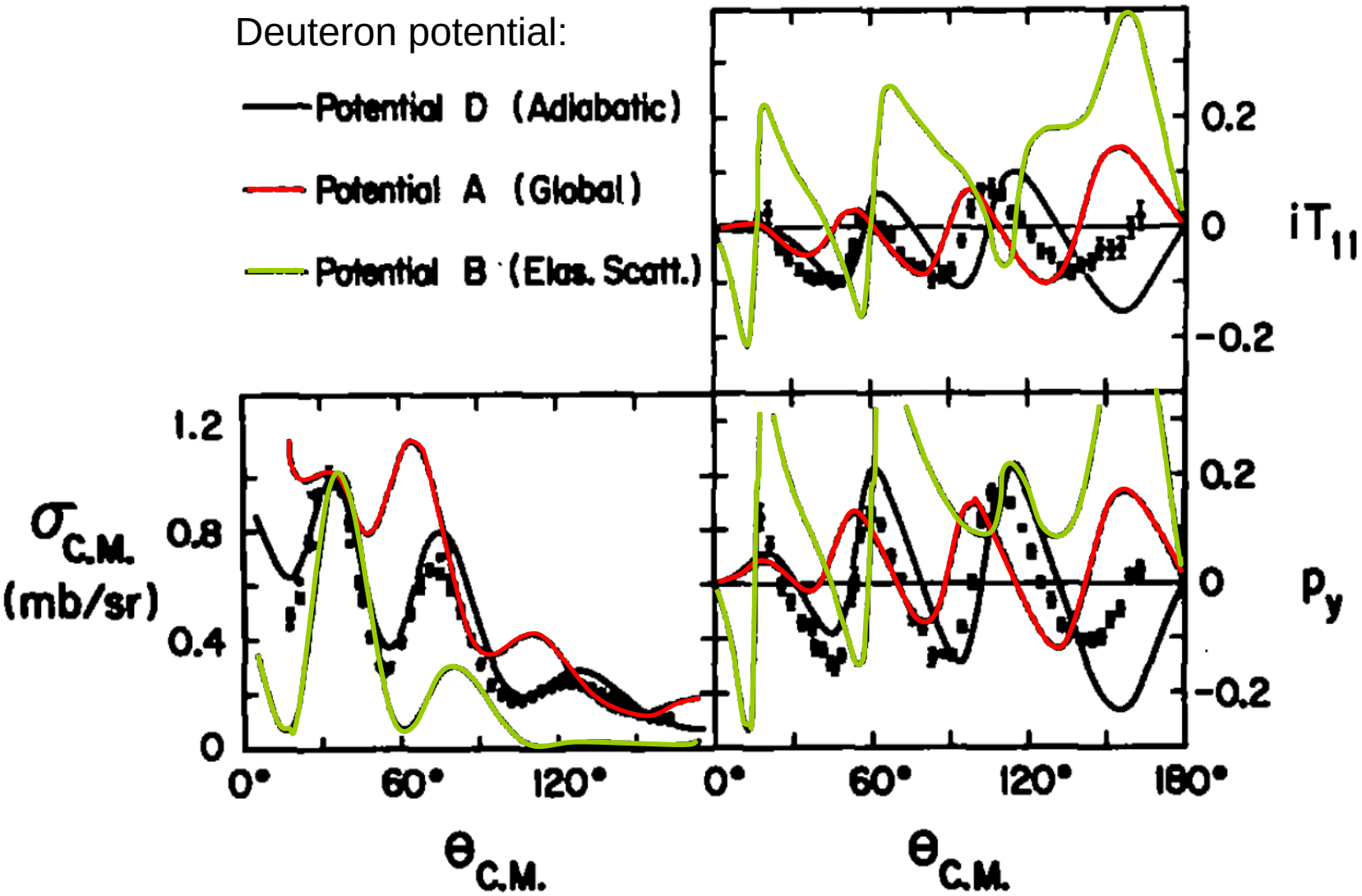
J.D.Harvy and R.C.Johnson, Phys. Rev. 3, 636 (1971)



Conventional potential has an absorptive part that has to account for deuteron breakup. In adiabatic approach, this breakup is explicitly included.

