Generalized Transverse-Momentum Dependent Parton Distributions

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Outline

- Quark-quark correlator
- Interpretation in the transverse plane
- Wigner distributions

Physical processes (not exhaustive)

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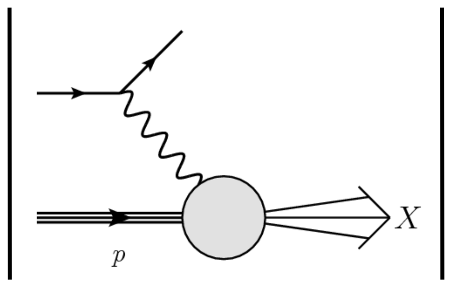

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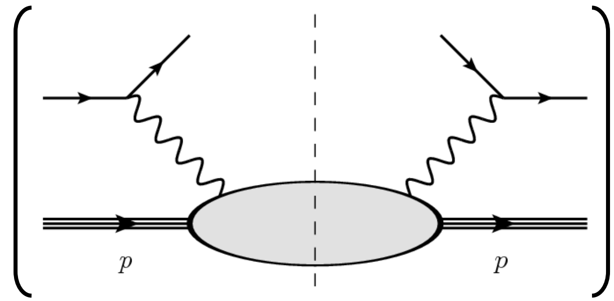

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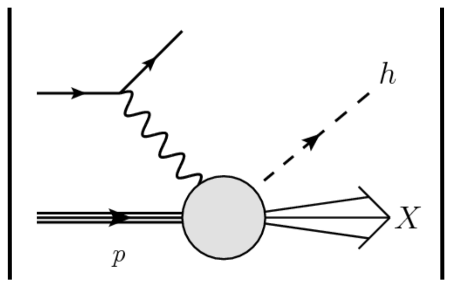

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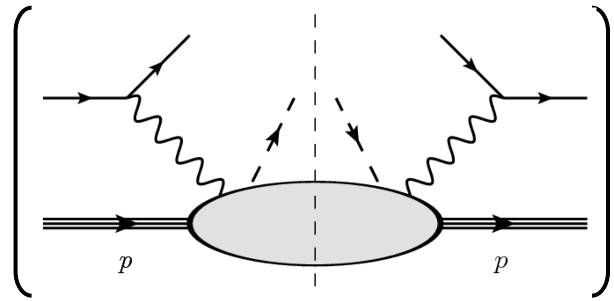

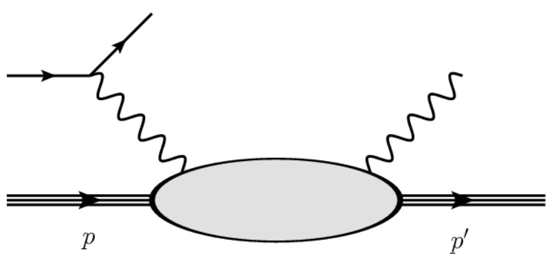

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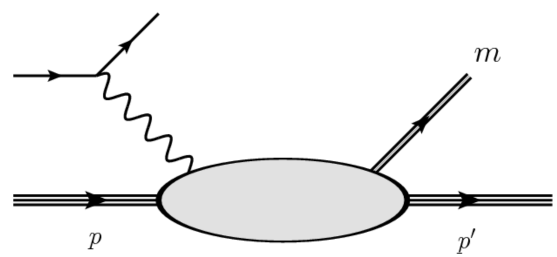

Hard exclusive meson leptoproduction

Handbag approximation

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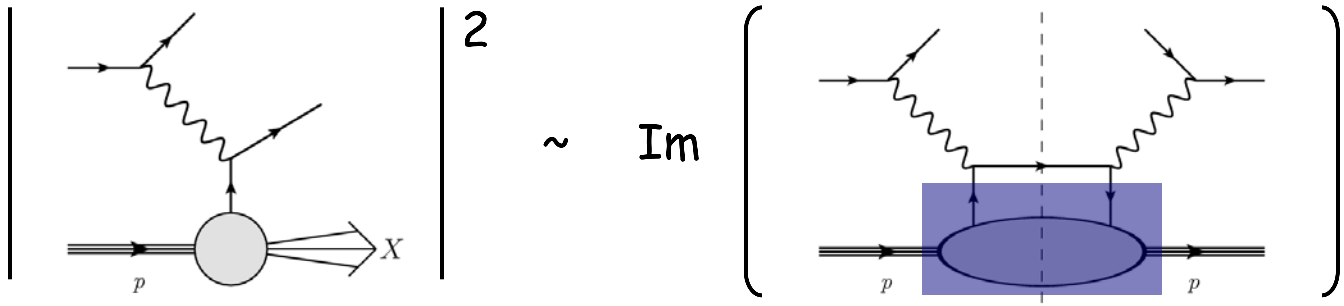

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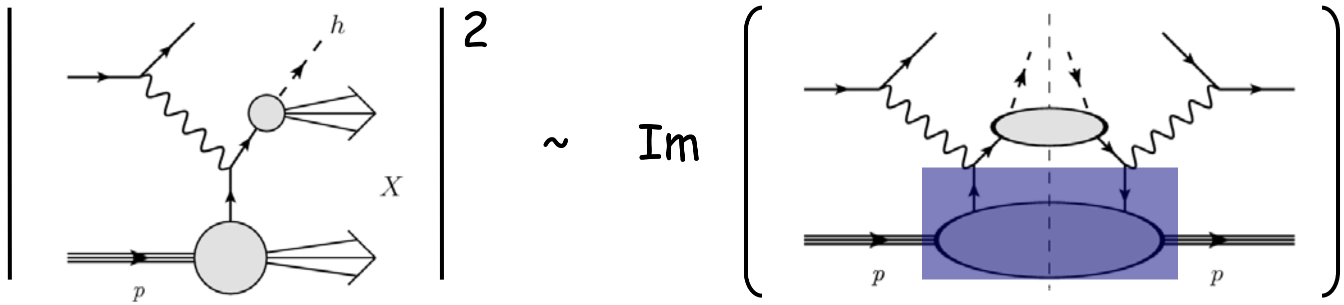

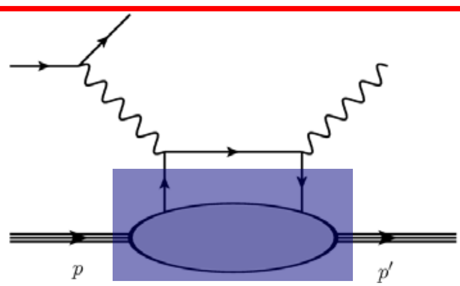

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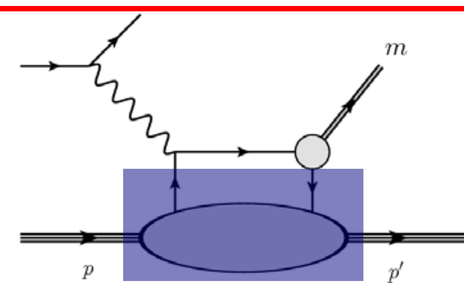
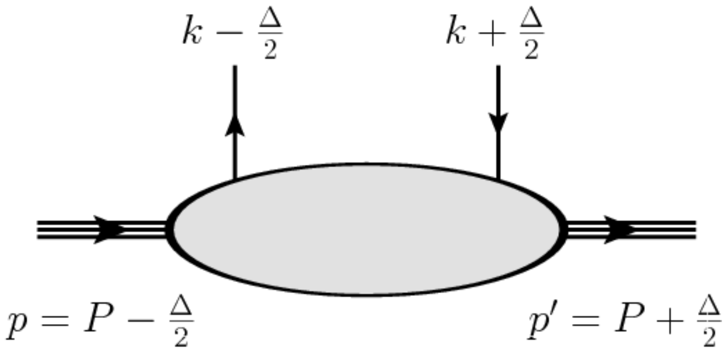


Hard exclusive meson leptoproduction

Quark-quark correlator



$$\frac{1}{2}\int \frac{d^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, \langle p', \Lambda' | \overline{\psi}(-\tfrac{z}{2}) \Gamma \mathcal{W}\, \psi(\tfrac{z}{2}) | p, \Lambda \rangle$$

