Methods to describe direct reactions.

I. Optical Model.

N.K. Timofeyuk
University of Surrey, UK

Content

- Elastic scattering. Optical model.
- Phenomenological parameterizations for optical potentials
- Theoretical justification for optical potential
- Folding model
- Microscopic model
- Dispersive optical model

What are direct reactions?

Direct reactions involve changes in motion only of few nucleons.
The rest remains unchanged.

Examples:

Elastic scattering : $A + a \rightarrow A + a$

Inelastic scattering: $A + a \rightarrow A^* + a^*$

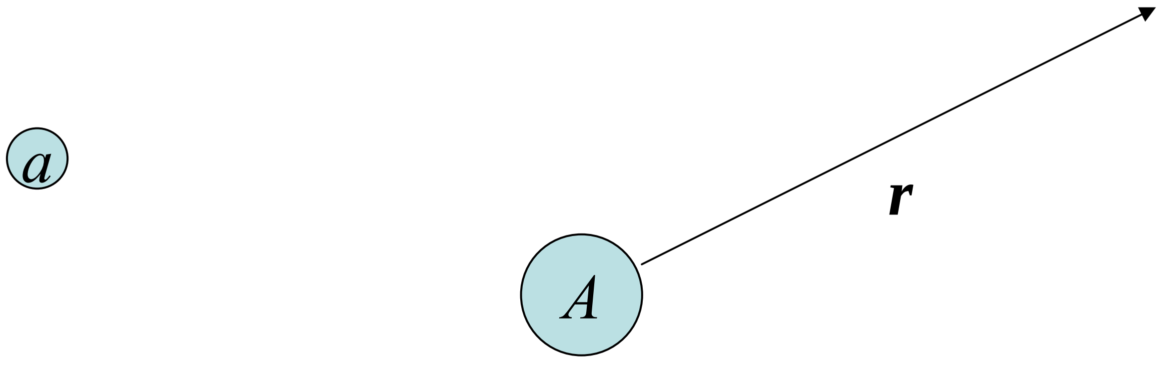
Transfer reactions: $A + a\ (=b+x) \rightarrow B\ (=A+x) + b$

Breakup reactions: $A + a\ (=b+x) \rightarrow A + x + b$

Radiative capture: $A + a \rightarrow B\ (=A+a) + \gamma$

Elastic scattering. Potential model.

$$A + a \rightarrow A + a$$



Only wave function of the relative motion is considered and it is assumed it satisfies the Schrödinger equation

$$\left(-\frac{\hbar^2}{2\mu}\vec{\nabla}^2 + V_{opt}(\boldsymbol{r}) - E\right)\Psi(\boldsymbol{r}) = 0$$

Boundary conditions and amplitude

$$\Psi^{(+)}(\mathbf{r})\Big|_{r\to\infty} \to e^{i\mathbf{k}\mathbf{r}} + f(\mathbf{k},\mathbf{k}')\frac{e^{ikr}}{r}$$

incident wave

amplitude

outgoing wave

$$f(\mathbf{k},\mathbf{k}') = -\frac{\mu}{2\pi\hbar^2}\int d\mathbf{r}\, e^{-i\mathbf{k}'\mathbf{r}} V_{opt}(\mathbf{r})\Psi^{(+)}(\mathbf{r})$$

Cross section:

$$\frac{d\sigma}{d\Omega} = \left|f(\mathbf{k},\mathbf{k}')\right|^2$$

It is assumed that $V_{opt}(r)$ is complex: $V_{opt}(r) = V(r) + i\,W(r)$

What does it mean?

$$\times\Psi^*(\boldsymbol{r},t) \qquad i\hbar\frac{\partial\Psi(\boldsymbol{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2\mu}\vec{\nabla}^2 + V(\boldsymbol{r}) + iW(\boldsymbol{r})\right)\Psi(\boldsymbol{r},t)$$

$$\times\Psi(\boldsymbol{r},t) \qquad -i\hbar\frac{\partial\Psi^*(\boldsymbol{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2\mu}\vec{\nabla}^2 + V(\boldsymbol{r}) - iW(\boldsymbol{r})\right)\Psi^*(\boldsymbol{r},t)$$

$$\frac{\partial\left|\Psi(\boldsymbol{r},t)\right|^2}{\partial t} = -\vec{\nabla}\boldsymbol{j}(\boldsymbol{r},t) + \frac{2W(\boldsymbol{r})}{\hbar}\left|\Psi(\boldsymbol{r},t)\right|^2$$

Change in probability density

flux

W > 0 generation of particle
W < 0 absorption of particles

Typical parameterization of optical potentials

real volume — imaginary volume — imaginary surface

$$V(r) = -V_r f_{ws}(r, R_0, a_0) - \mathrm{i}W_v f_{ws}(r, R_w, a_w) - \mathrm{i}W_s(-4a_w)\frac{\mathrm{d}}{\mathrm{d}r} f_{ws}(r, R_w, a_w)$$

$$-2(V_{so} + \mathrm{i}W_{so})\left(\frac{-1}{r}\frac{\mathrm{d}}{\mathrm{d}r} f_{ws}(r, R_{so}, a_{so})\boldsymbol{l}\cdot\boldsymbol{\sigma}\right)$$ ← spin-orbit

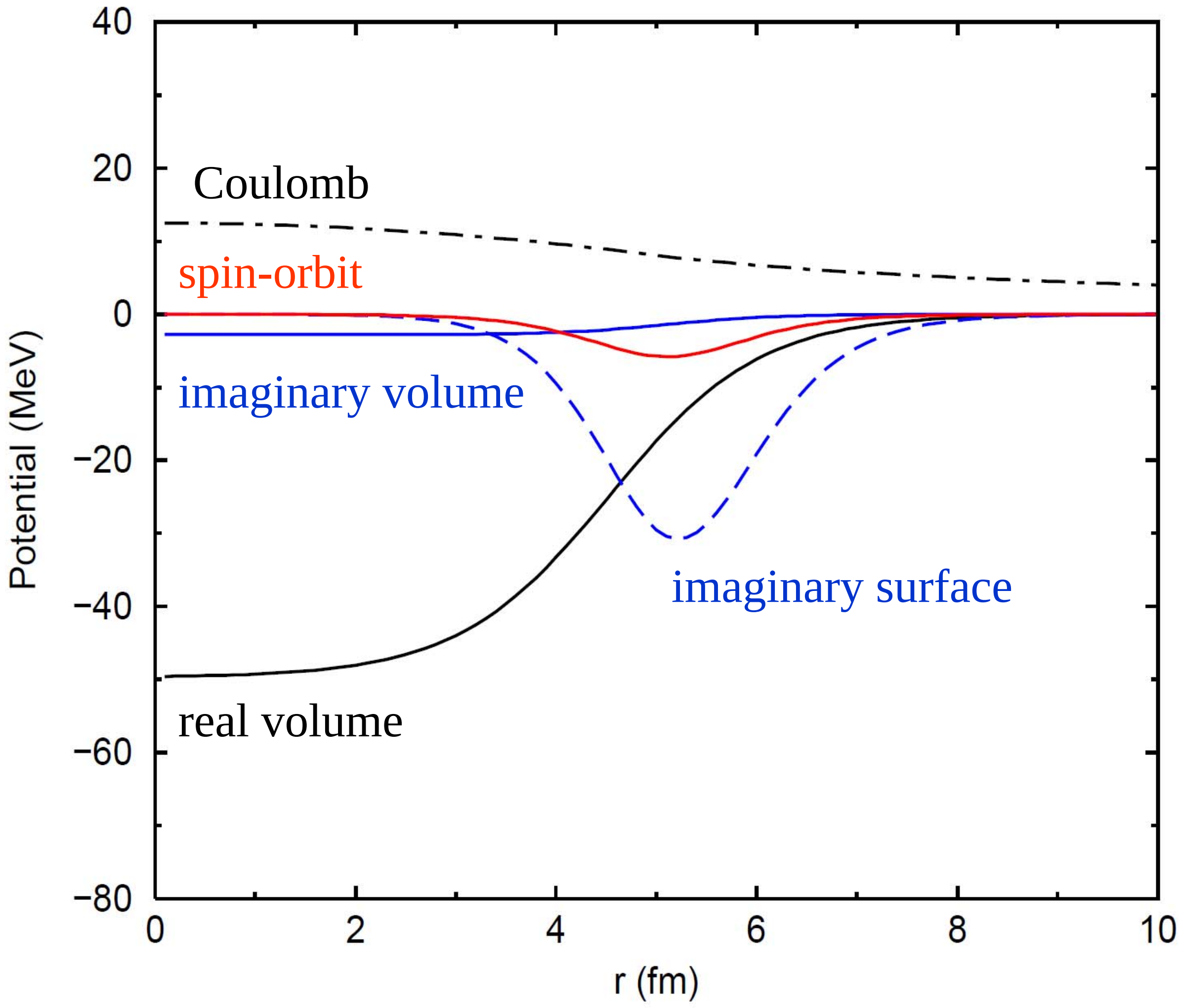
$$+\begin{cases} \dfrac{Ze^2}{r}, & r \ge R_c \\ \dfrac{Ze^2}{2R_c}\left(3 - \dfrac{r^2}{R_c^2}\right), & r \le R_c \end{cases} \quad \text{for incident protons},$$ ← Coulomb

$$+0 \qquad \text{for incident neutrons}.$$

Woods-Saxon form factor

$$f_{ws}(r, R, a) = \frac{1}{1 + \exp[(r - R)/a]}$$

An example of optical potential. p+^{58}Ni at E = 25 MeV



Optical model codes

- MINOPT (R.L. Varner, OPRL)
  - Optical model + minimization
- DWUCK (P.D. Kunz)
  - Elastic scattering + DWBA
- CHUCK (Kunz)
  - Coupled channels code
- SPI-GENOA (F. Perey)
  - Optical model + minimization
- ECIS (J. Raynal, Saclay)
  - Coupled channel code
- FRESCO (I.J.Thompson, LLNL)
  - General coupled reaction channels code

Global optical potentials

Global optical potentials assume that depths, radii and diffusenesses of optical potentials have a simple dependence on incident energy and mass of the target.