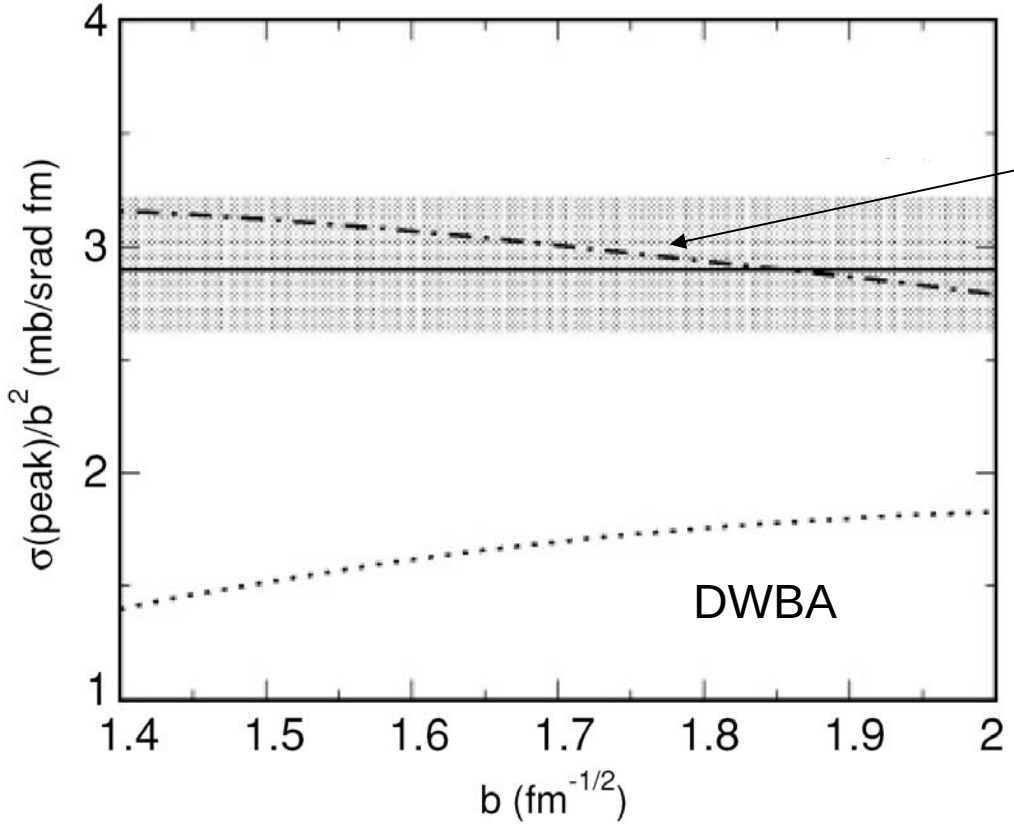
$^{116}Sn(d,p)^{117}Sn$ E_d=8.22 MeV

R.R. Cadmus Jr.,and W. Haeberli, Nucl. Phys. A327, 419 (1979)



$^{12}C(d,p)^{13}C$ at $E_d = 51$ MeV

*A.M. Mukhamedzhanov and F. Nunes, Phys. Rev. C **72**, 017602 (2005)*



Calculated using adiabatic d-^{12}C potential

It is not possible to determine b (and therefore S) from this graph.

Adiabatic cross sections are “more peripheral” than the conventional DWBA cross sections, the contribution from partial waves with low relative orbital momentum is suppressed

Remnant term in the transfer reaction amplitude.

$$T_{\beta\alpha}^{DWBA}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = \left\langle \chi_\beta^{(-)} \psi_b \psi_B \middle| V_\beta - U_\beta(\boldsymbol{r}_\beta) \middle| \chi_\alpha^{(+)} \psi_a \psi_A \right\rangle$$

$$V_\beta = V_{bB} = \sum_{i\in b} \sum_{j\in B} v_{ij} = \sum_{i\in b} \left\{ \sum_{j\in x} + \sum_{j\in A} \right\} v_{ij} = V_{bx} + V_{bA}.$$

The DWBA reaction amplitude has two terms:

$$T_{\beta\alpha}^{DWBA}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = \left\langle \chi_\beta^{(-)} \psi_b \psi_B \middle| V_{bx} \middle| \chi_\alpha^{(+)} \psi_a \psi_A \right\rangle$$

1) The main term that factorizes via SFs or ANCs and depends on small r_{xb}

$$+ \langle \chi_\beta^{(-)} \psi_b \psi_B \mid \sum_{i\in b, j\in A} V_{ij} - U_\beta(\boldsymbol{r}_\beta) \mid \chi_\alpha^{(+)} \psi_a \psi_A \rangle$$

2) The remnant term that does not factorize via SFs or ANCs and is not bounded by small r_{xb}

Adiabatic calculation always assume that the remnant term can be neglected.