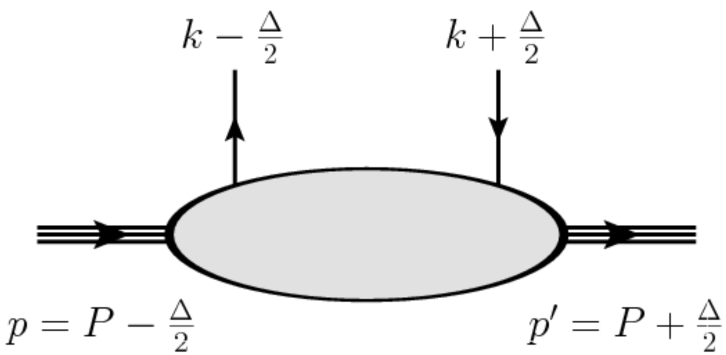
Γ : Dirac matrices

$\mathcal{W}$: Wilson line ensuring gauge invariance

We want a snapshot

Pictures are not equal- x^0
but equal- $x^+ = (x^0 + x^3)/\sqrt{2}$

GTMDs (Generalized Transverse-Momentum dependent parton Distributions)



$$W^{[\Gamma]}_{\Lambda'\Lambda}(P,x,\vec{k}_\perp,\Delta)=\frac{1}{2}\int\frac{\mathrm{d}z^-\,\mathrm{d}^2z_\perp}{(2\pi)^3}\,e^{ik\cdot z}\,\langle p',\Lambda'|\overline{\psi}(-\tfrac{z}{2})\Gamma\mathcal{W}\,\psi(\tfrac{z}{2})|p,\Lambda\rangle\Big|_{z^+=0}$$

Parameterized by GTMDs

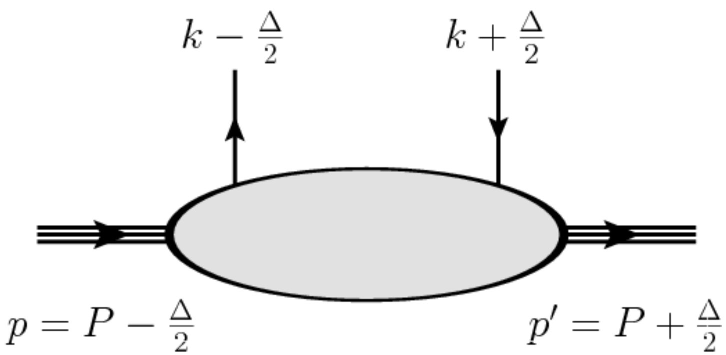
$$X(x,\xi,\vec{k}_\perp^2,\vec{k}_\perp\cdot\vec{\Delta}_\perp,\vec{\Delta}_\perp^2;\eta)\in\mathbb{C}$$

[Meißner, Metz, Schlegel (2009)]

Struck quark variables	Mean momentum	Momentum transfer
Longitudinal fraction	$x=\frac{k^+}{P^+}$	$\xi=-\frac{\Delta^+}{2P^+}$
Transverse	$\vec{k}_\perp$	$\vec{\Delta}_\perp$

Physical interpretation?

Connection with Form Factors



$$\int \mathrm{d}x\, \mathrm{d}^2k_\perp\, W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2P^+} \langle p', \Lambda' | \overline{\psi}(0)\Gamma\psi(0) | p, \Lambda \rangle$$

Parameterized by the FFs !

Spatial distribution of partons

Example : electromagnetic FFs

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Sachs Dirac Pauli

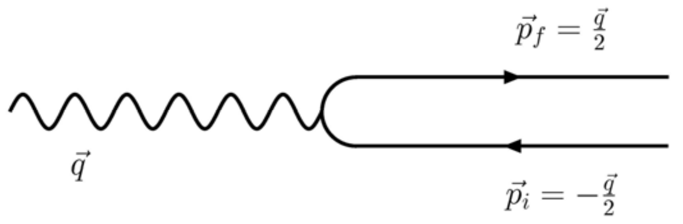
NB: $\Delta \equiv q = p' - p$

$$Q^2 = -q^2$$

Standard Picture

[Ernst, Sachs, Wali (1960)]
[Sachs (1962)]

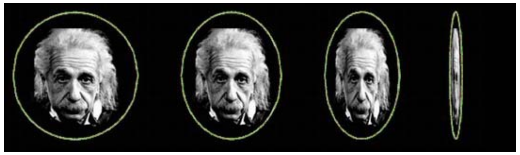
Breit frame



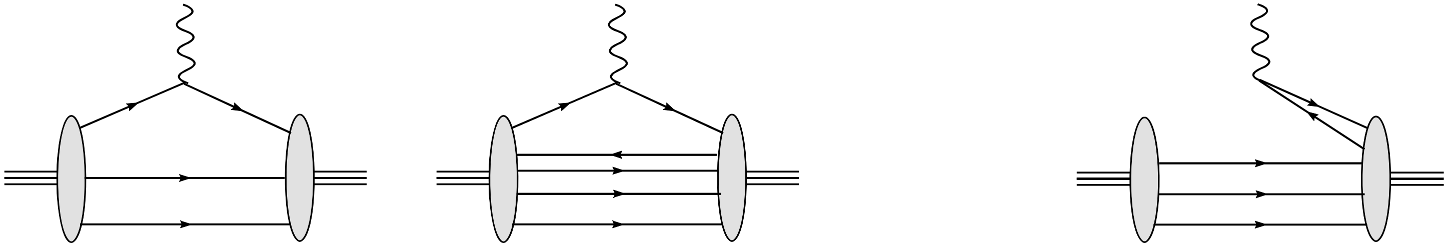
$$\rho(\vec{r}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}} G_E(Q^2)$$

BUT

- Lorentz contraction
- Creation/annihilation of pairs



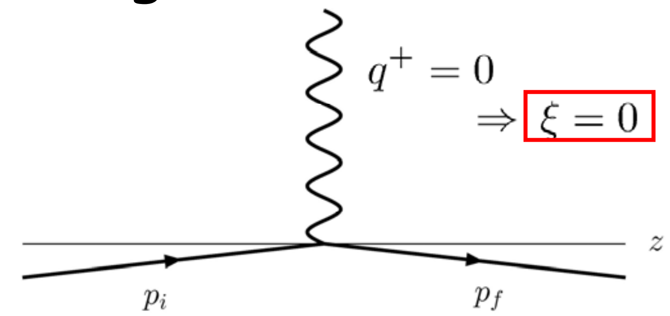
NO probability/charge density interpretation



Correct Picture

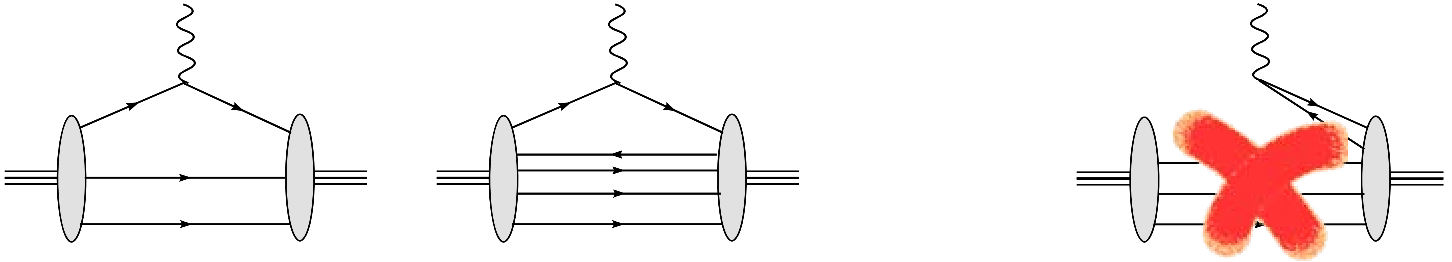
[Soper (1977)]
[Burkardt (2000,2003)]

Light-front frame

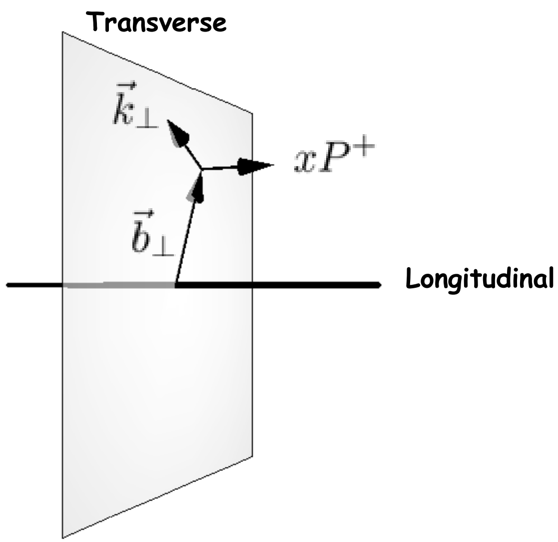


$$\rho_\lambda(\vec{b}) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} \frac{e^{-i\vec{q}_\perp \cdot \vec{b}}}{2p^+} \langle p^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | p^+, -\frac{\vec{q}_\perp}{2}, \lambda \rangle$$

- Extreme Lorentz contraction → 2D picture
- No creation/annihilation of massive pairs $p^+ > 0$



Wigner Distributions



Fourier transform of GTMDs are distributions in both $\vec{b}_\perp$ and $\vec{k}_\perp$ at $\xi = 0$

$$\rho(x, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}\, X(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \vec{\Delta}_\perp^2)$$

→ Quantum phase-space distributions !