Typical assumptions are:

$V = V_0 - V_1 E + \alpha (N\text{-}Z) /A$

$W = W_0 + W_1 E$

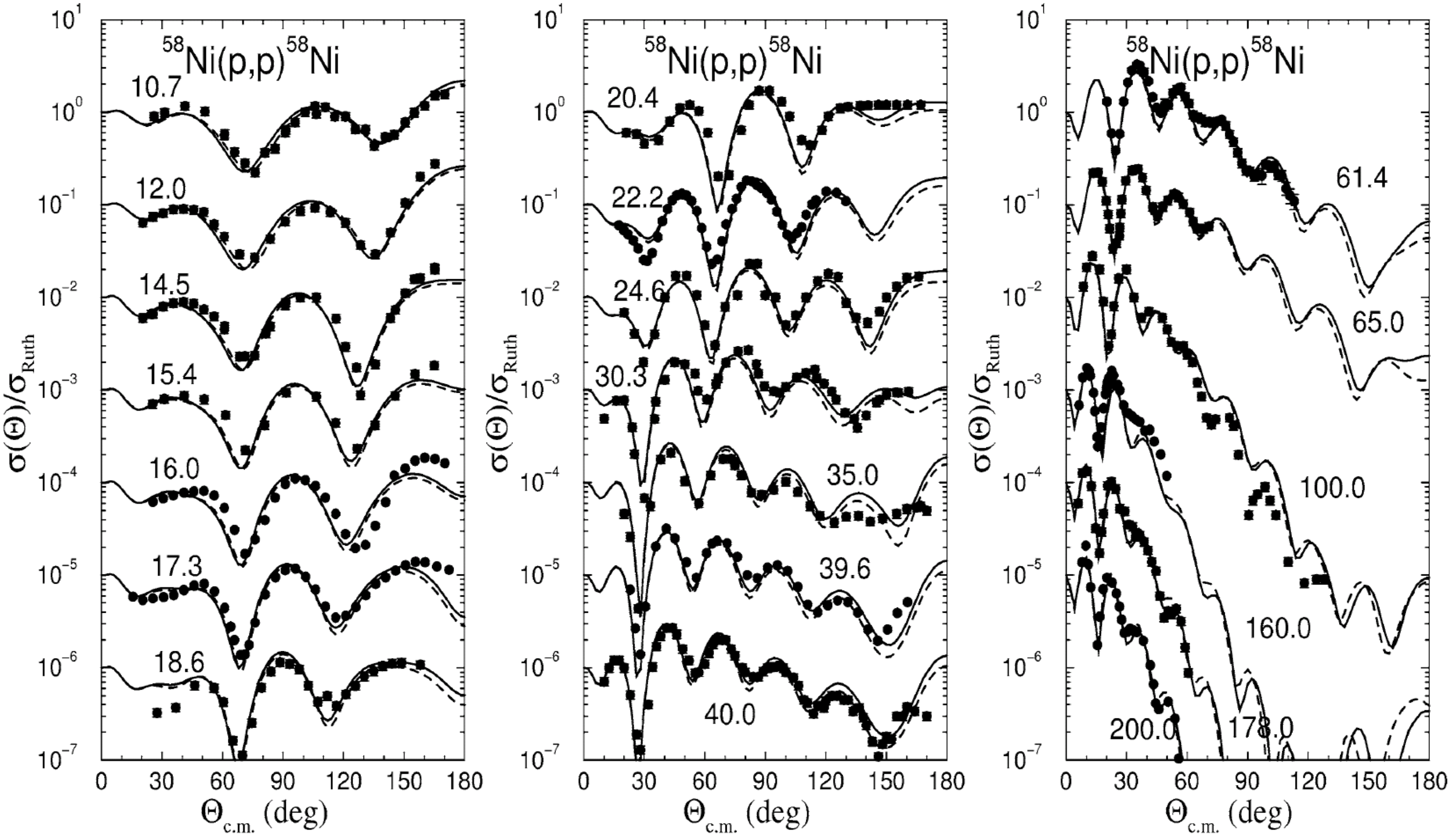
$R = r_0 A^{1/3}$ (r_0 *can have mild energy dependence)*

Global nucleon-nucleus optical potentials

Systematics	Mass range	Energy range
Becchetti-Greenless *Phys. Rev 182, 1190 (1969)*	$A \geq 40$	$E \leq 40$ MeV
Watson-Singh-Segel *Phys. Rev 182, 977 (1969)*	$6 \leq A \leq 16$	$10 \leq E \leq 50$ MeV
CH89 *Phys. Rep. 201, 57 (1989)*	$40 \leq A \leq 209$	$10 \leq E_p \leq 65$ MeV $10 \leq E_n \leq 26$ MeV
KD02 *Nucl. Phys. A 713, 231 (2003)*	$24 \leq A \leq 209$	1 keV $\leq E \leq$ 200 MeV
WP *Phys. Rev C 80, 034608 (2009)*	$12 \leq A \leq 60$ *(aims on studies with radioactive beams)*	$30 \leq E \leq 160$ MeV

Examples for proton elastic scattering calculated with KD02 global potential

(taken from *A.J. Koning, J.P. Delaroche, Nucl. Phys. A 713, 231 (2003))*

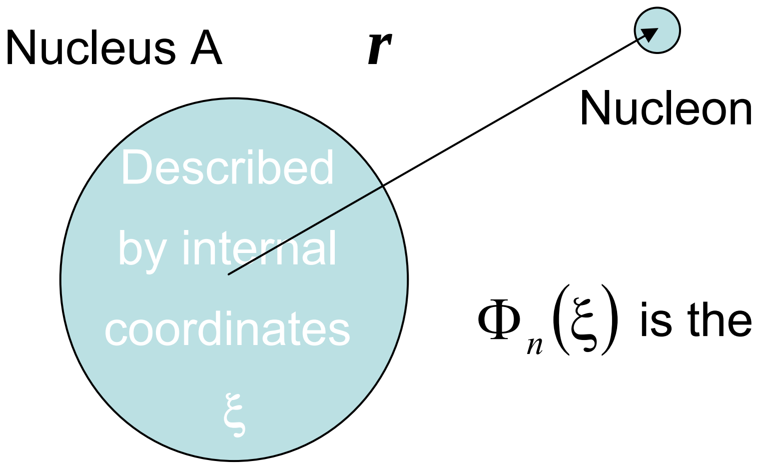


Other global optical potentials

Systematics	Mass range	Energy range
Deuteron + nucleus	$27 \leq A \leq 238$	$11.8 \leq E \leq 90$ MeV
Daehnick et al, Phys. Rev C21, 2253 (1980)		
Deuteron + nucleus	$12 \leq A \leq 238$	$11.8 \leq E \leq 200$ MeV
H.An and C. Cai, Phys. Rev C73, 054605 (2006)		
^{3}He + nucleus	$10 \leq A \leq 208$	$10 \leq E \leq 220$ MeV
H.J. Trost et al, Nucl. Phys. A462, 333 (1987)		
^{3}He + nucleus	$40 \leq A \leq 209$	$30 \leq E \leq 217$ MeV
D.Y.Pang et al, Phys. Rev. C79, 024615 (2006)		
^{4}He + nucleus	$12 \leq A \leq 208$	Coulomb barrier $\leq E \leq 140$ MeV
A.Kumar et al, Nucl. Phys. A776, 105 (2006)		

Theoretical justification of the optical model

Coupled channel approach



Total wave function $\Psi(\xi, \boldsymbol{r})$

$$\Psi(\xi, \boldsymbol{r}) = \sum_n u_n(\boldsymbol{r})\, \Phi_n(\xi)$$

$\Phi_n(\xi)$ is the wave function of the *n*-th excited state in A

$\Psi(\xi, \boldsymbol{r})$ satisfies the Schrodinger equation

$$\left(\hat{T} + V(\xi, \boldsymbol{r}) - E\right) \Psi(\xi, \boldsymbol{r}) = 0$$

Coupled-channel set of differential equations:

$$\left(\hat{h}_n - E\right)u_n(\boldsymbol{r}) = -\sum_{m\neq n} V_{nm}(\boldsymbol{r})\,u_m(\boldsymbol{r})$$

$$\hat{h}_n = \frac{\hat{p}^2}{2m} + \varepsilon_n + V_{nn}(\boldsymbol{r})$$

$$V_{nm}(\boldsymbol{r}) = V^*_{mn}(\boldsymbol{r}) \equiv \left\langle n \middle| \hat{V} \middle| m \right\rangle = \int \mathrm{d}\xi\; \Phi^*_n(\xi)\,\hat{V}(\xi,\boldsymbol{r})\,\Phi_m(\xi)$$

Asymptotic conditions:

$$u_n(\boldsymbol{r})\Big|_{r\to\infty} \to \begin{cases} e^{i\boldsymbol{kr}} + \text{ outgoing wave, if } n = 1 \\ \text{outgoing wave, if } n > 1 \text{ and } E > \varepsilon_n \\ \text{decaying wave, if } n > 1 \text{ and } E < \varepsilon_n \end{cases}$$

Let us keep the elastic channel w.f. $u_1(\boldsymbol{r})$ and combine all the other channel wave functions $u_i(\boldsymbol{r})$ into an object

$$\mathbf{\Phi} \equiv \begin{pmatrix} u_2(\boldsymbol{r}) \\ u_3(\boldsymbol{r}) \\ .... \\ .... \end{pmatrix}$$

Then let us introduce a row $\hat{V} \equiv (V_{12}(\boldsymbol{r}), V_{13}(\boldsymbol{r}), ...)$,

a column $\hat{V}^+ \equiv \begin{pmatrix} V_{21}(\boldsymbol{r}) \\ V_{31}(\boldsymbol{r}) \\ .... \\ .... \end{pmatrix}$ and a matrix $\hat{h} \equiv \begin{pmatrix} \hat{h}_{11} & \hat{h}_{12} & ... & ... \\ \hat{h}_{21} & \hat{h}_{22} & ... & ... \\ ... & ... & ... & ... \\ ... & ... & ... & ... \end{pmatrix}$

where $\hat{h}_{nm} \equiv \hat{h}_n \delta_{nm} + V_{nm}(\boldsymbol{r})$

Then the coupled system of differential equations reads:

$$\left(\hat{h}_1 - E\right)u_1 = -\hat{V}\,\boldsymbol{\Phi},$$

$$\left(\hat{h} - E\right)\boldsymbol{\Phi} = -\hat{V}^+ u_1$$

The formal solution of the second equation is $\boldsymbol{\Phi} = \left(E + i\varepsilon - \hat{h}\right)^{-1} \hat{V}^+ u_1$

Substituting it into the first equation we get

$$\left(\hat{h}_1 - E\right)u_1 = -\hat{V}\,\frac{1}{E + i\varepsilon - \hat{h}}\,\hat{V}^+ u_1$$

or $\left(\dfrac{\vec{p}^{\,2}}{2m} + V_{\text{eff}} - E\right)u_1 = 0$

where $V_{\text{eff}} = V_{11} + \hat{V}\,\dfrac{1}{E + i\varepsilon - \hat{h}}\,\hat{V}^+$

H. Feshbach, Ann. Phys. 5, 357 (1958)

Properties of the effective (or optical) potential $V_{\mathrm{eff}} \equiv V_{\mathrm{opt}}$

$$V_{\mathrm{opt}} = \langle \Phi_1 | V | \Phi_1 \rangle + \hat{V} \frac{1}{E + i\varepsilon - \hat{h}} \hat{V}^{+}$$

- ❑ Contains folding potential $\langle \Phi_1 | V | \Phi_1 \rangle$

- ❑ Non-local (as $\hat{h}$ contains kinetic energy and $\Rightarrow$ includes differential operators)

$$V_{\mathrm{opt}}\, u_1(\boldsymbol{r}) = \int d\boldsymbol{r}'\, V_{\mathrm{opt}}(\boldsymbol{r}, \boldsymbol{r}') u_1(\boldsymbol{r}')$$

- ❑ Non-Hermitian, $\langle V u(\boldsymbol{r}) | u(\boldsymbol{r}) \rangle \neq \langle u(\boldsymbol{r}) | V | u(\boldsymbol{r}) \rangle$, V_{opt} contains real and imaginary part because

$$\lim_{\varepsilon \to 0} \frac{1}{E + i\varepsilon - \hat{h}} = \mathrm{P}\, \frac{1}{E - \hat{h}} - i\pi\delta(E - \hat{h})$$

- ❑ Energy dependent

A non-local potential model and equivalent local potential model

F. Perey and B.Buck, Nucl. Phys. ***32****, 353 (1962)*

Non-local model:

$$\left(\frac{\hbar^2}{2\mu}\vec{\nabla}^2 + E\right)\Psi_N(\boldsymbol{r}) = \int d\boldsymbol{r}'\, V(\boldsymbol{r},\boldsymbol{r}')\Psi_N(\boldsymbol{r}')$$