Approximate way to include remnant term

$$\langle \chi_{\beta}^{(-)} \psi_b \psi_B \,|\, \sum_{i\in b, j\in A} V_{ij} - U_{\beta}(\boldsymbol{r}_{\beta}) \,|\, \chi_{\alpha}^{(+)} \psi_a \psi_A \rangle$$

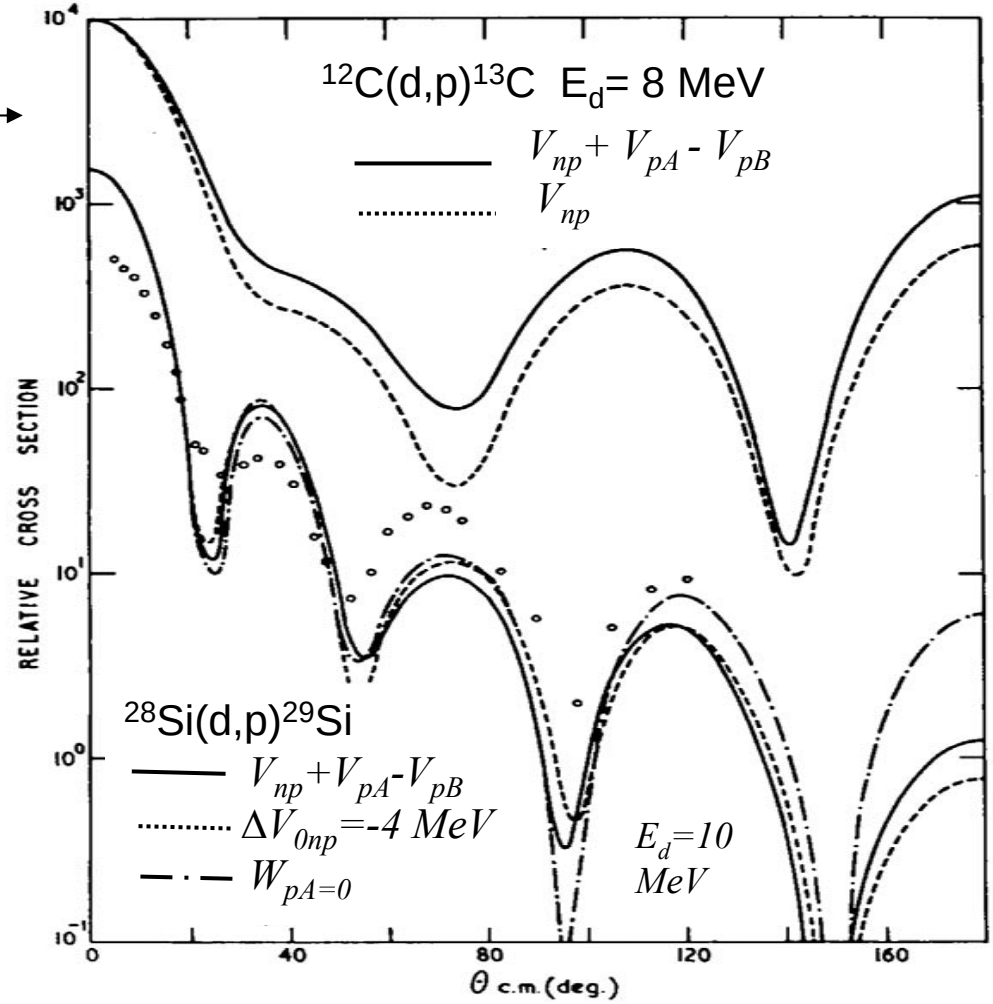
$$\approx \langle \chi_{\beta}^{(-)} \psi_b \psi_B \,|\, V_{bB}(\boldsymbol{r}_{bB}) - U_{\beta}(\boldsymbol{r}_{\beta}) \,|\, \chi_{\alpha}^{(+)} \psi_a \psi_A \rangle$$

- Often V_{bB} is chosen as a complex optical potential between b and B.
- No theoretical justification is given to this choice.
- Remnant term is important for heavy particle transfer

W.R. Smith, Nucl. Phys. A130, 657 (1969)

Comparing remnant and no-remnant calculations

Comparing various verions of remnant calculations



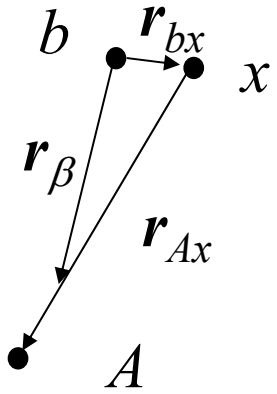
Avoiding calculation of the remnant term

$$T_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = \left\langle \chi_\beta^{(-)} \psi_b \psi_B \middle| V_{bx}(\boldsymbol{r}_{bx}) + V_{bA}(\boldsymbol{r}_{bA}) - U_\beta(\boldsymbol{r}_\beta) \middle| \chi_\alpha^{(+)} \psi_a \psi_A \right\rangle$$