[Wigner (1932)] QM

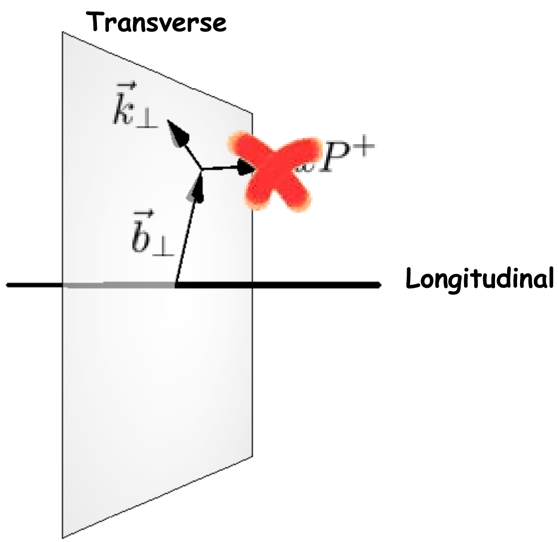
[Belitsky, Ji, Yuan (2004)] QFT (Breit Frame)

Heisenberg's uncertainty principle → Quasi-probabilistic interpretation

Phase-space distributions are very important in :

- Statistical Mechanics
- Quantum chemistry
- Classical and quantum optics
- Signal analysis
- …

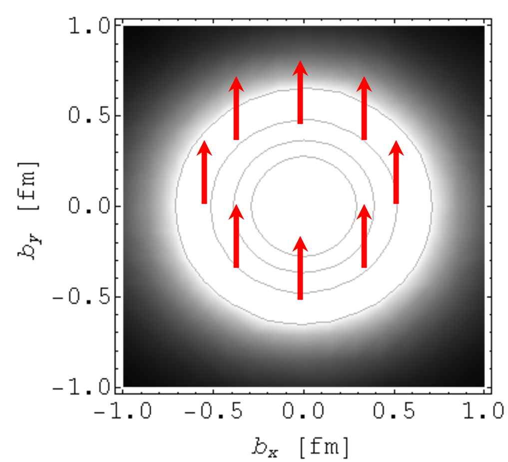
Wigner Distributions (Preliminary results)

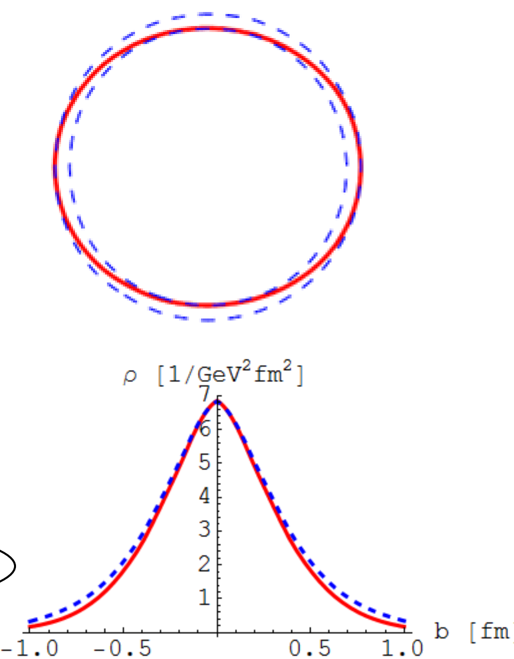


Fourier transform of GTMDs are distributions in both $\vec{b}_\perp$ and $\vec{k}_\perp$ at $\xi = 0$

$$\rho_{2D}(\vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2) = \int \mathrm{d}x \, \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \, X(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \vec{\Delta}_\perp^2)$$

fixed $\vec{k}_\perp$





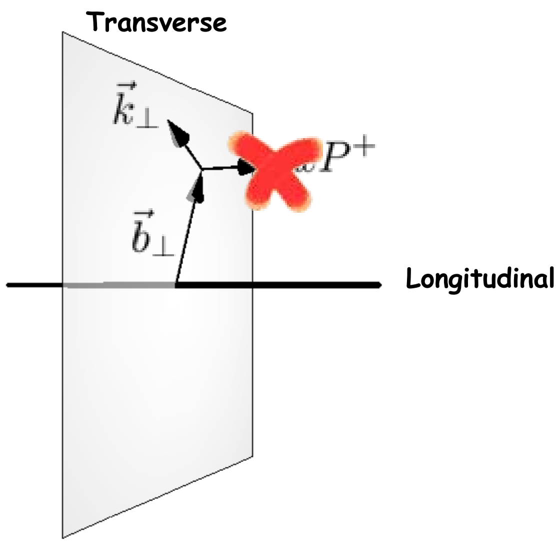
Unpolarized u quark in unpolarized proton

Based on a 3Q light-cone wave function (model)

[C.L., Pasquini (in preparation)]

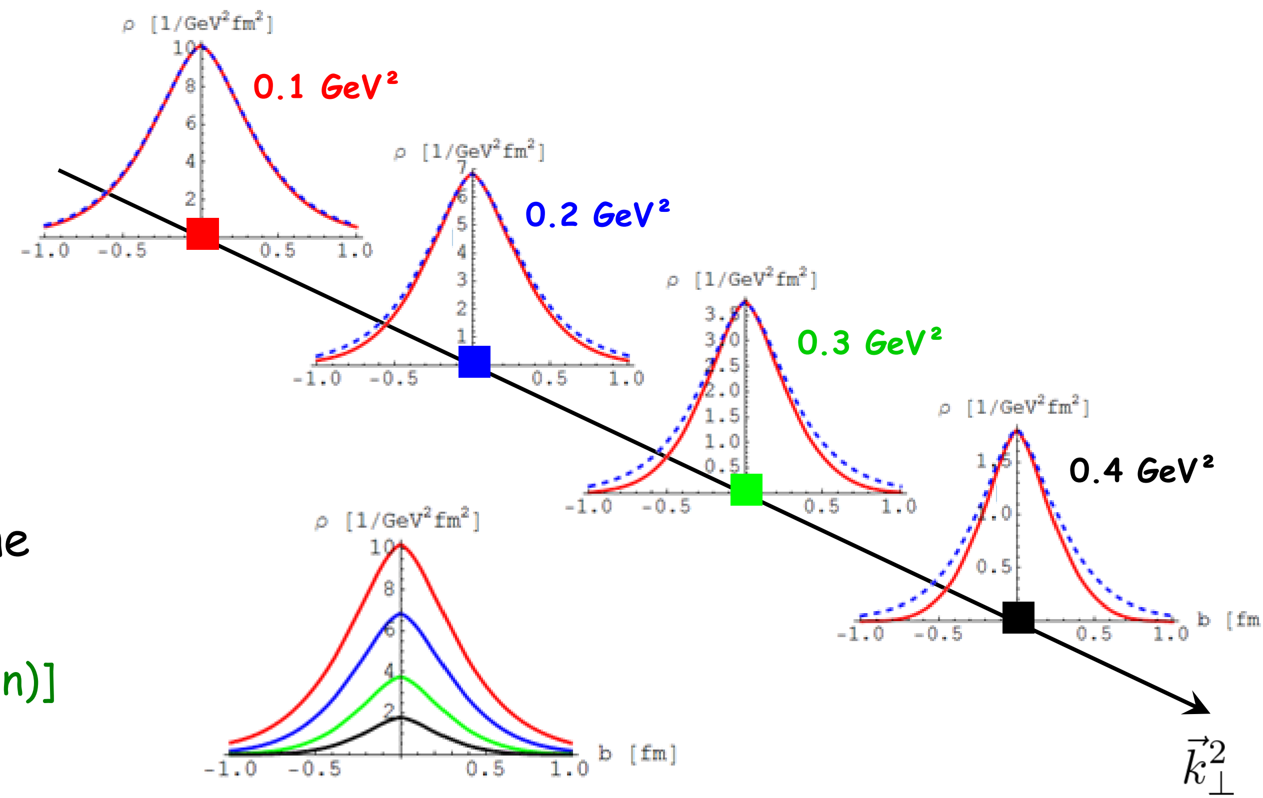
Orbital angular momentum?