Assume that

$$V(\boldsymbol{r},\boldsymbol{r}') = U_N\left(\frac{|\boldsymbol{r}+\boldsymbol{r}'|}{2}\right)\frac{\exp\left(-(\boldsymbol{r}-\boldsymbol{r}')^2/\beta^2\right)}{\pi^{3/2}\beta^3}$$

Local model

$$\left(\frac{\hbar^2}{2\mu}\vec{\nabla}^2 + E\right)\Psi_L(\boldsymbol{r}) = U_L(\boldsymbol{r})\Psi_L(\boldsymbol{r})$$

When

$$U_L(r)\exp\left[\frac{\mu\beta^2}{2\hbar^2}\left(E - U_L(r)\right)\right] = U_N(r)$$

Then $\Psi_N \approx \Psi_L$

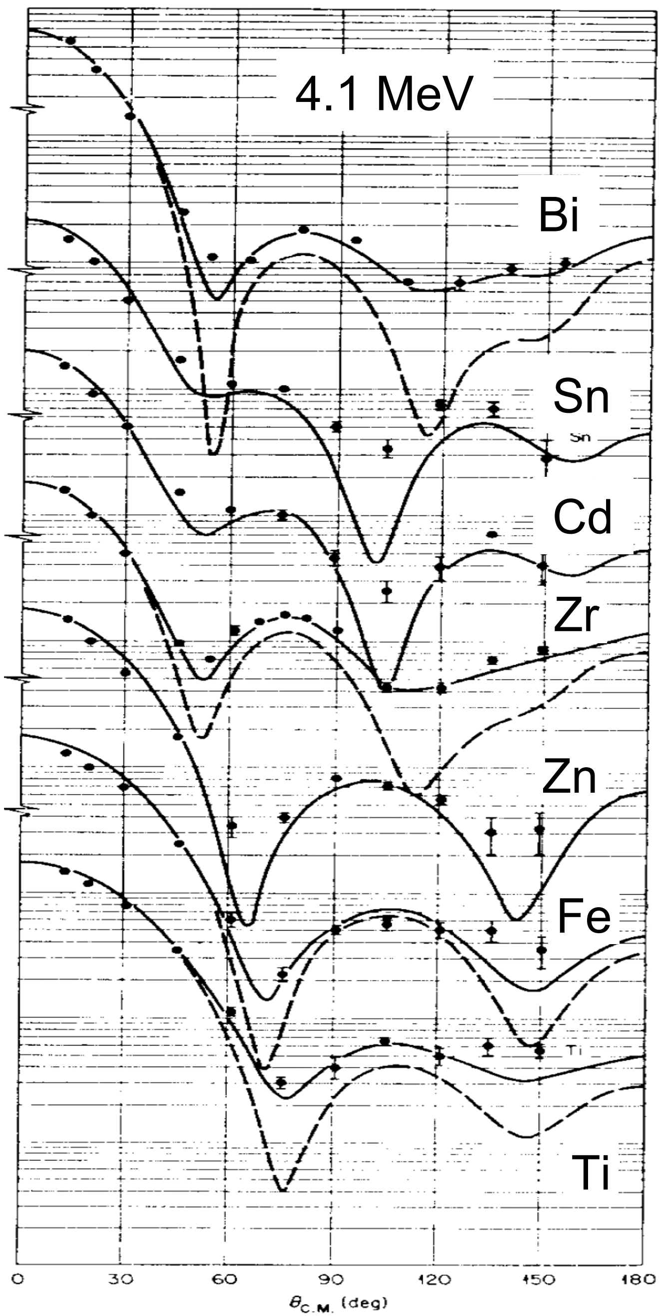
$$-U(p) = [V+iW_I]f_S(p)+iW_D f_D(p),$$

$$f_S(p) = \left[1+\exp\left(\frac{p-R}{a_S}\right)\right]^{-1}, \quad R = r_0 A^{\frac{1}{3}},$$

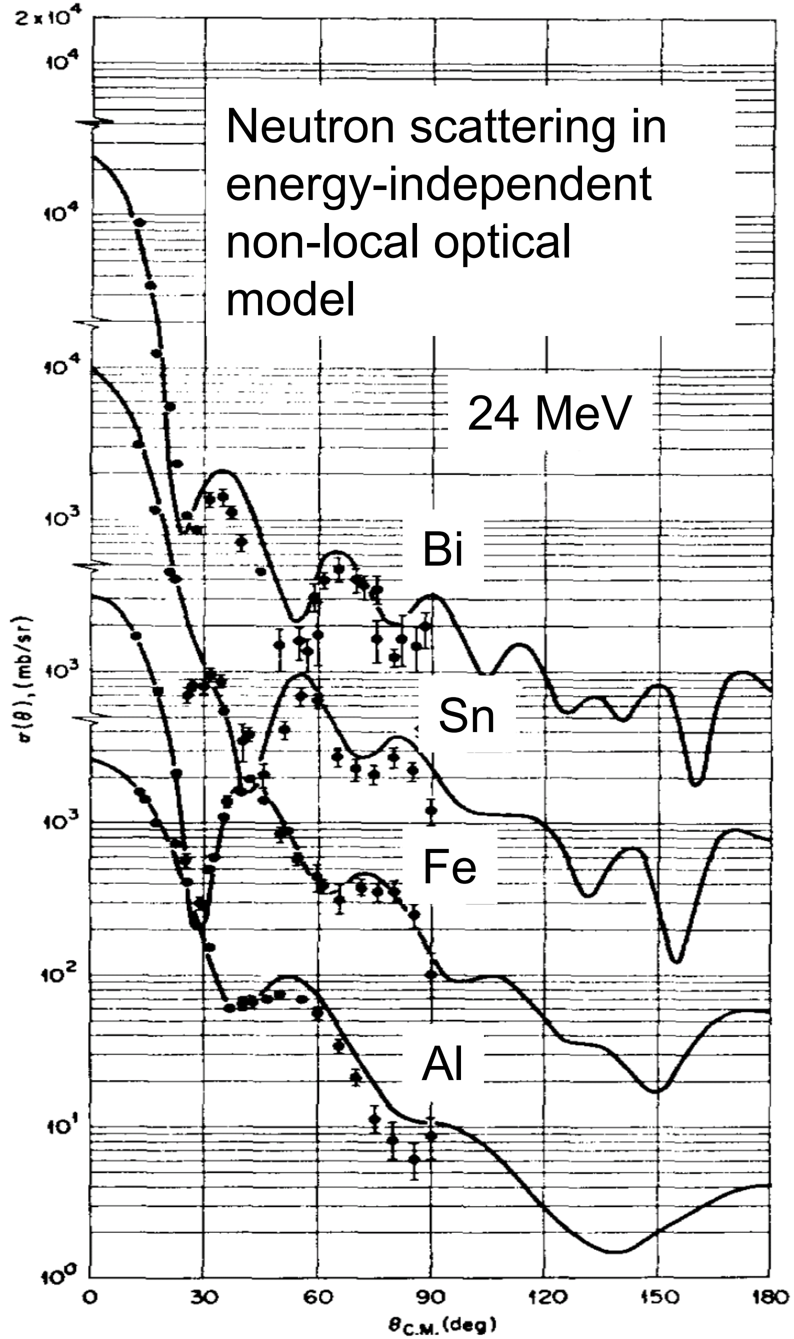
$$f_D(p) = 4\exp\left(\frac{p-R}{a_D}\right)\left[1+\exp\left(\frac{p-R}{a_D}\right)\right]^{-2}$$

Optical parameters for *n+Al,Fe,Cu,Zn,Zr,Cd,Sn,Ta,Pb,Bi* for $4 \le E \le 26$

		Non-local	Equivalent local parameters			
E	(MeV)	All energies	4.1	7.0	14	26
V	(MeV)	70.00	41.35	40.31	38.00	34.80
r_0	(fm)	1.25	1.32	1.32	1.32	1.31
a_S	(fm)	0.65	0.62	0.62	0.62	0.62
W_D	(MeV)	7.00	3.95	3.94	3.35	3.34
a_D	(fm)	0.65	0.65	0.65	0.65	0.65
β	(fm)	1.00				

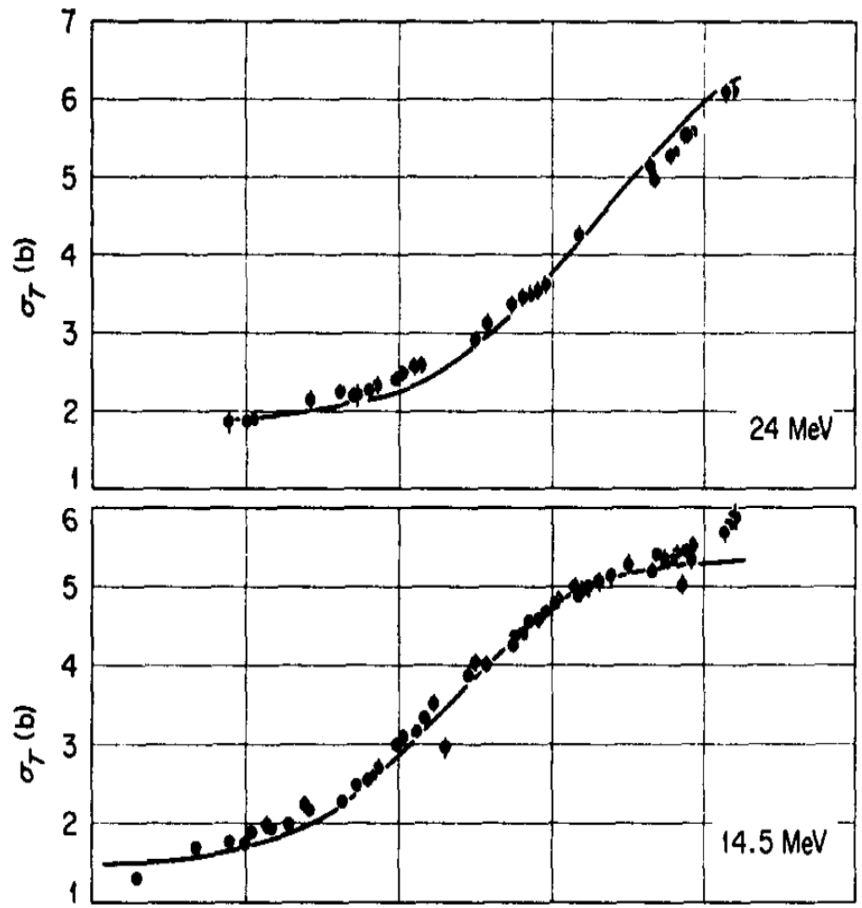


4.1 MeV
Bi
Sn
Cd
Zr
Zn
Fe
Ti
0 30 60 90 120 150 180
θC.M. (deg)

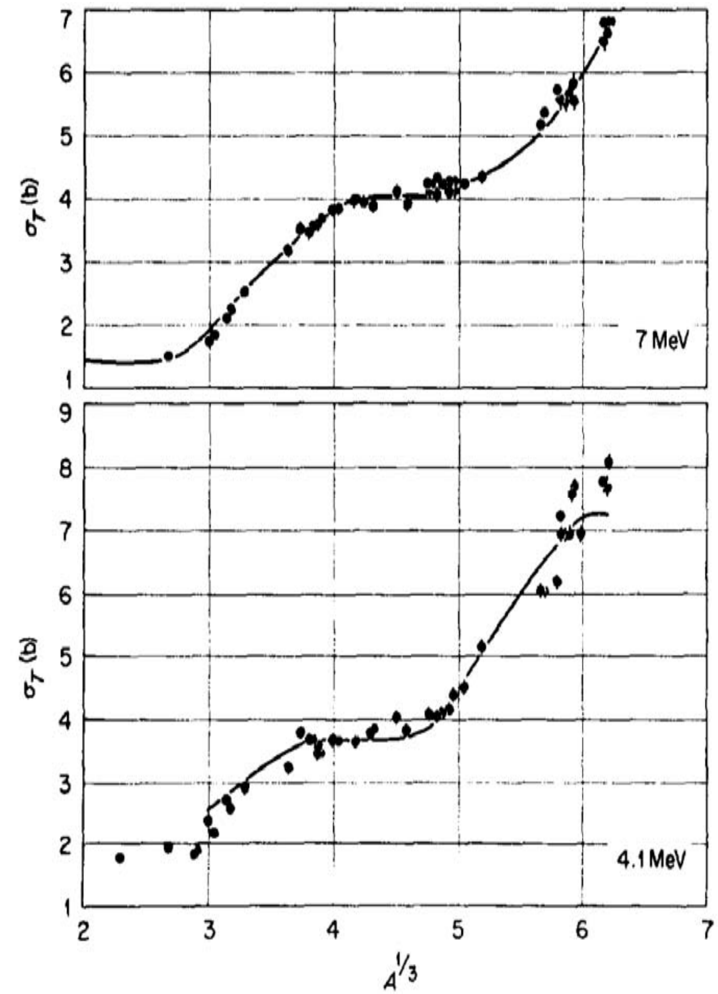


Neutron scattering in energy-independent non-local optical model
24 MeV
Bi
Sn
Fe
Al
σ(θ), (mb/sr)
0 30 60 90 120 150 180
θC.M. (deg)