Reminder: derivation of reaction amplitude:

$$\left(E - H_b - H_A - H_x - \frac{\boldsymbol{p}_\beta^2}{2\mu_\beta} - \frac{\boldsymbol{p}_{Ax}^2}{2\mu_{Ax}} - V_{bA} - V_{xA} \right) \Psi_\alpha^{(+)} = V_{bx} \Psi_\alpha^{(+)},$$

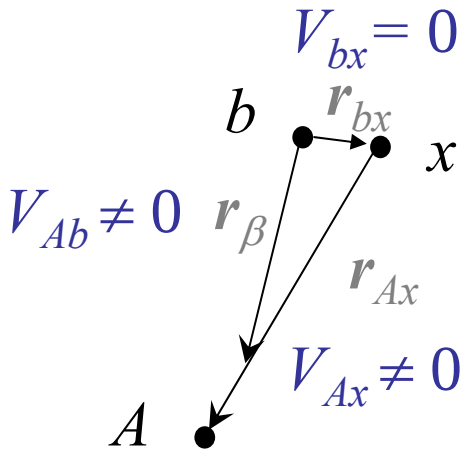
$U_\beta(\boldsymbol{r}_\beta)$ is arbitrary. Let us choose $\sum_{i\in b, j\in A} V_{ij}$ instead of $U_\beta(\boldsymbol{r}_\beta)$



$$T_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = \left\langle \tilde{\phi}_\beta^{(-)} \middle| V_{bx} \middle| \Psi_\alpha^{(+)} \right\rangle$$

$$\left(E - H_b - H_A - H_x - \frac{\boldsymbol{p}_\beta^2}{2\mu_\beta} - \frac{\boldsymbol{p}_{Ax}^2}{2\mu_{Ax}} - V_{bA} - V_{xA} \right) \tilde{\phi}_\beta^{(+)} = 0$$

Recoil excitation and breakup



$$T_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta,\boldsymbol{k}_\beta)=\left\langle \tilde{\phi}_\beta^{(-)}\middle|V_{bx}\middle|\Psi_\alpha^{(+)}\right\rangle$$

$$\left(T+V_{bA}+V_{xA}-E\right)\tilde{\phi}_\beta^{(+)}=0$$

What does it mean?

- Breakup of B is included in $\tilde{\phi}_\beta^{(+)}$ as its overlap with any continuum state of B is not zero.

- The breakup of B occurs because A inside B interacts with b, gets a recoil and then passes it via V_{Ax} to x.

- The price to pay for getting rid of the remnant term is to include recoil excitation and breakup of B.

Wave function $\tilde{\phi}_{\beta}^{(+)}$ in the adiabatic model:

$$\tilde{\phi}_{\beta}^{(+)} = \chi_{bA}^{(+)}(\boldsymbol{k}_{\beta}, \boldsymbol{r}_{bA})\psi_{Ax}(\boldsymbol{r}_{Ax})e^{-i\mu \boldsymbol{k}_{\beta}\boldsymbol{r}_{Ax}}, \qquad \mu = m_x/(m_x + m_A)$$

In the zero-range approximation, $V_{bx}(\boldsymbol{r}_{bx})\psi_{bx}(\boldsymbol{r}_{bx}) = D_0\delta(\boldsymbol{r}_b - \boldsymbol{r}_x)$