Wigner Distributions (Preliminary results)



Fourier transform of GTMDs are distributions in both $\vec{b}_\perp$ and $\vec{k}_\perp$ at $\xi = 0$

$$\rho_{2D}(\vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2) = \int \mathrm{d}x \, \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \, e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} \, X(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \vec{\Delta}_\perp^2)$$



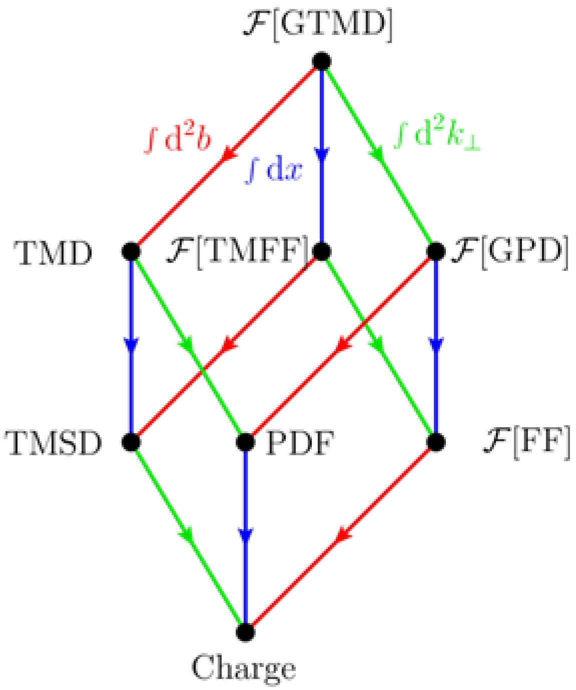
Unpolarized u quark in unpolarized proton

Based on a 3Q light-cone wave function (model)

[C.L., Pasquini (in preparation)]

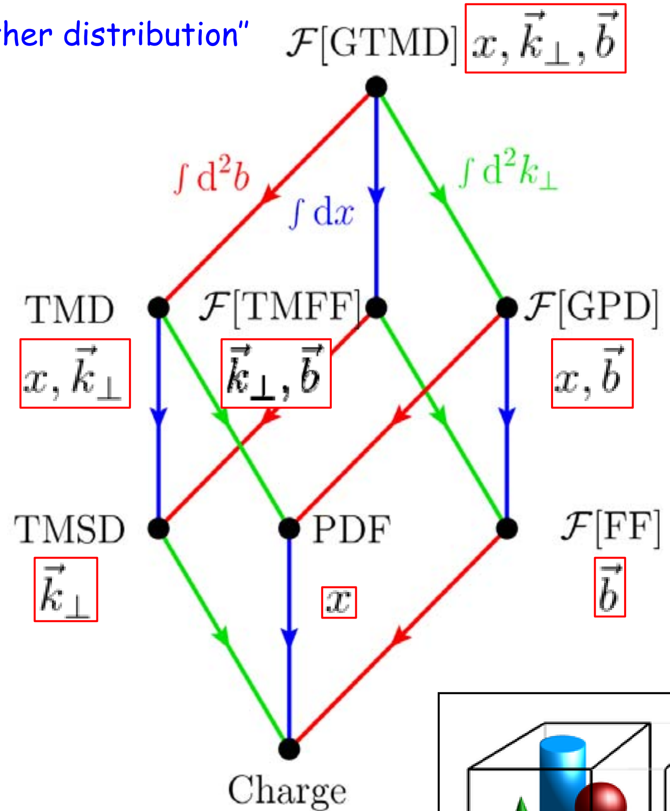
Summary

- Quark-quark correlator
  - Most complete information on hadron structure
  - GTMDs are "mother" distribution
- 2D-Fourier transform on the light cone
  - Correct interpretation of FFs
  - GTMDs can be related to Wigner distributions
- Wigner distributions in a LCQM
  - Non-trivial structure for unpolarized quark in unpolarized hadron
  - Connection with quark orbital angular momentum



Backup

Physical interpretation

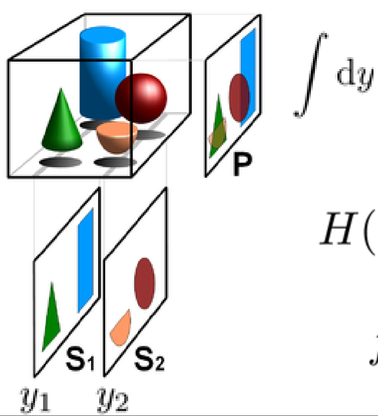


(Quasi-)probabilistic interpretation in impact-parameter space with $\xi = 0$

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

$\vec{\Delta}_\perp \leftrightarrow \vec{b}$

$$\int d^2b \simeq \vec{\Delta}_\perp = \vec{0}_\perp$$



Example : unpolarized quark in unpolarized hadron

$$H(x,0,\vec{\Delta}_\perp^2) = \int d^2k_\perp \, F_{11}(x,0,\vec{k}_\perp^2,\vec{k}_\perp\cdot\vec{\Delta}_\perp,\vec{\Delta}_\perp^2)$$

$$f_1(x,\vec{k}_\perp^2) = F_{11}(x,0,\vec{k}_\perp^2,0,0)$$

Physical interpretation

Twist-2 operators (leading order in P^+)

$$\Gamma = \gamma^+,\ i\sigma^{+1}\gamma_5,\ i\sigma^{+2}\gamma_5,\ \gamma^+\gamma_5$$

Effect on quark LC helicity

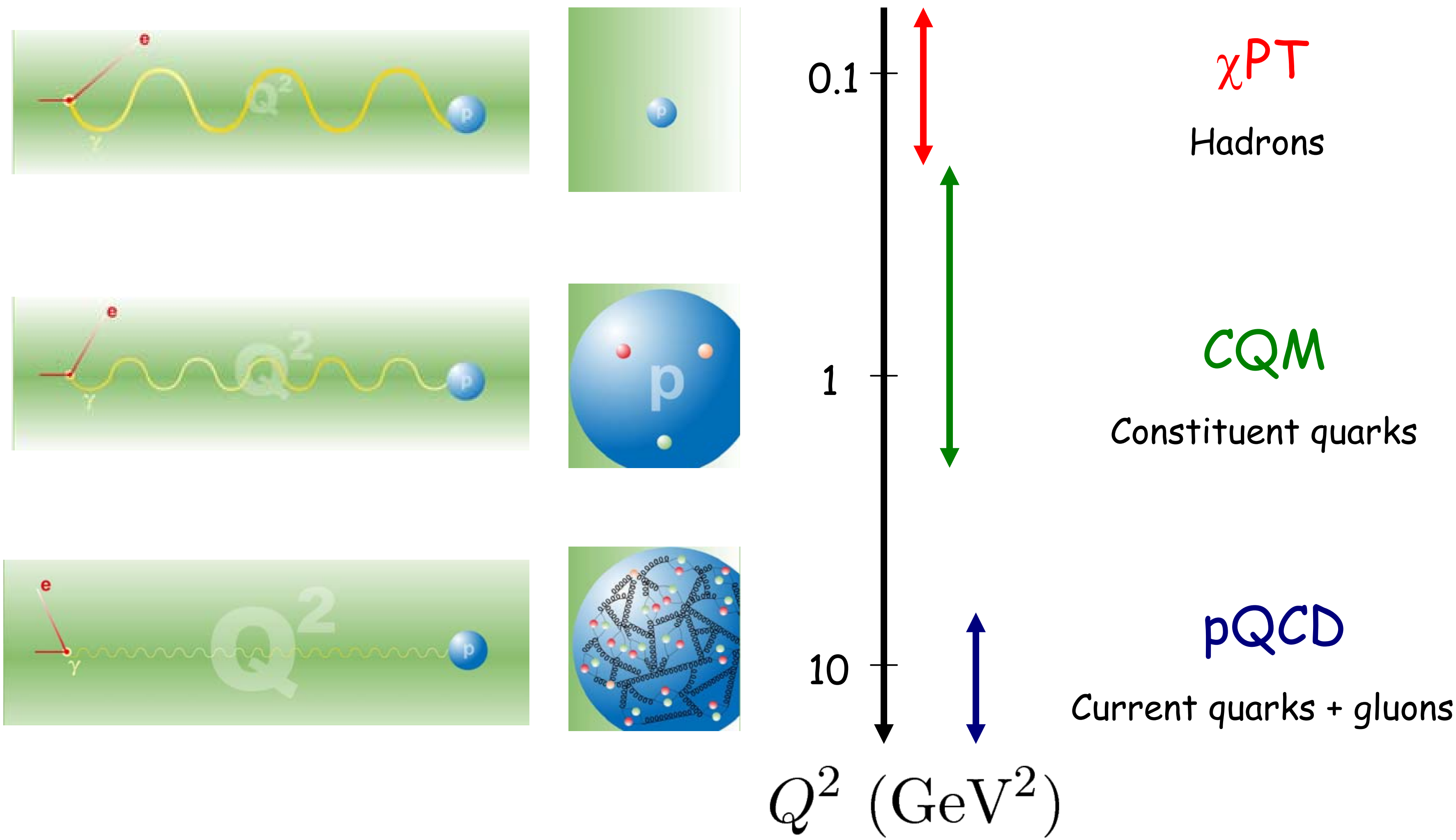
$$(\bar\sigma^\nu)_{\lambda'\lambda} \equiv \left(\mathbb{1}, \sigma^1, \sigma^2, \sigma^3\right)_{\lambda'\lambda}$$

Convenient tensor notation $[\Gamma], \Lambda', \Lambda \quad \rightarrow \quad \mu, \nu$

$$W^{\mu\nu} = \frac{1}{2}\sum_{\Lambda'\Lambda}(\bar\sigma^\mu)_{\Lambda\Lambda'} W^\nu_{\Lambda'\Lambda} \qquad W^\nu_{\Lambda'\Lambda} = \left(W^{[\gamma^+]}_{\Lambda'\Lambda}, W^{[i\sigma^{+1}\gamma_5]}_{\Lambda'\Lambda}, W^{[i\sigma^{+2}\gamma_5]}_{\Lambda'\Lambda}, W^{[\gamma^+\gamma_5]}_{\Lambda'\Lambda}\right)$$