Total neutron cross sections calculated in energy-independent non-local optical model



$A^{1/3}$



Folding model

The optical potential $V_{\text{opt}} = \langle \Phi_1 | V | \Phi_1 \rangle + \hat{V} \dfrac{1}{E + i\varepsilon - \hat{h}} \hat{V}^{+}$

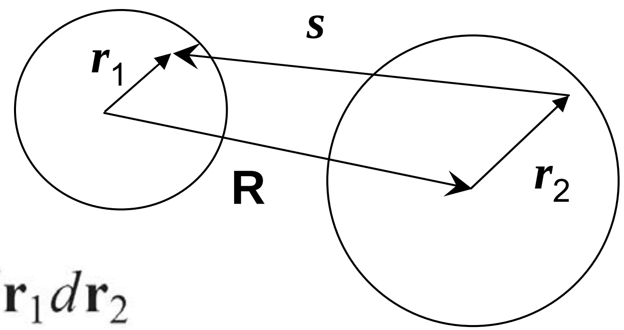
contains a simple term

$$\left\langle \Phi_1 \middle| \hat{V} \middle| \Phi_1 \right\rangle = \int \mathrm{d}\xi \, \Phi_n^*(\xi) \hat{V}(\xi, \boldsymbol{r}) \Phi_m(\xi)$$

which can be evaluated if nuclear densities and NN interactions are known.

Analysis in the folding model assumes that the real part of the optical potential is proportional to the folding potential so that only imaginary part is fitted.

Folding potential for $a + A$



$$V(R) = \lambda V_F(R)$$

$$= \lambda \int \int \rho_a(\mathbf{r}_1)\rho_A(\mathbf{r}_2)\, v_{\text{eff}}(E, \rho_a, \rho_A, s)\, d\mathbf{r}_1 d\mathbf{r}_2$$

$$s = |\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1|$$

Densities ρ_a and ρ_A can be derived from

- measured charge distributions (for stable nuclei)
- model calculations (e.g. Hartree-Fock, shell model *etc*)

λ can be obtained from

- fitting to angular distributions of elastic scattering
- fitting to bound states energies (if applied for neutron capture)
- fitting to thermal total cross sections (for neutrons)
- fitting to resonance energies (if applied for neutron capture)

DDM3Y (density dependent) NN effective potential

A.M.Kobos, B.A.Brown, R.Lindsay, G.R.Satchler, Nucl.Phys. A425, 205 (1984)

$$v_{eff}(E,\rho_a,\rho_A, s) = v_{M3Y}(E,s)\ f(E,\rho_a+\rho_A)$$

$$v_{M3Y}(E,s) = 7999\ \exp(-4s)/4s - 2134\ \exp(-2.5s)/2.5s + J_{00}(E)\ \delta(s)$$

Exchange part

$$J_{00}(E) = -276\ (1 - 0.005E/A_a)\ (\mathrm{MeV\cdot fm^3})$$

Density dependent part:

$$f(E,\rho) = C(E)(1+\alpha(E)e^{-\beta(E)\rho})$$

Coefficients $C(E)$, $\alpha(E)$ and $\beta(E)$ are determined by fitting volume integral of $v_{eff}(E,\rho_a,\rho_A, s)$ to the strength of the real part of a G-matrix effective interaction obtained from Bruekner-Hartree-Fock calculations for nuclear matter of various densities and at various energies.

Microscopic nucleon-nucleus optical potential. JLM model.

J.P. Jeukenne, A. Lejeune and C. Mahaux, Phys. Rev. C10, 1391 (1974)
J.P. Jeukenne, A. Lejeune and C. Mahaux, Phys. Rev. C16, 80 (1977)
E.Bauge, J.P.Delaroche, M.Girod, Phys.Rev. C58, 1118 (1998)
E.Bauge, J.P.Delaroche, M.Girod, Phys.Rev. C63, 024607 (2001)

- Optical nucleus-nucleus potential $U_{NM}(\rho,E)$ for infinite nuclear matter is derived in the Bruekner-Hartree-Fock approximation (as a function of density ρ).

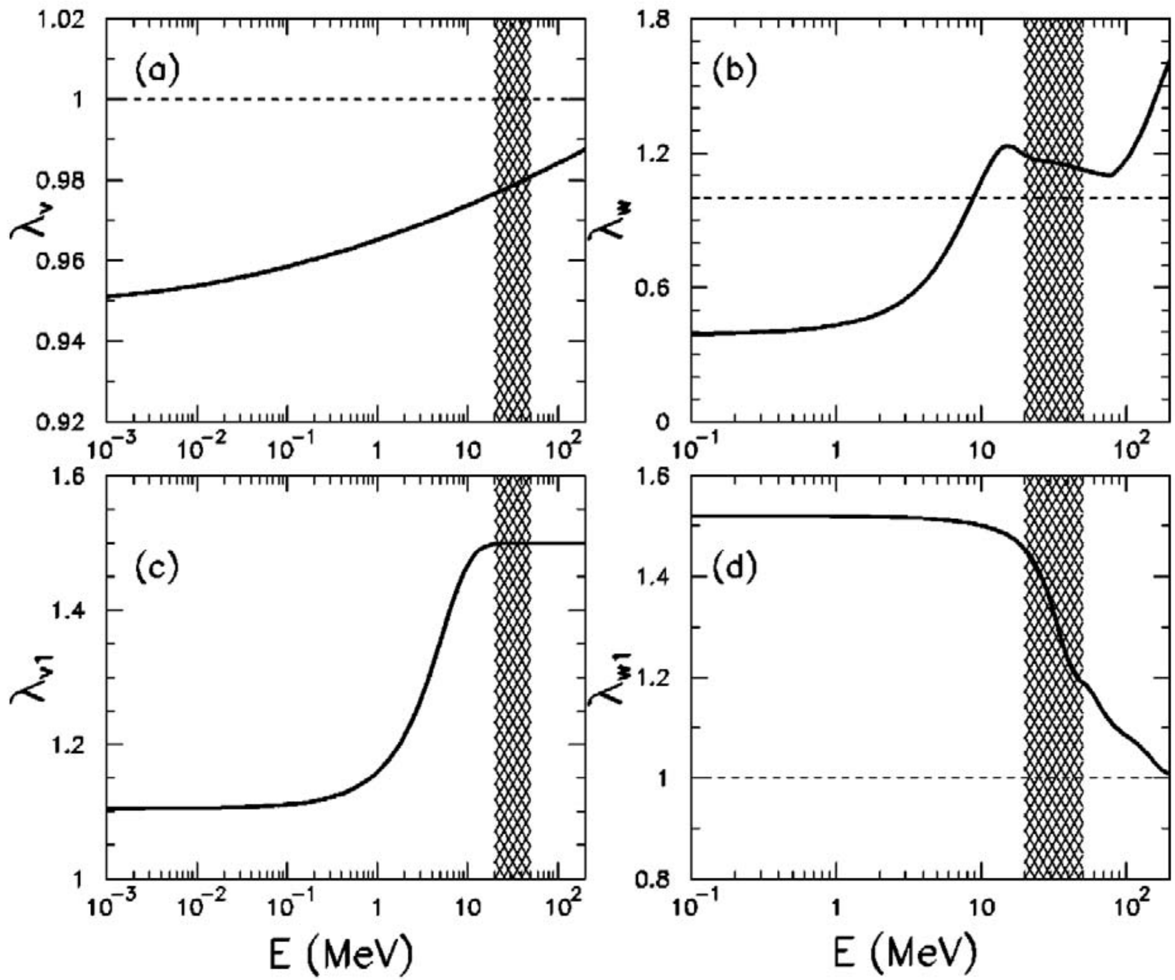
- $U_{NM}(\rho,E)$ is parameterised in terms of densities and energies

$$U_{NM}(\rho, E) \sim \sum_{i,j} C_{ij}\rho^i E^{j-1}$$

- For finite nuclei, local optical nucleon-nucleus potential $V_{\rm opt}(r,E)$ is related to $U_{NM}(\rho(r),E)$

$$U_{NM}(\rho,E)=\lambda_V(E)[V_0(\rho,E-E_C)\pm\lambda_{V_1}(E)\frac{\rho_n-\rho_p}{\rho}V_1(\rho,E-E_C)]$$

$$+i\lambda_W(E)[W_0(\rho,E-E_C)\pm\lambda_{W_1}(E)\frac{\rho_n-\rho_p}{\rho}W_1(\rho,E-E_C)]$$



E.Bauge, J.P.Delaroche, M.Girod, Phys.Rev. C63, 024607 (2001)