the $A(a,b)B$ reaction amplitude becomes

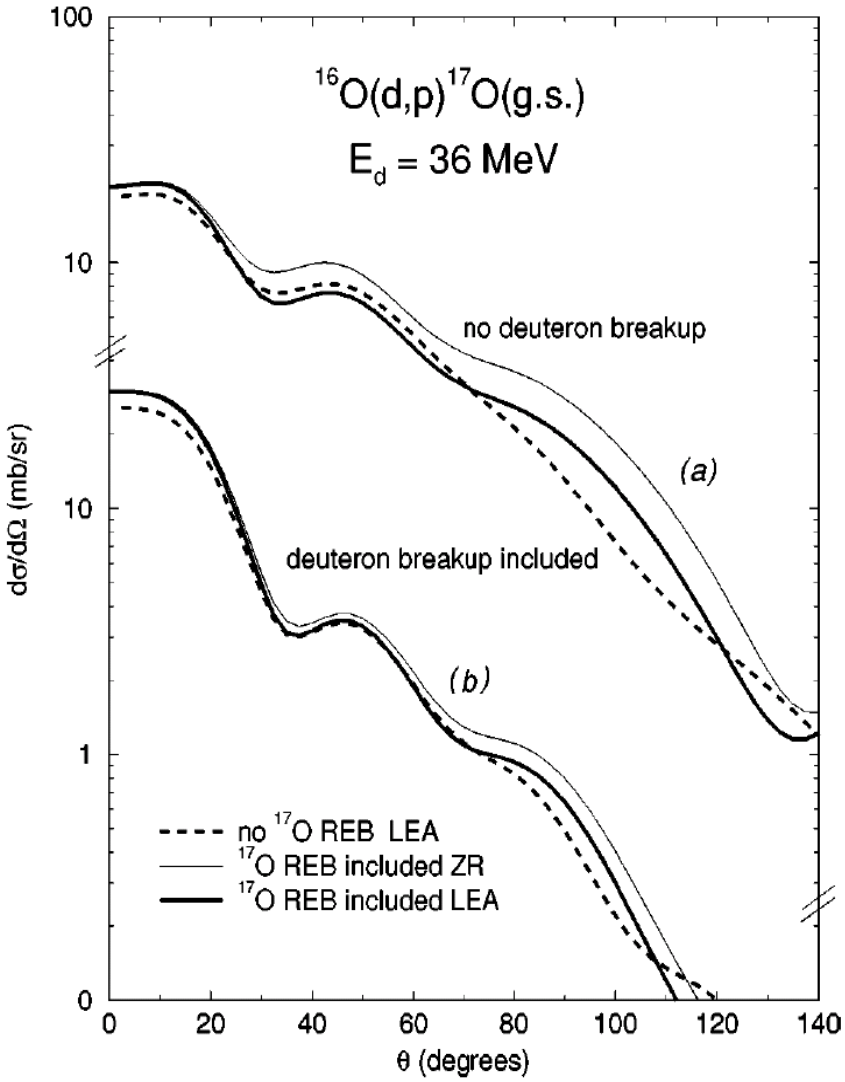
$$T_{ad}^{ZR} = D_0 \int d\boldsymbol{r}\ \chi_{bA}^{(-)*}(\boldsymbol{k}_{\beta}, \boldsymbol{r}) e^{i\mu \boldsymbol{k}_{\beta}\boldsymbol{r}} \psi_{Ax}(\boldsymbol{r}) \chi_{\alpha}^{(+)}(\boldsymbol{k}_{\alpha}, \boldsymbol{r})$$

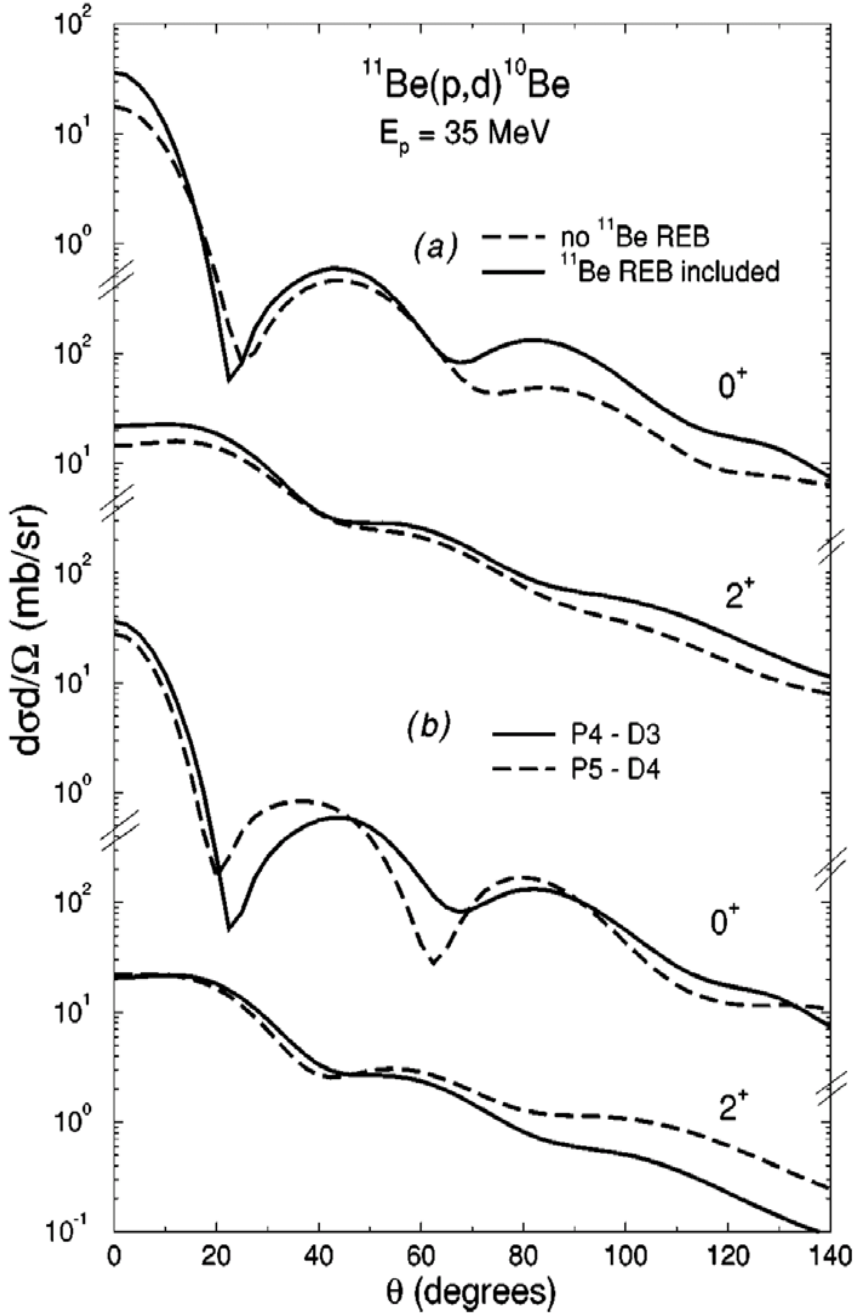
In the conventional approach (that assume that the remnant term is small)