LC helicity $\neq$ canonical spin

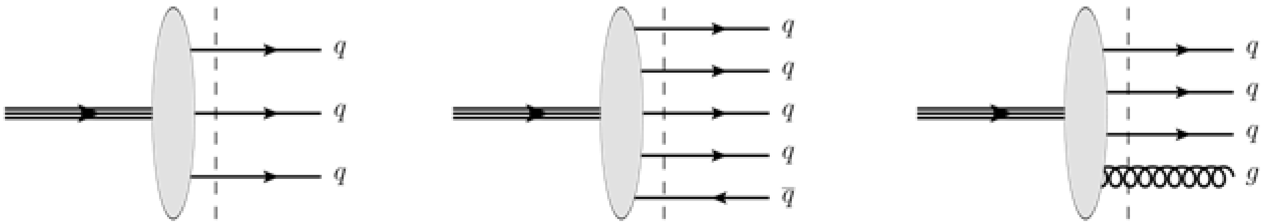
Degrees of freedom



Fock expansion

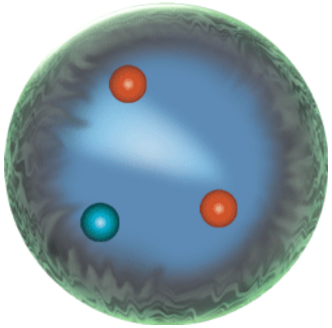
QCD

$$|\Psi\rangle = \Psi_{3q}|qqq\rangle + \Psi_{3q(q\bar{q})}|qqq(q\bar{q})\rangle + \Psi_{3qg}|qqqg\rangle + \cdots$$



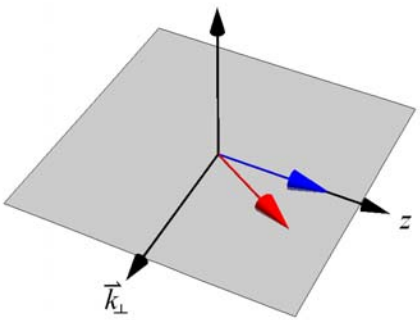
Constituent quark models

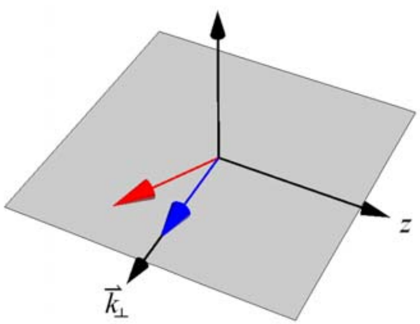
$$|\Psi\rangle = \Psi_{3Q}|QQQ\rangle \quad (+\cdots)$$

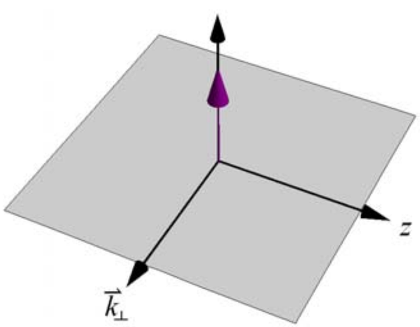


Assumptions :
- Lowest component (3Q)
- Non mutually interacting quarks

LC helicity and Canonical spin







- Non mutually interacting quarks

$$q_\lambda^{LC}(k) = \sum_s D^{(1/2)*}_{\lambda s}(k)\, q_s^C(k)$$

LC helicity

Canonical spin

$$D^{(1/2)*}_{\lambda s}(k) = \frac{1}{|\vec{K}|}\begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}$$

Light-Cone Quark Model

$$\mathcal{M}_0^2 = \sum_i \frac{m_i^2 + \vec{k}_{i\perp}^2}{x_i}$$

$$K_z = m + x\mathcal{M}_0 \qquad \vec{K}_\perp = \vec{k}_\perp \qquad k_z = x\mathcal{M}_0 - \sqrt{\vec{k}^2 + m^2}$$

(Melosh rotation)

Chiral Quark-Soliton Model

$$K_z = h(|\vec{k}|) + \frac{k_z}{|\vec{k}|}\, j(|\vec{k}|) \qquad \vec{K}_\perp = \frac{\vec{k}_\perp}{|\vec{k}|}\, j(|\vec{k}|) \qquad k_z = x\mathcal{M}_N - E_{\text{lev}}$$

S-wave P-wave

Bag Model

$$K_z = t_0(|\vec{k}|) + \frac{k_z}{|\vec{k}|}\, t_1(|\vec{k}|) \qquad \vec{K}_\perp = \frac{\vec{k}_\perp}{|\vec{k}|}\, t_1(|\vec{k}|) \qquad k_z = x\mathcal{M}_N - \omega/R_0$$

S-wave P-wave

Light front- and instant-form WFs

		$l_z=-1$	$l_z=0$			$l_z=+1$			$l_z=+2$
		$\Psi^{\uparrow}_{\uparrow\uparrow\uparrow}$	$\Psi^{\uparrow}_{\uparrow\uparrow\downarrow}$	$\Psi^{\uparrow}_{\uparrow\downarrow\uparrow}$	$\Psi^{\uparrow}_{\downarrow\uparrow\uparrow}$	$\Psi^{\uparrow}_{\downarrow\downarrow\uparrow}$	$\Psi^{\uparrow}_{\downarrow\uparrow\downarrow}$	$\Psi^{\uparrow}_{\uparrow\downarrow\downarrow}$	$\Psi^{\uparrow}_{\downarrow\downarrow\downarrow}$
$l_z=-1$	ψ^{+}_{+++}	$z_1z_2z_3$	$z_1z_2l_3$	$z_1l_2z_3$	$l_1z_2z_3$	$l_1l_2z_3$	$l_1z_2l_3$	$z_1l_2l_3$	$l_1l_2l_3$
$l_z=0$	ψ^{+}_{++-}	$-z_1z_2r_3$	$z_1z_2z_3$	$-z_1l_2r_3$	$-l_1z_2r_3$	$-l_1l_2r_3$	$l_1z_2z_3$	$z_1l_2z_3$	$l_1l_2z_3$
	ψ^{+}_{+-+}	$-z_1r_2z_3$	$-z_1r_2l_3$	$z_1z_2z_3$	$-l_1r_2z_3$	$l_1z_2z_3$	$-l_1r_2l_3$	$z_1z_2l_3$	$l_1z_2l_3$
	ψ^{+}_{-++}	$-r_1z_2z_3$	$-r_1z_2l_3$	$-r_1l_2z_3$	$z_1z_2z_3$	$z_1l_2z_3$	$z_1z_2l_3$	$-r_1l_2l_3$	$z_1l_2l_3$
$l_z=+1$	ψ^{+}_{--+}	$r_1r_2z_3$	$r_1r_2l_3$	$-r_1z_2z_3$	$-z_1r_2z_3$	$z_1z_2z_3$	$-z_1r_2l_3$	$-r_1z_2l_3$	$z_1z_2l_3$
	ψ^{+}_{-+-}	$r_1z_2r_3$	$-r_1z_2z_3$	$r_1l_2r_3$	$-z_1z_2r_3$	$-z_1l_2r_3$	$z_1z_2z_3$	$-r_1l_2z_3$	$z_1l_2z_3$
	ψ^{+}_{+--}	$z_1r_2r_3$	$-z_1r_2z_3$	$-z_1z_2r_3$	$l_1r_2r_3$	$-l_1z_2r_3$	$-l_1r_2z_3$	$z_1z_2z_3$	$l_1z_2z_3$
$l_z=+2$	ψ^{+}_{---}	$-r_1r_2r_3$	$r_1r_2z_3$	$r_1z_2r_3$	$z_1r_2r_3$	$-z_1z_2r_3$	$-z_1r_2z_3$	$-r_1z_2z_3$	$z_1z_2z_3$

$$z_i \equiv K^i_z, \quad l_i \equiv K^i_L, \quad r_i \equiv K^i_R$$

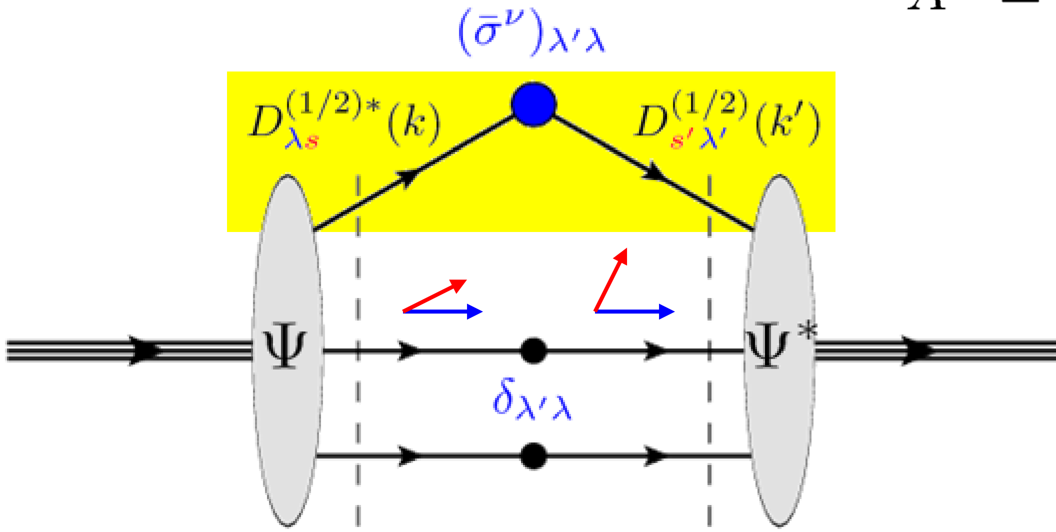
Assumption : ➢ $l_z = 0$ in instant form (automatic w/ spherical symmetry)