Spin-orbit optical potential

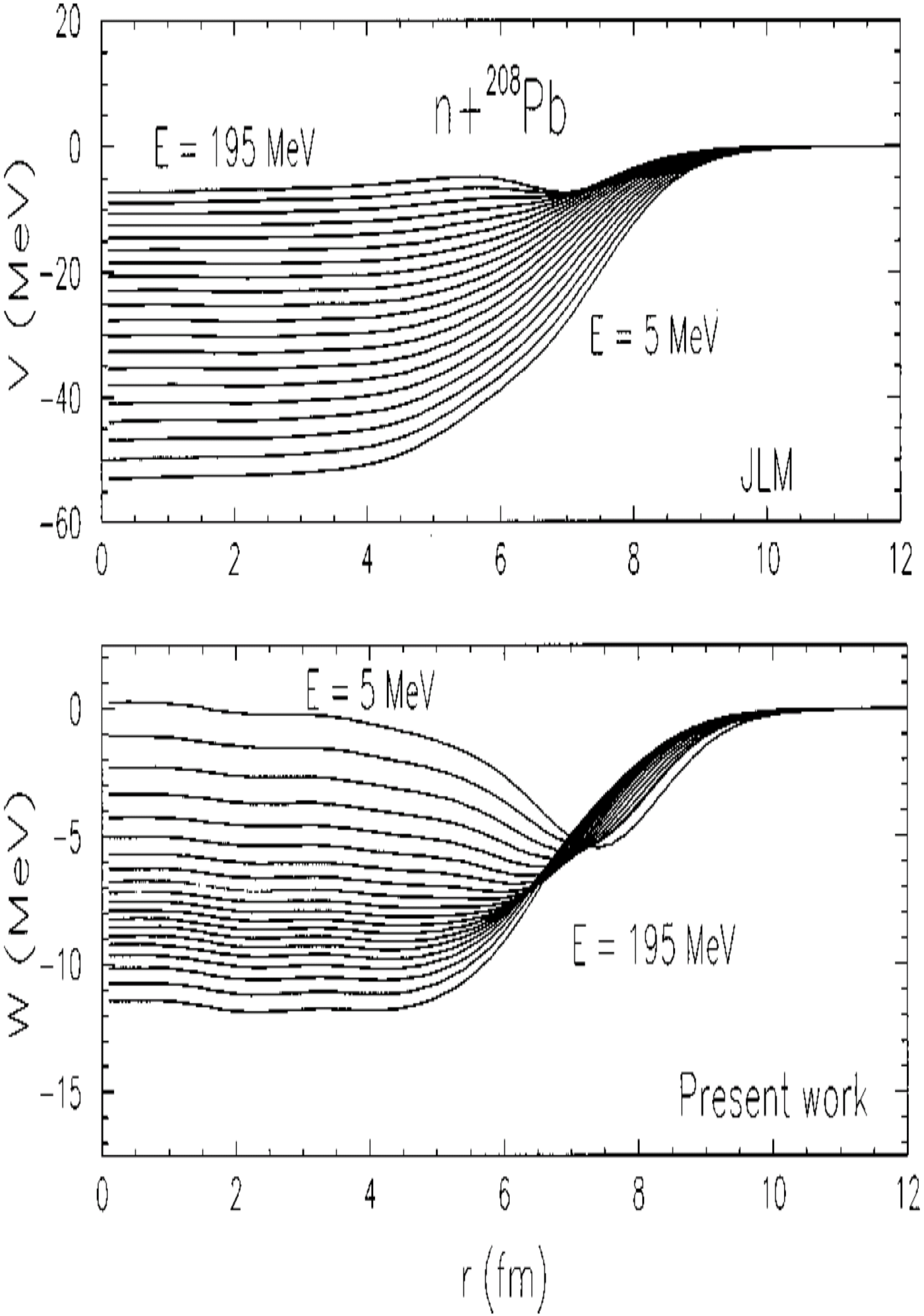
E.Bauge, J.P.Delaroche, M.Girod, Phys.Rev. C58, 1118 (1998)

$$\frac{\hbar^2}{2m^2c^2}\vec{\ell}\cdot\vec{\sigma}[\lambda_{v_{\rm so}}V_{\rm so}(r)+i\lambda_{w_{\rm so}}W_{\rm so}(r)]$$

$$V^{\rm so}_{n(p)}(r)=\lambda_{v_{\rm so}}U^{\rm so}_{n(p)}(r),$$

$$W^{\rm so}_{n(p)}(r)=\lambda_{w_{\rm so}}U^{\rm so}_{n(p)}(r)$$

$$U^{\rm so}_{n(p)}(r)=\frac{1}{r}\frac{d}{dr}\left(\frac{2}{3}\rho_{p(n)}+\frac{1}{3}\rho_{n(p)}\right)$$



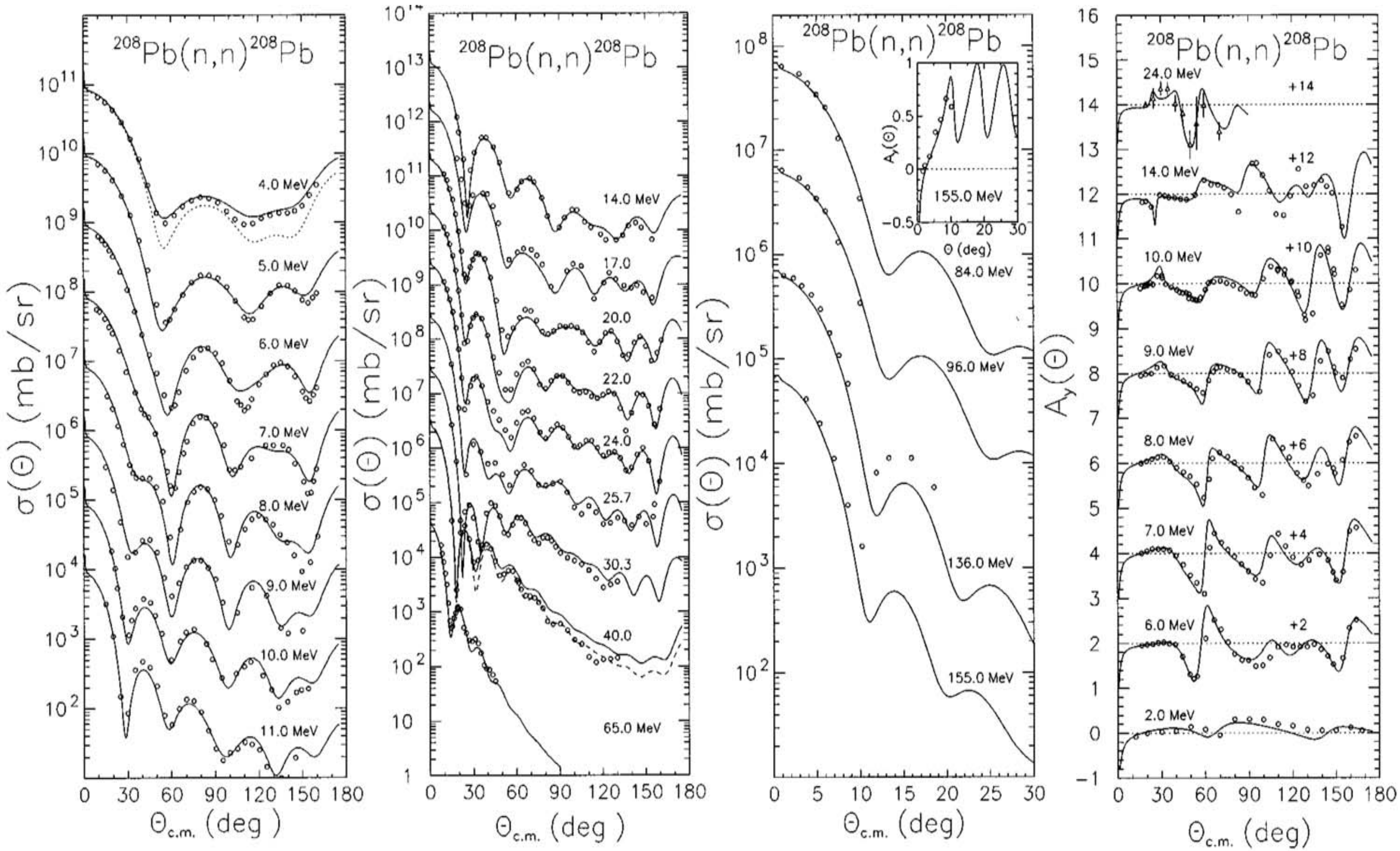
Example: optical JLM potential for *n*+^{208}*Pb* calculated in *E.Bauge, J.P.Delaroche, M.Girod, Phys.Rev. C58, 1118 (1998)*

← Real part

← Imaginary part

Example: $n+^{208}Pb$ *for* $4 \leq E \leq 155$ *MeV* calculated with optical potential from

E.Bauge, J.P.Delaroche, M.Girod, Phys.Rev. C58, 1118 (1998)

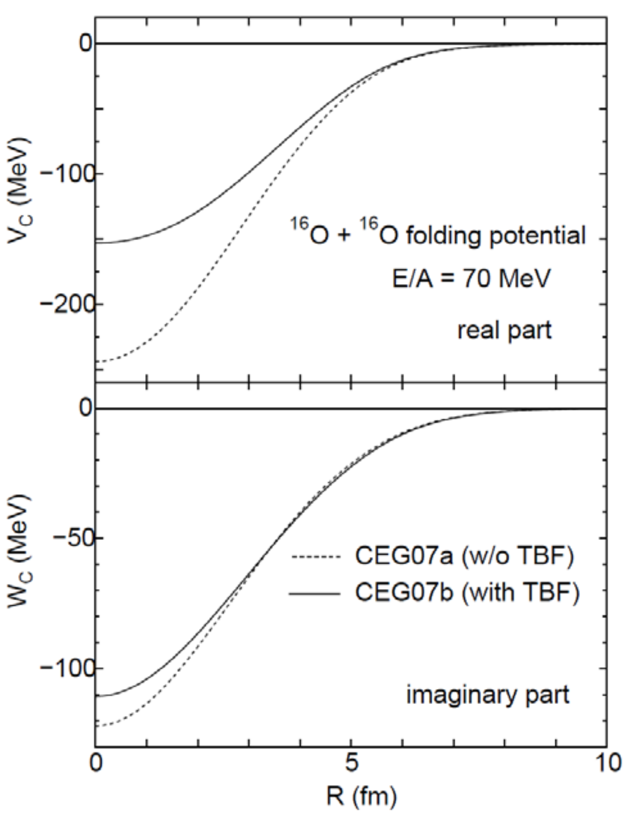


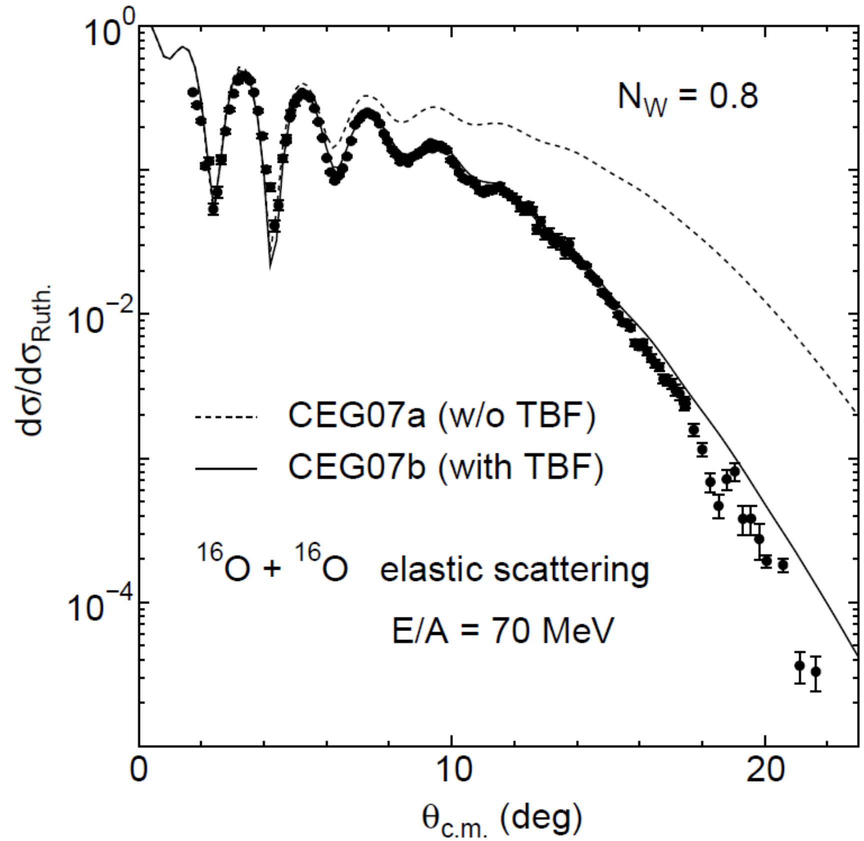
Role of three-body force in elastic scattering

T. Furumoto, Y. Sakuragi, Y. Yamamoto, Phys.Rev. C 82, 044612 (2010)

Two-body interaction is not sufficient to describe many-body systems. Three-body force is important in many-body structure calculations.