$$T_{standard}^{ZR} = D_0 \int d\boldsymbol{r}\ \chi_{\beta}^{(-)*}(\boldsymbol{k}_{\beta}, \tilde{\mu}\boldsymbol{r}) \psi_{Ax}(\boldsymbol{r}) \chi_{\alpha}^{(+)}(\boldsymbol{k}_{\alpha}, \boldsymbol{r})$$

$$\tilde{\mu} = m_A/(m_x + m_A)$$

N.K. Timofeyuk and R.C.Johnson, Phys. Rev. C 59, 1545 (1999)





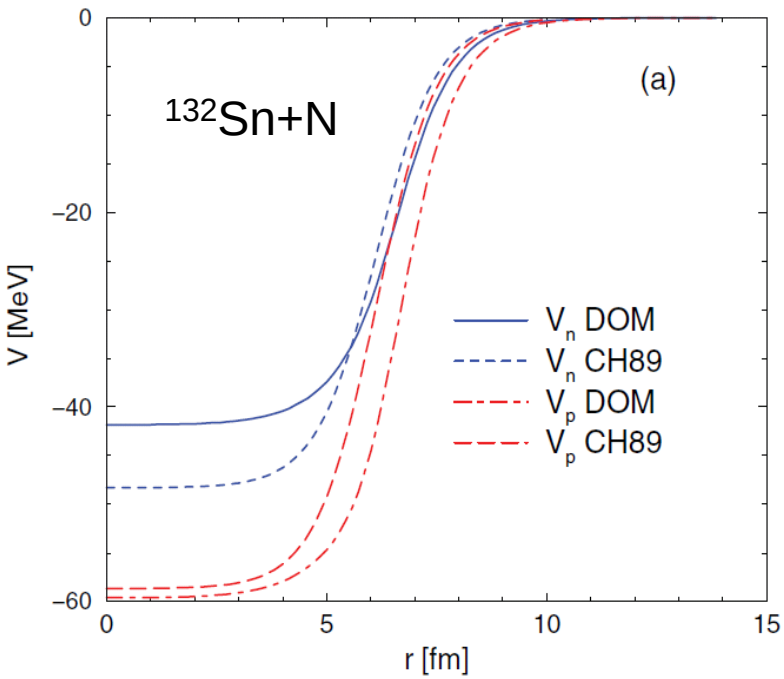
Transfer reactions with dispersive optical potentials