↳ More convenient to work in canonical spin basis

3Q overlap representation

$$A^+ = 0 \quad \Rightarrow \quad \mathcal{W} \sim 1$$

$$W^{\mu\nu} =$$



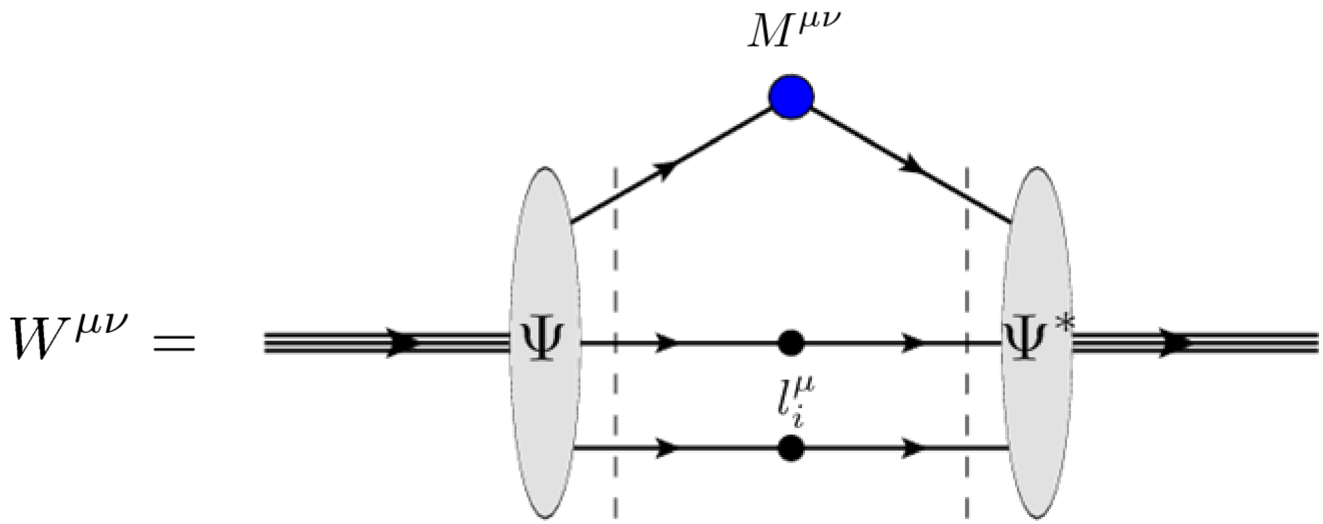
Quark line :

$$\sum_{\lambda'\lambda} D^{(1/2)}_{s'\lambda'}(k')\,(\bar{\sigma}^\nu)_{\lambda'\lambda}\,D^{(1/2)*}_{\lambda s}(k) = (\sigma_\mu)_{s's}\,M^{\mu\nu}(k',k)$$

$$M^{\mu\nu}(k',k) = \frac{1}{|\vec{K}'||\vec{K}|}\begin{pmatrix} \vec{K}'\cdot\vec{K} & i(\vec{K}'\times\vec{K})_x & i(\vec{K}'\times\vec{K})_y & -i(\vec{K}'\times\vec{K})_z \\ i(\vec{K}'\times\vec{K})_x & \vec{K}'\cdot\vec{K}-2K'_xK_x & -K'_xK_y-K'_yK_x & K'_xK_z+K'_zK_x \\ i(\vec{K}'\times\vec{K})_y & -K'_yK_x-K'_xK_y & \vec{K}'\cdot\vec{K}-2K'_yK_y & K'_yK_z+K'_zK_y \\ i(\vec{K}'\times\vec{K})_z & -K'_zK_x-K'_xK_z & -K'_zK_y-K'_yK_z & -\vec{K}'\cdot\vec{K}+2K'_zK_z \end{pmatrix}$$

Active quark :	Spectator quarks :
$M^{\mu\nu}(k'_1,k_1)$	$l^\mu_i \equiv M^{\mu 0}(k'_i,k_i)$

3Q amplitude

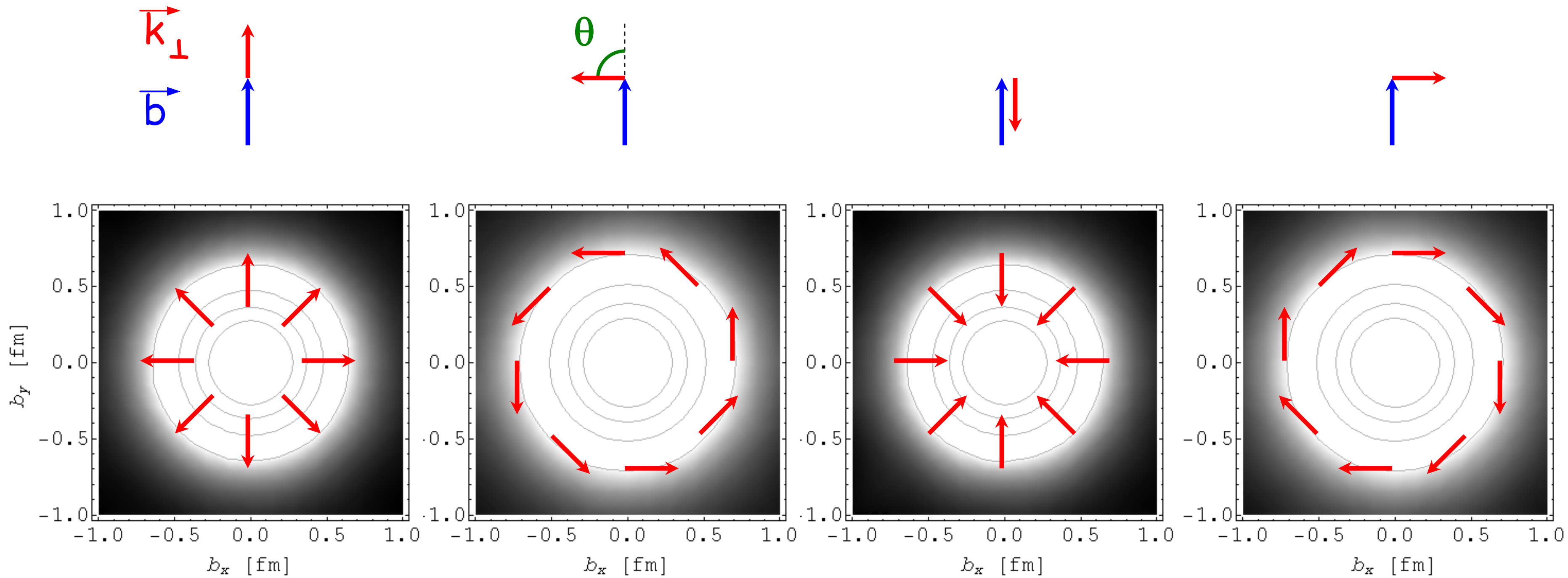


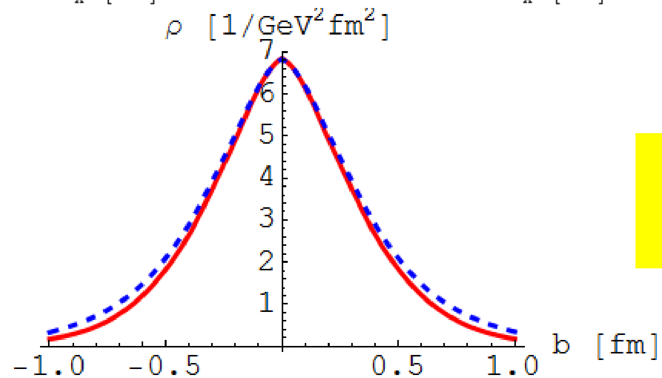
$$W^{\mu\nu} = \int (\mathrm{d}\boldsymbol{k}_2)\,(\mathrm{d}\boldsymbol{k}_3)\,\Psi^*(\{k_i'\})\,\Psi(\{k_i\})$$
$$\times \{c_1\, M^{\mu\nu}\,(l_2 \cdot l_3) + c_2\,[l_2^\mu\,(l_3 \cdot M)^\nu + l_3^\mu\,(l_2 \cdot M)^\nu]\}$$