First calculations of elastic scattering with three-body force. Complex NN interaction is used. ⇒ the folding potential is complex.





Dispersion relations for optical potentials

Optical potential has a general representation:

$$V_{opt}(\boldsymbol{r},\boldsymbol{r}',E)=V_0(\boldsymbol{r},\boldsymbol{r}')+\Delta V(\boldsymbol{r},\boldsymbol{r}',E)+iW(\boldsymbol{r},\boldsymbol{r}',E)$$

The energy-dependent part $\Delta V(\boldsymbol{r},\boldsymbol{r}',E)$ is related to $W(\boldsymbol{r},\boldsymbol{r}',E)$

$$\Delta V(\boldsymbol{r},\boldsymbol{r}',E)=\frac{\mathrm{P}}{\pi}\int dE' \; \frac{W(\boldsymbol{r},\boldsymbol{r}',E')}{E-E'}$$

This relation is a consequence of

- Causality requirement that a scattered wave cannot be emitted before the arrival of the incident wave

- Only outgoing waves are present in the non-elastic channels.

C. Mahaux and R. Sartor, Adv. Nucl. Phys. 20, 1 (1991)
C. Mahaux, H. Ngo, G.R. Satchler, Nucl. Phys. A449, 354 (1986)

Causality principle and threshold anomaly of nucleus-nucleus potential

C. Mahaux, H. Ngo, G.R. Satchler, Nucl. Phys. A449, 354 (1986)

Dispersion relation are extendable for volume integrals of $\Delta V(\mathbf{r},\mathbf{r}',E)$ and $W(\mathbf{r},\mathbf{r}',E)$

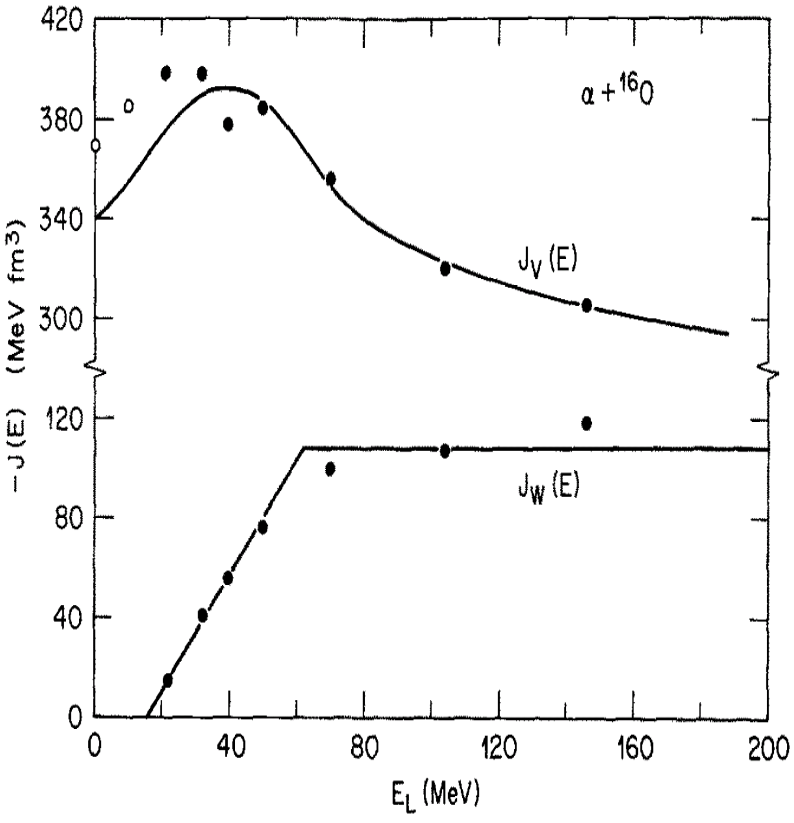
$$J_{\Delta V}(E) = \frac{4\pi}{A_p A_t} \int_0^\infty dr r^2 \, \Delta V(r,E) \qquad J_W(E) = \frac{4\pi}{A_p A_t} \int_0^\infty dr r^2 \, W(r,E)$$

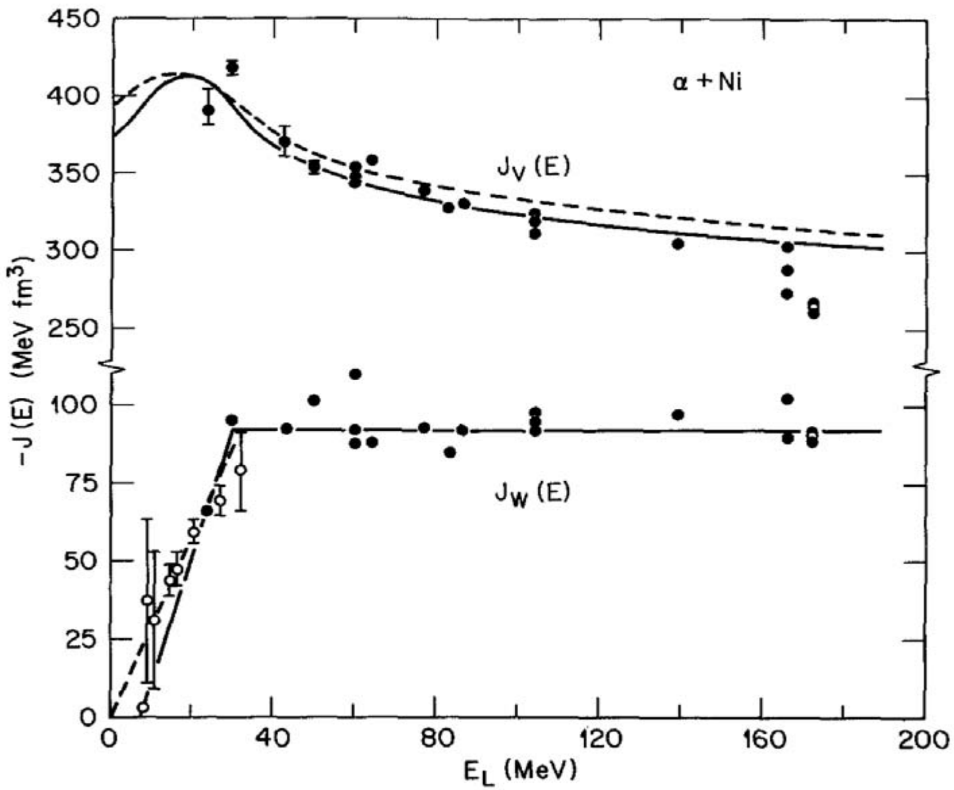
$$J_{\Delta V}(E) = \frac{\mathrm{P}}{\pi} \int dE' \; \frac{J_W(E')}{E - E'}$$

$J_W(E)$ has very simple dependence on energy.

Threshold anomaly in α –nucleus scattering

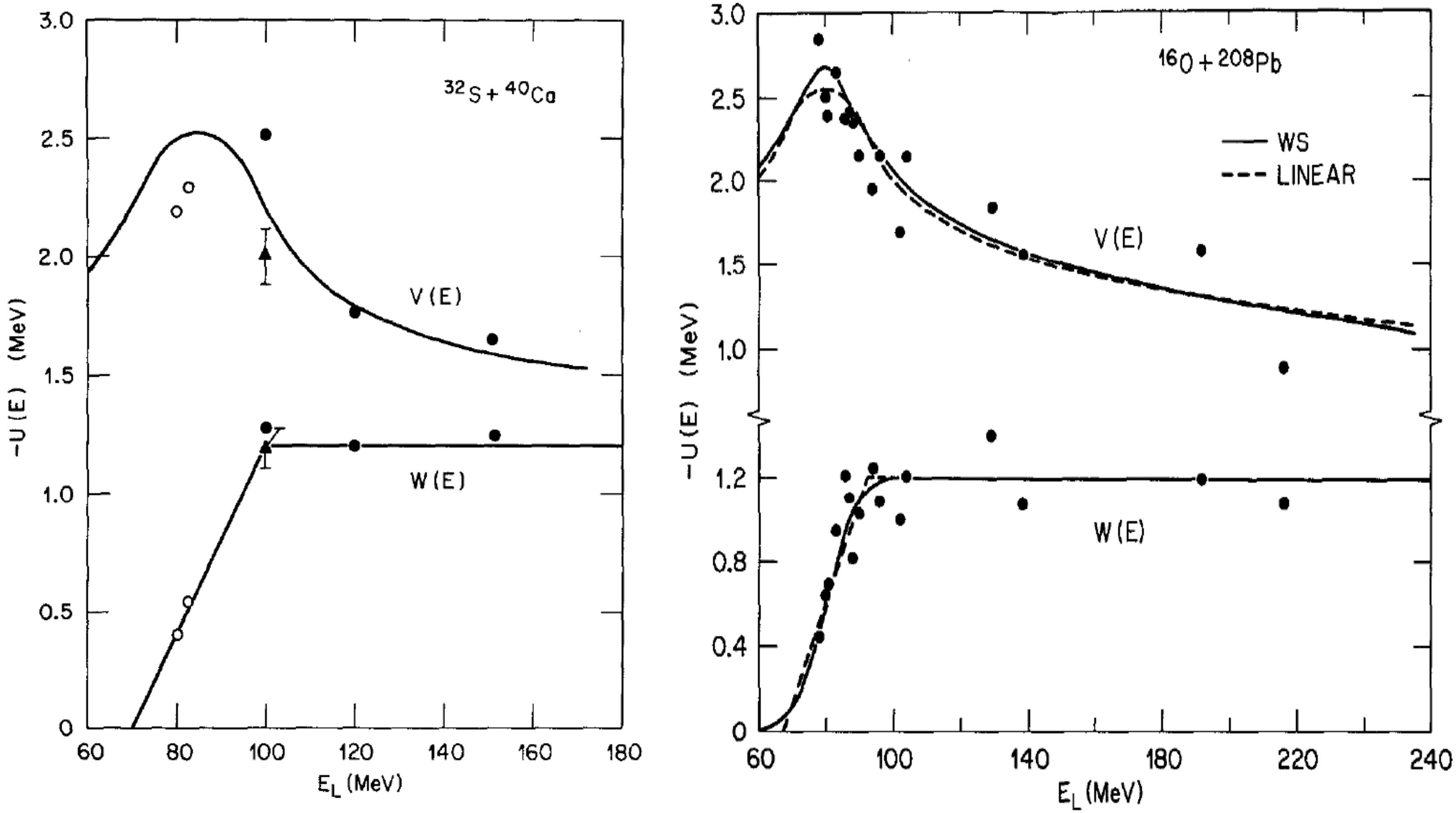
C.Mahaux, H.Ngo, G.R.Satchler, Nucl.Phys. A449, 354 (1986)





Threshold anomaly in nucleus –nucleus scattering

C.Mahaux, H.Ngo, G.R.Satchler, Nucl.Phys. A449, 354 (1986)



Dispersive optical model (DOM) analysis of a large volume of data:

J.M. Mueller et al, Phys. Rev. C ***83****, 064605 (2011)*

Data involved:

$n + {}^{40,48}Ca, {}^{54}Fe, {}^{58,60}Ni, {}^{92}Mo, {}^{116,118,120,124}Sn, {}^{208}Pb,$