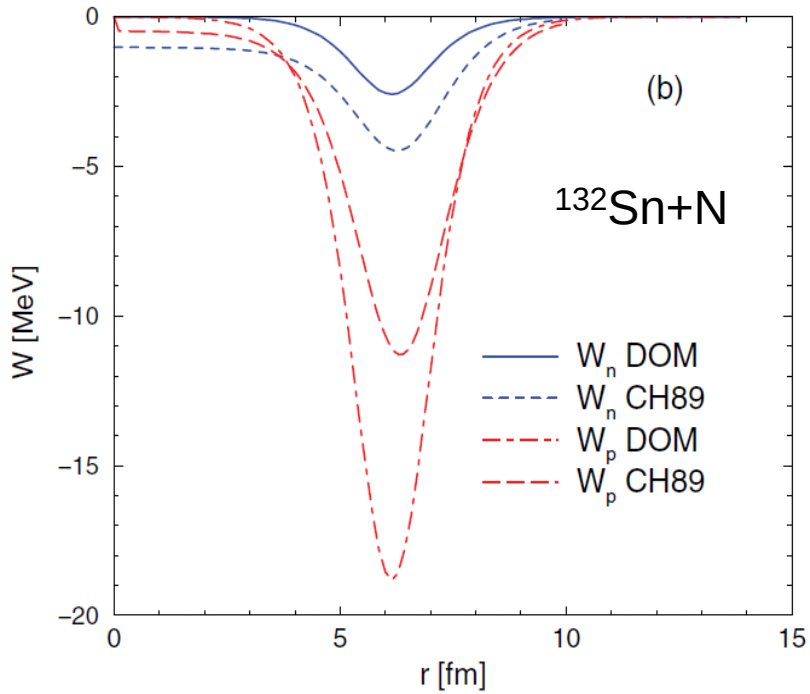
N. B. Nguyen et al, Phys. Rev. C84, 044611 (2011)

$$V_{opt}(\boldsymbol{r},\boldsymbol{r}',E) = V_0(\boldsymbol{r},\boldsymbol{r}') + \Delta V(\boldsymbol{r},\boldsymbol{r}',E) + iW(\boldsymbol{r},\boldsymbol{r}',E)$$

$$\Delta V(\boldsymbol{r},\boldsymbol{r}',E) = \frac{\mathrm{P}}{\pi}\int dE' \, \frac{W(\boldsymbol{r},\boldsymbol{r}',E')}{E-E'}$$

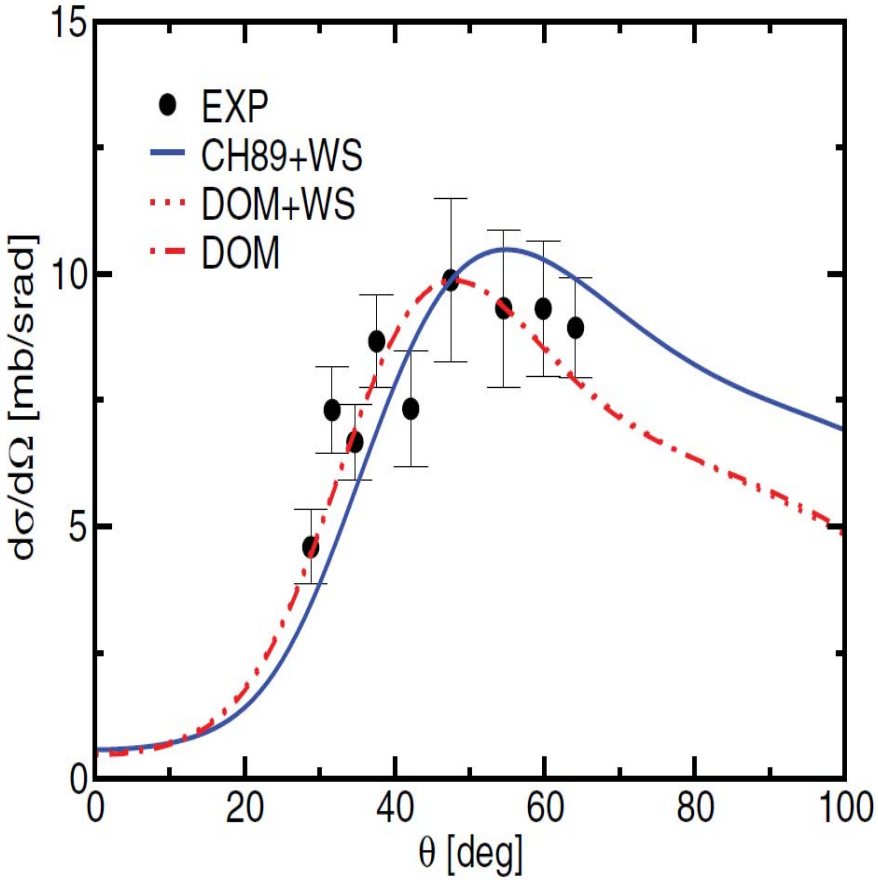
DOM from has been described in terms of 32 parameters used to fit data sets for 40 ≤A ≤ 208 and 4 ≤ E ≤200 MeV (taken from *J.M. Mueller et al, Phys. Rev. C* ***83****, 064605 (2011))*