<u>Assumption :</u> ➢ $SU(6)$ symmetry ➡ $c_1^u = 1, \quad c_2^u = 4$
$c_1^d = 2, \quad c_2^d = -1$

[C.L., Pasquini, Vdh (in preparation)]

Wigner Distributions



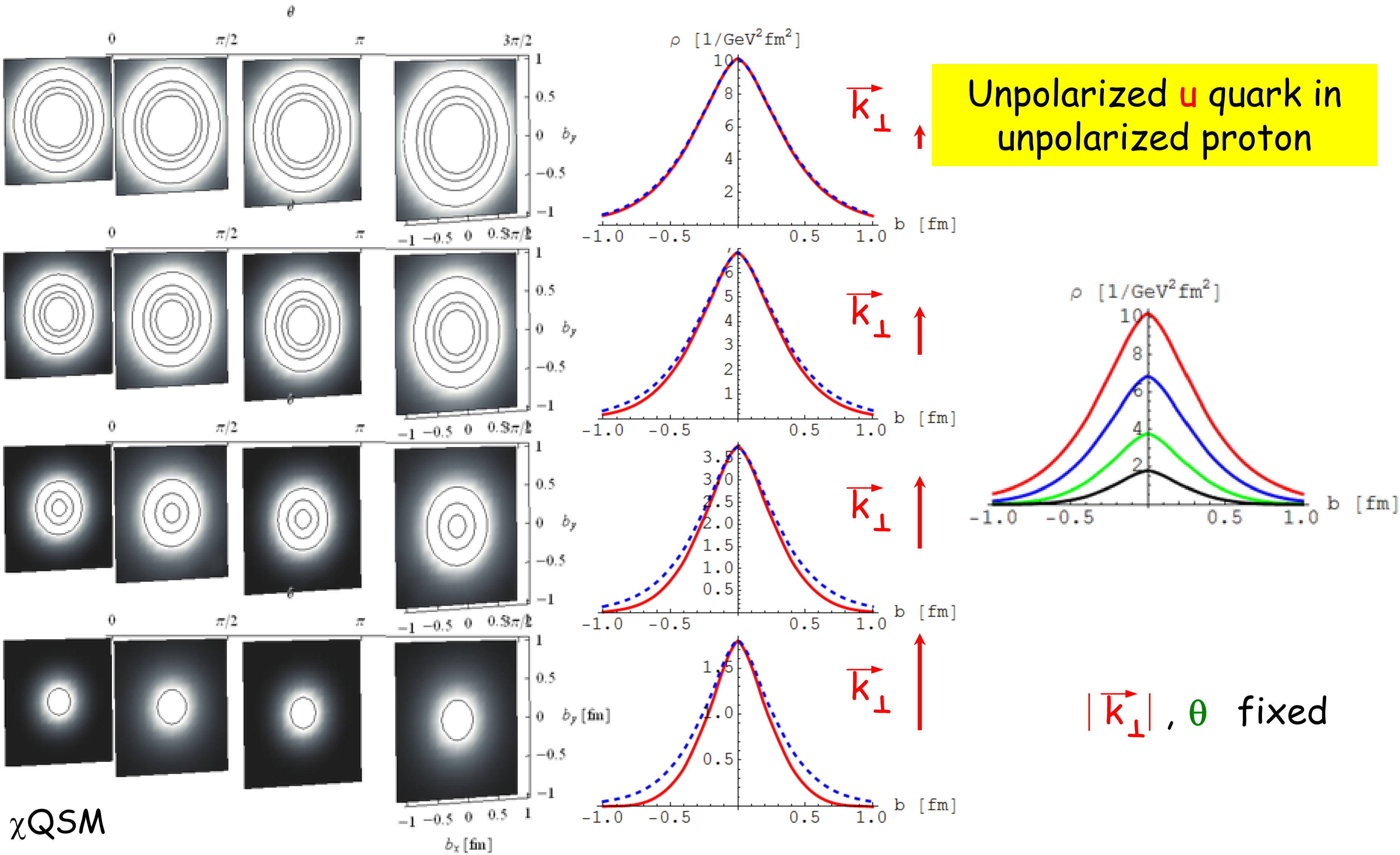


$|\vec{k}_\perp|$, θ fixed

Unpolarized u quark in unpolarized proton

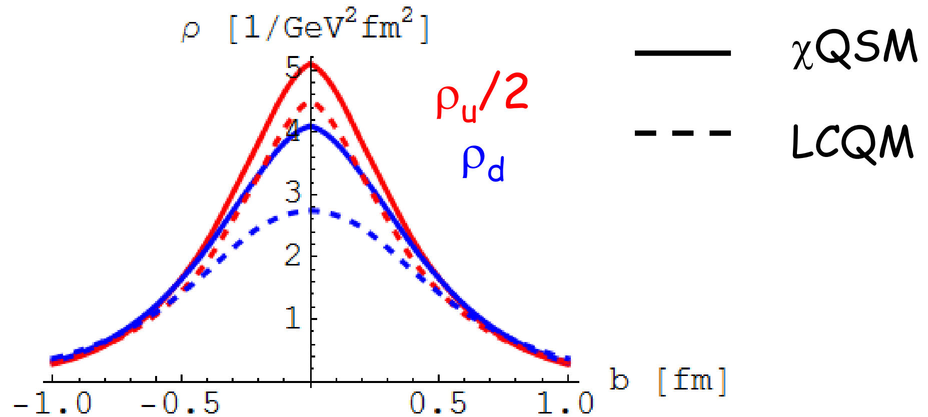
χQSM

Wigner Distributions

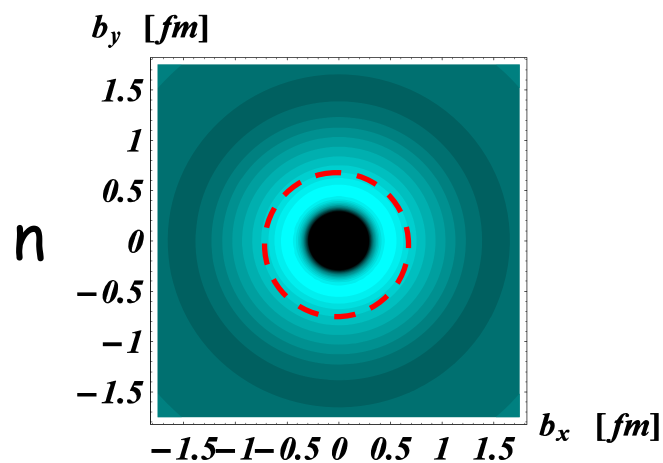


Wigner Distributions

Unpolarized u and d quarks in unpolarized proton

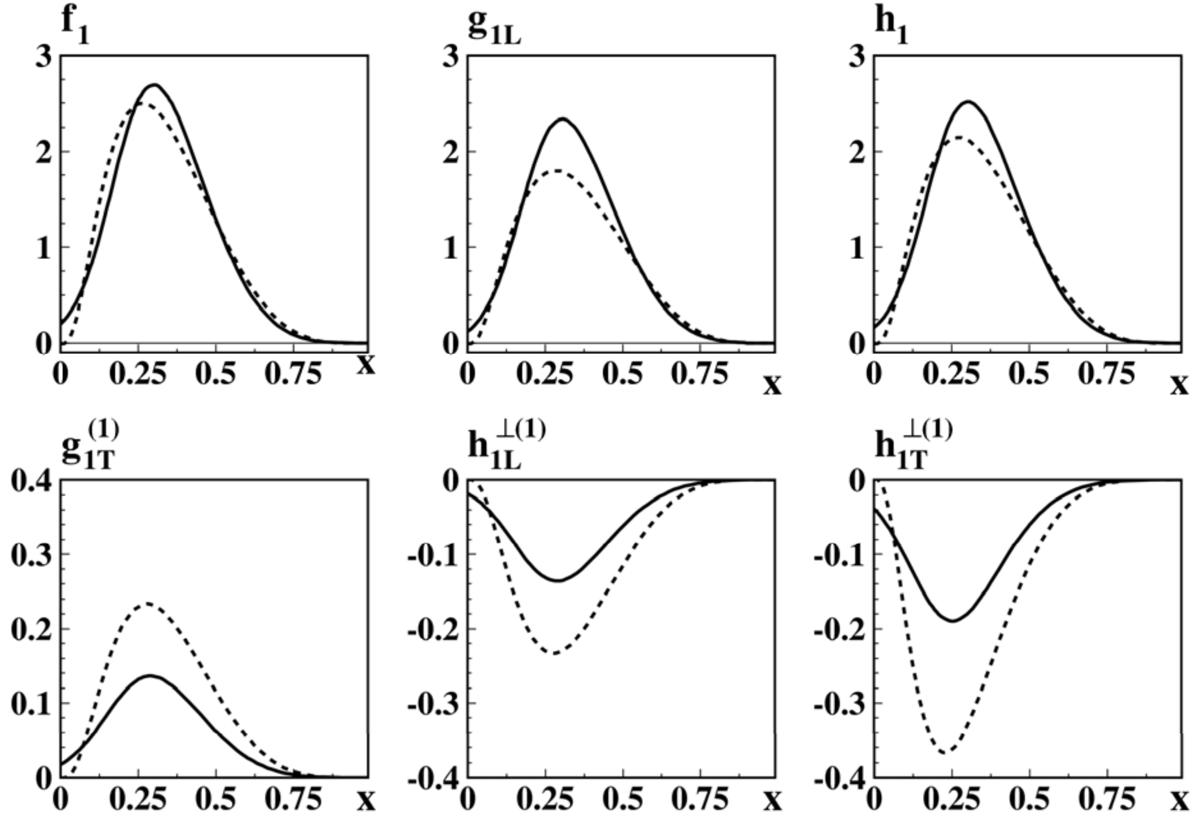


More u than d in central region!