$p + {}^{40,42,44,48}Ca, {}^{50}Ti, {}^{52}Cr, {}^{54}Fe, {}^{58}Ni, {}^{60,62,64}Ni, {}^{90}Zr, {}^{92}Mo,$
${}^{114,116,118,120,122,124}Sn, {}^{208}Pb$

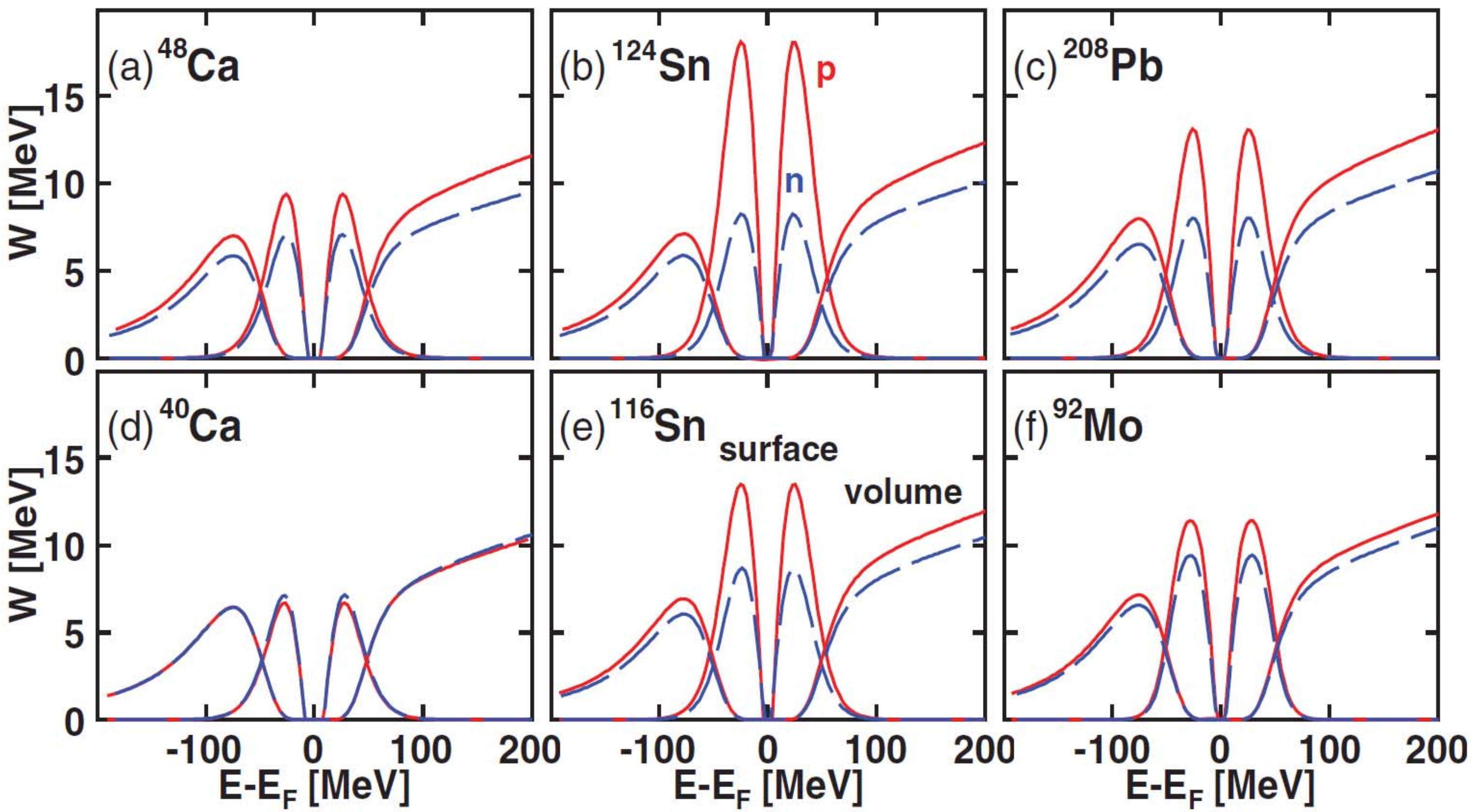
available for energy range $4 \le E \le 200$ MeV.

Dispersive optical potential

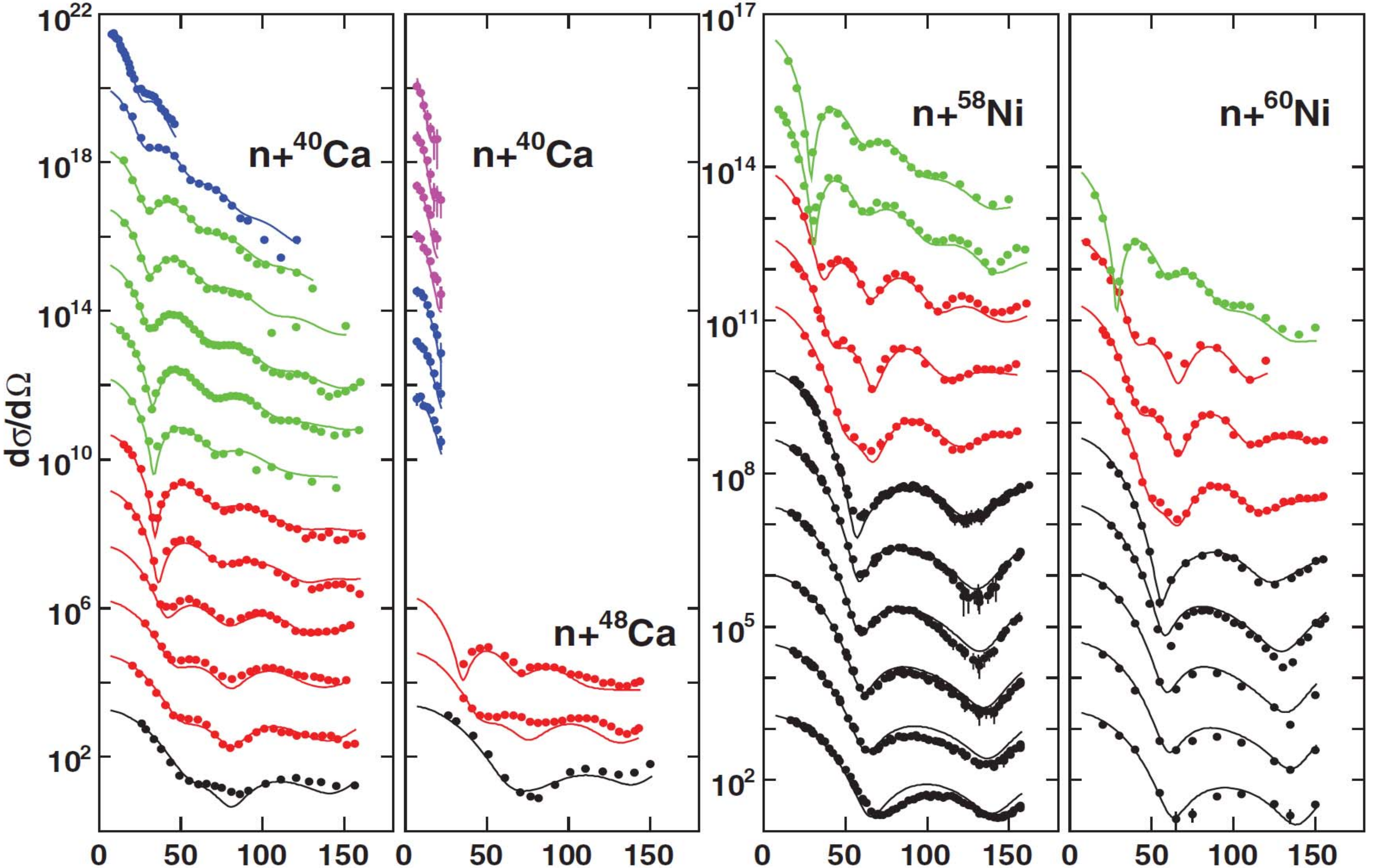
$$V_{opt}(\mathbf{r}, E) = V_0(\mathbf{r}, E) + \Delta V(\mathbf{r}, E) + iW(\mathbf{r}, E)$$

has been described in terms of 32 parameters

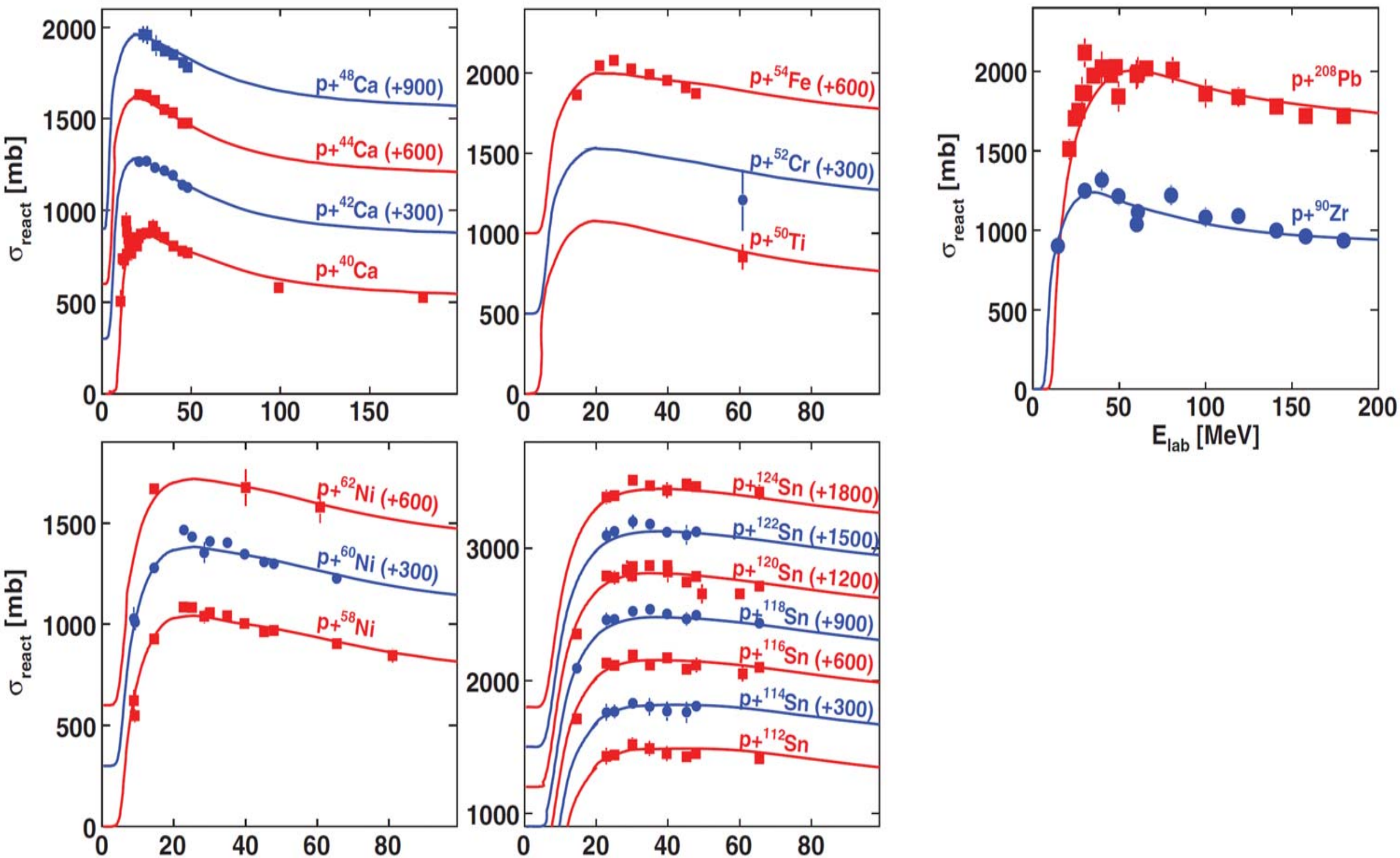
Fitted energy dependencies of magnitudes of the imaginary volume and surface potentials



Examples of elastic scattering description within the DOM.
Differential cross sections.