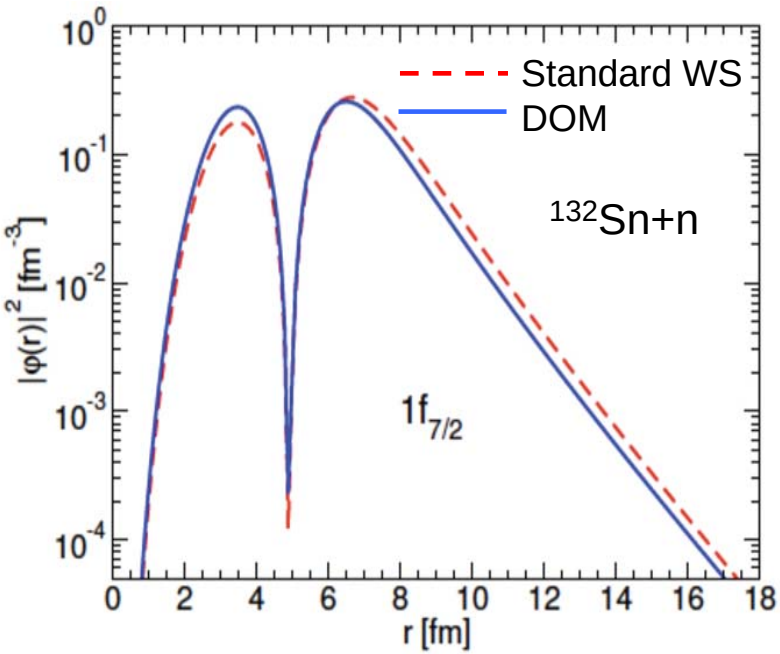


$^{132}Sn(d,p)^{133}Sn$, E_d = 9 .46 MeV

Johnson-Tandy adiabatic model has been used to calculate transfer cross sections, remnant term is neglected.



DOM can also predict potential well for neutron bound state



ANCs obtained from transfer reactions using

- Global systematic of nucleon optical potentials CH89
- DOM

		Woods-Saxon potential used for neutron bound state		DOM used for neutron bound state	
Nucleus	E_d (MeV)	CH89 + WS	DOM + WS	DOM	DOM(th)
^{41}Ca	20	5.0	4.4	4.4	2.8
	56	4.6	3.8	3.8	
^{49}Ca	2	31.7	24.4	24.4	29.6
	13	27.9	22.7	22.6	
	19.3	26.0	23.1	23.0	
	56	35.8	23.5	23.2	
^{133}Sn	9.46	0.78	0.71	0.49	0.56
^{209}Pb	8	4.5	4.1	4.2	2.5
	20	2.4	1.7	1.7	

Spectroscopic factors obtained using

- Global systematic of nucleon optical potentials CH89
- DOM

			Woods-Saxon potential used for neutron bound state		DOM used for neutron bound state	
Nucleus	E_d	Data	CH89 + WS	DOM + WS	DOM	DOM(th)
^{41}Ca	20	[29]	0.96	0.85	0.86	0.75
	56	[30]	0.88	0.73	0.74	
^{49}Ca	2	[31]	0.94	0.72	0.66	0.80
	13	[32]	0.82	0.67	0.61	
	19.3	[32]	0.77	0.68	0.62	
	56	[33]	1.1	0.70	0.62	
^{133}Sn	9.46	[1]	1.1	1.0	0.72	0.80
^{209}Pb	8	[34]	1.7	1.5	1.2	0.76
	20	[35]	0.89	0.61	0.51	

Summary

- Exact amplitude can be written using distorted waves

$$T_{\beta\alpha}(\hat{\boldsymbol{r}}_\beta, \boldsymbol{k}_\beta) = \left\langle \chi_\beta^{(-)} \psi_b \psi_B \middle| V_{bx} + V_{bA} - U_\beta(\boldsymbol{r}_\beta) \middle| \Psi_\alpha^{(+)} \right\rangle$$

- Born approximation for exact wave function can be introduced that selects only one channel of interest

- Transfer reactions probe overlap functions. Many transfer reactions are peripheral, they are sensitive only to the asymptotic part of the overlap integral given by ANCs. If potential well for bound state are reliably determined then spectroscopic factors can be studied in transfer reactions as well.

- In the A(d,p)B reaction deuteron breakup channels play important role. They can approximately be taken into account using Johnson-Soper and Johnson-Tandy approximations. These approximations allow available DWBA codes to be used.

- Remnant term can be exactly excluded from transfer reaction calculations but then recoil excitation and breakup effects in the final nucleus should be taken into account

- Transfer reactions benefit from using optical potentials obtained from DOM. Then ANCs and SFs are less dependent on incident energies.