Examples of elastic scattering description within the DOM.
Reaction cross sections.

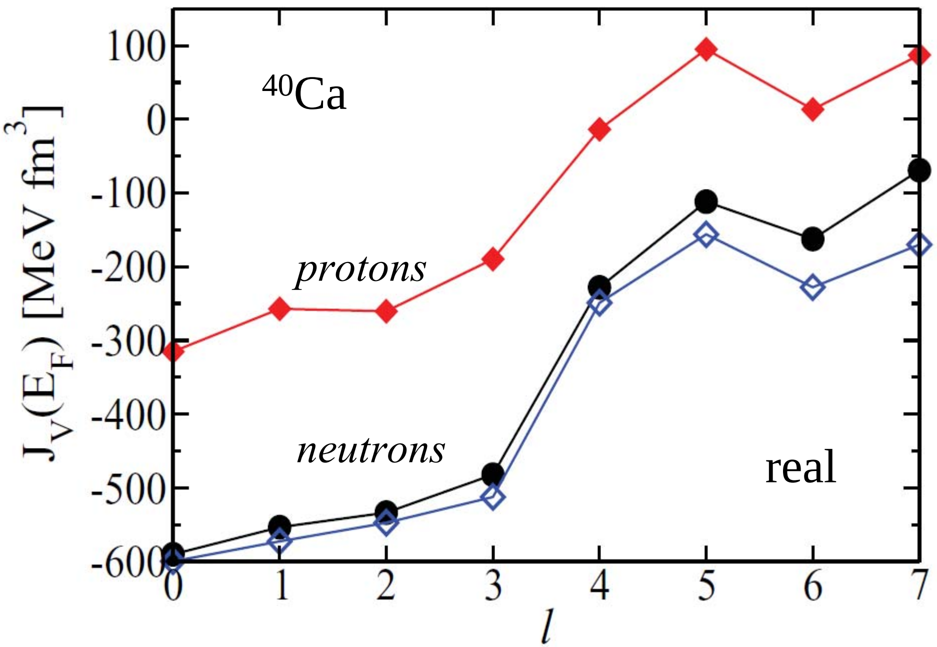


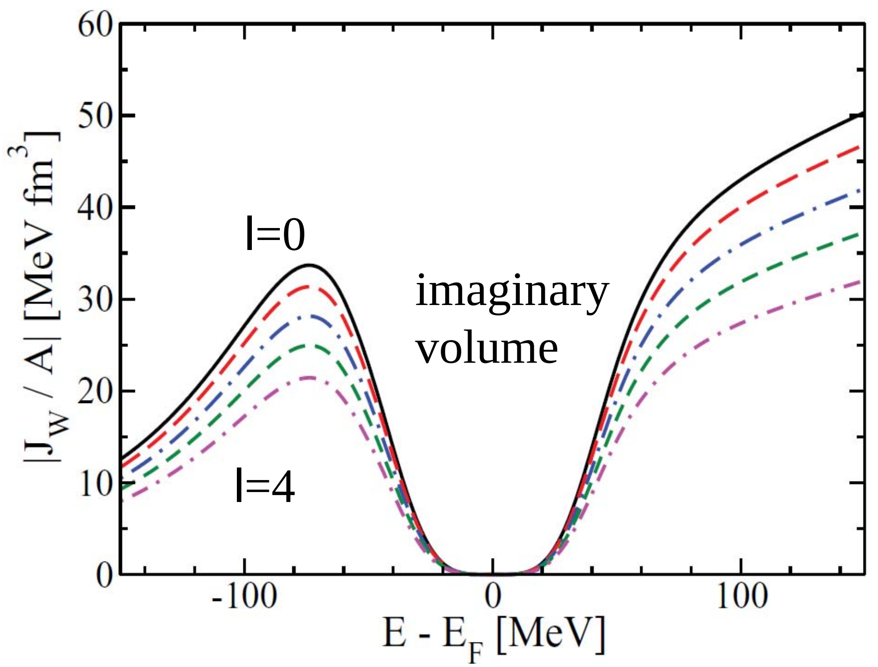
Microscopic ab-initio calculations of optical potential

S. J. Waldecker, C. Barbieri and W. H. Dickhoff, Phys. Rev. C 84, 034616 (2011)

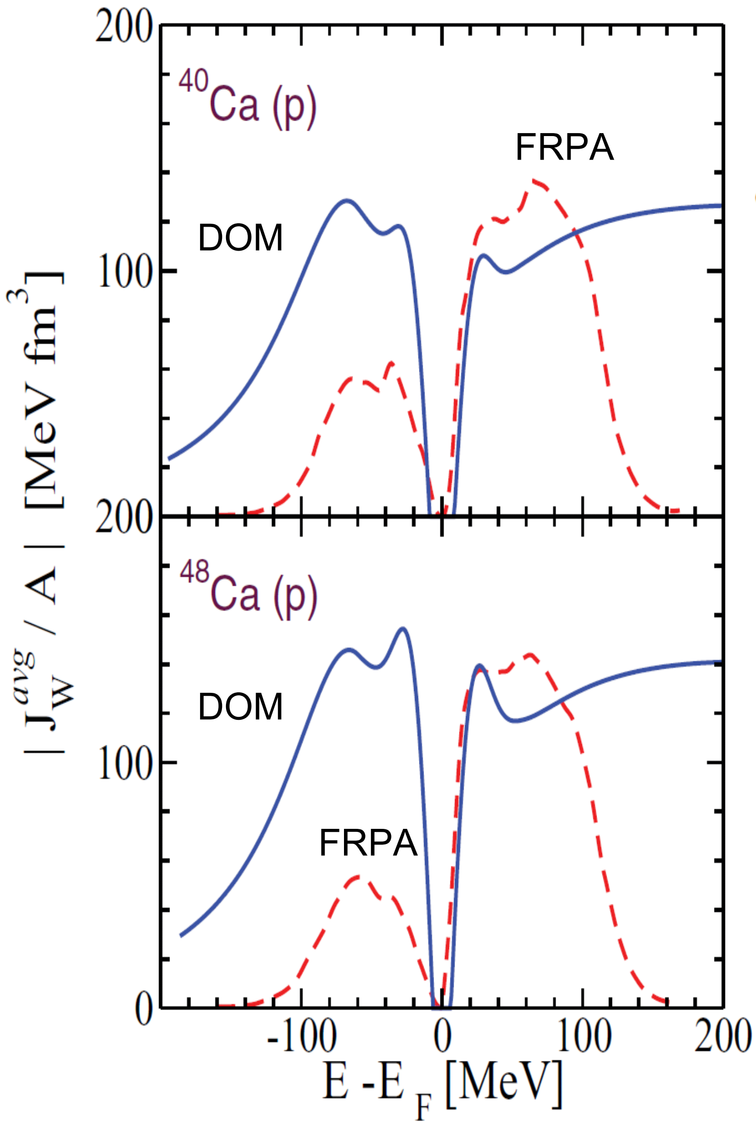
Faddeev-random-phase approximation approach using realistic Argonne AV18 potential

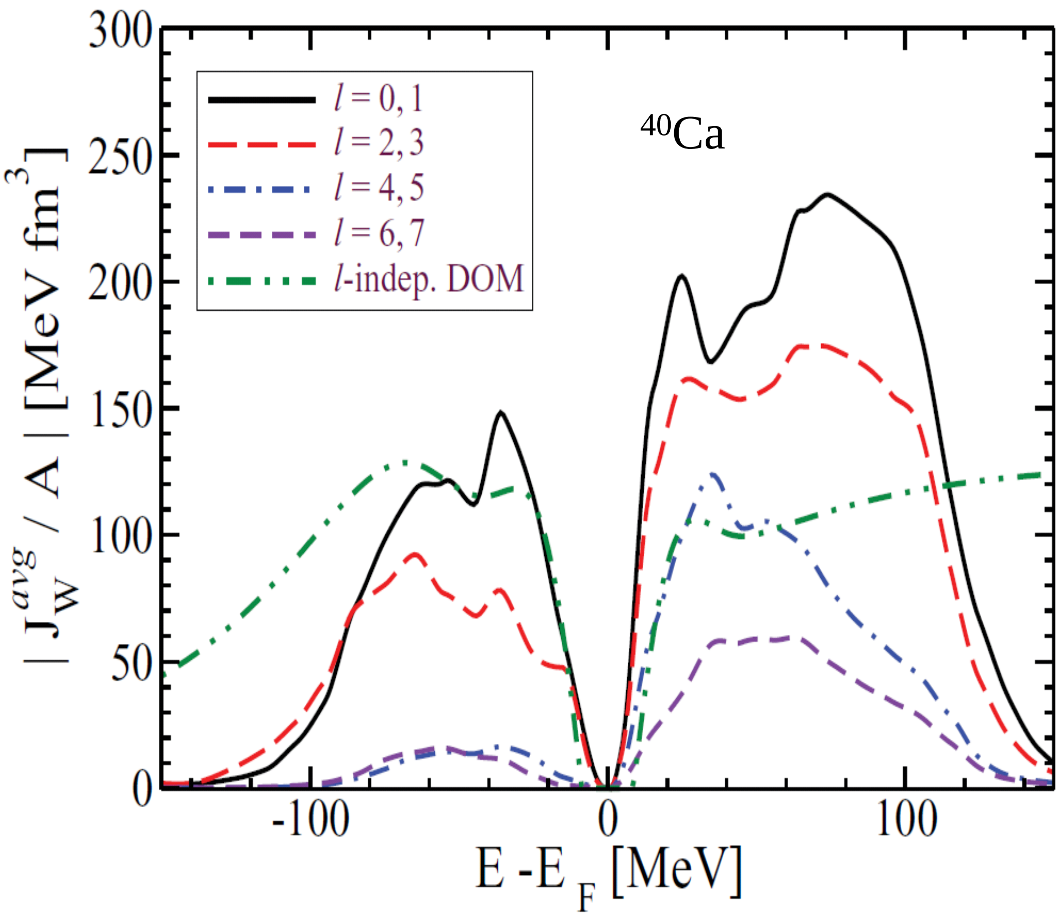
Results:
Optical potentials are different for partial waves with different orbital momentum l.





Comparison between microscopic (FRPA) and dispersive (DOM) optical potentials





Summary

- Elastic scattering can be described by optical model
- Optical potential has a general representation

$$V_{\text{opt}} = \langle \Phi_1 | V | \Phi_1 \rangle + \hat{V} \frac{1}{E + i\varepsilon - \hat{h}} \hat{V}^+$$

- Optical potential is non-local, energy-dependent, non-Hermitian
- For a non-local optical model an equivalent local model can exist
- Microscopic models for optical potential exist
  - Folding model
  - JLM model
  - FRPA
- Energy-dependent real part $\Delta V(\boldsymbol{r},\boldsymbol{r}',E)$ and imaginary part $W(\boldsymbol{r},\boldsymbol{r}',E)$ of the optical potential are related by dispersion relation which is a consequence of causality
- Many phenomenological parameterizations of optical potentials are available