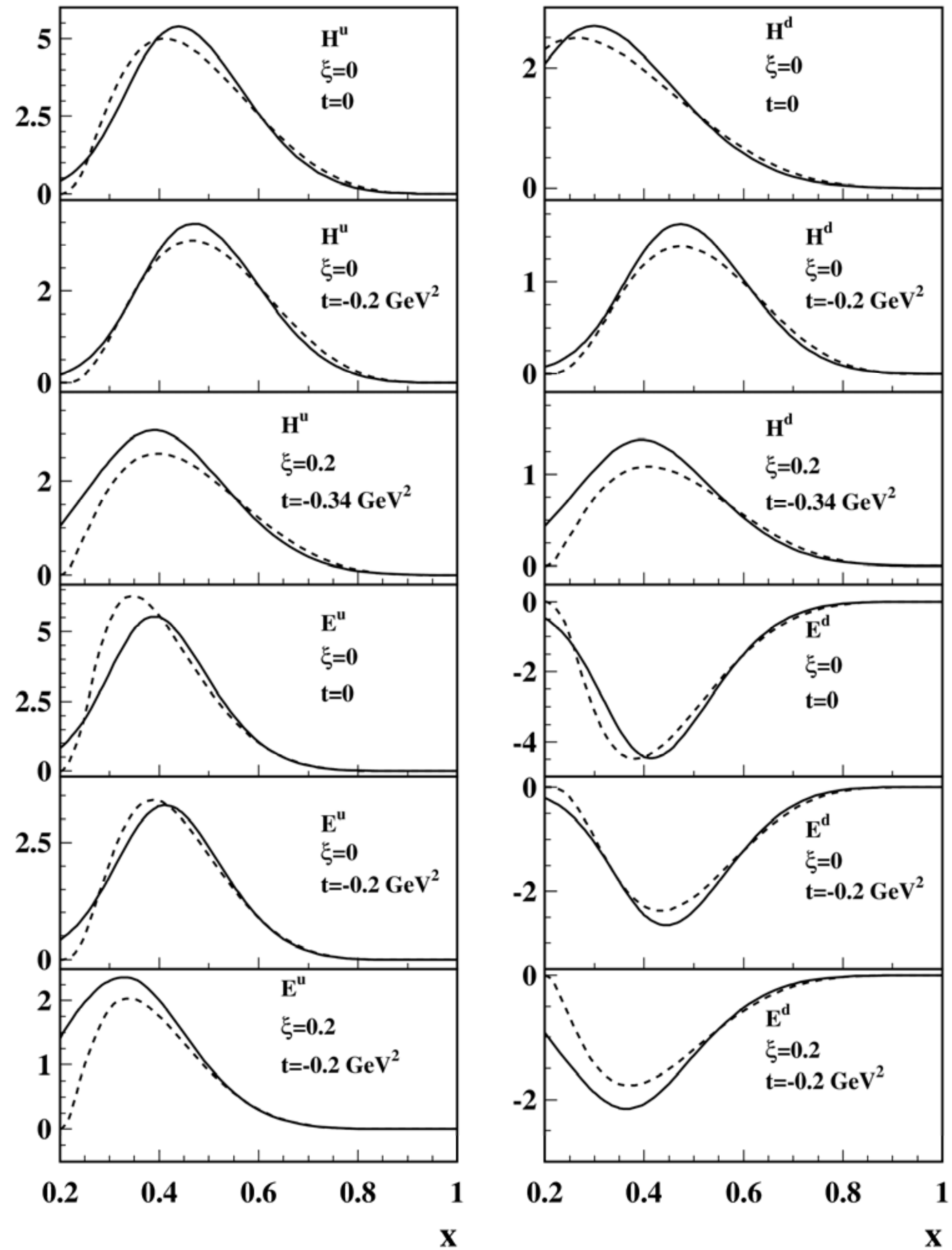


[Miller (2007)]

TMDs

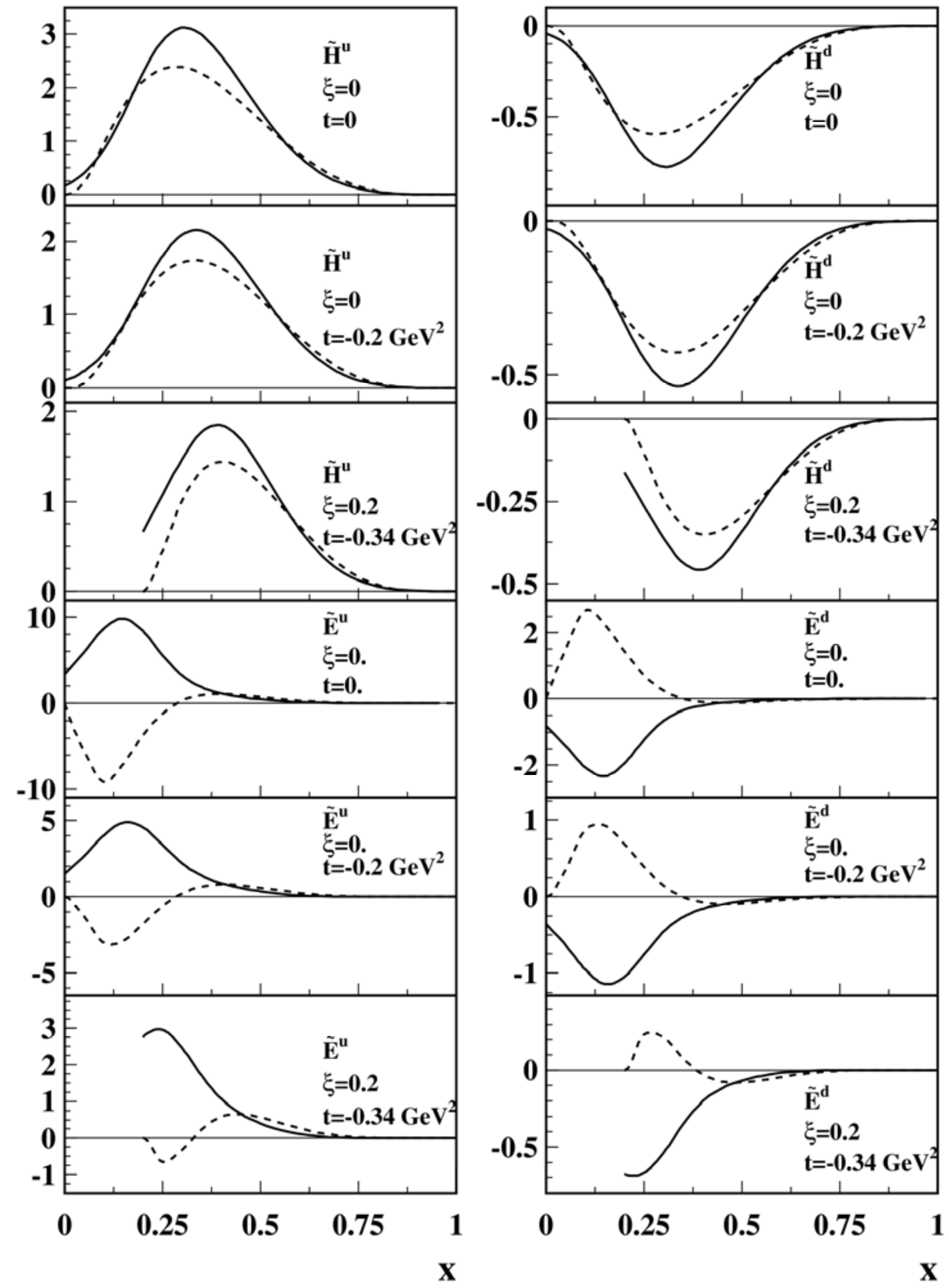


GPDs



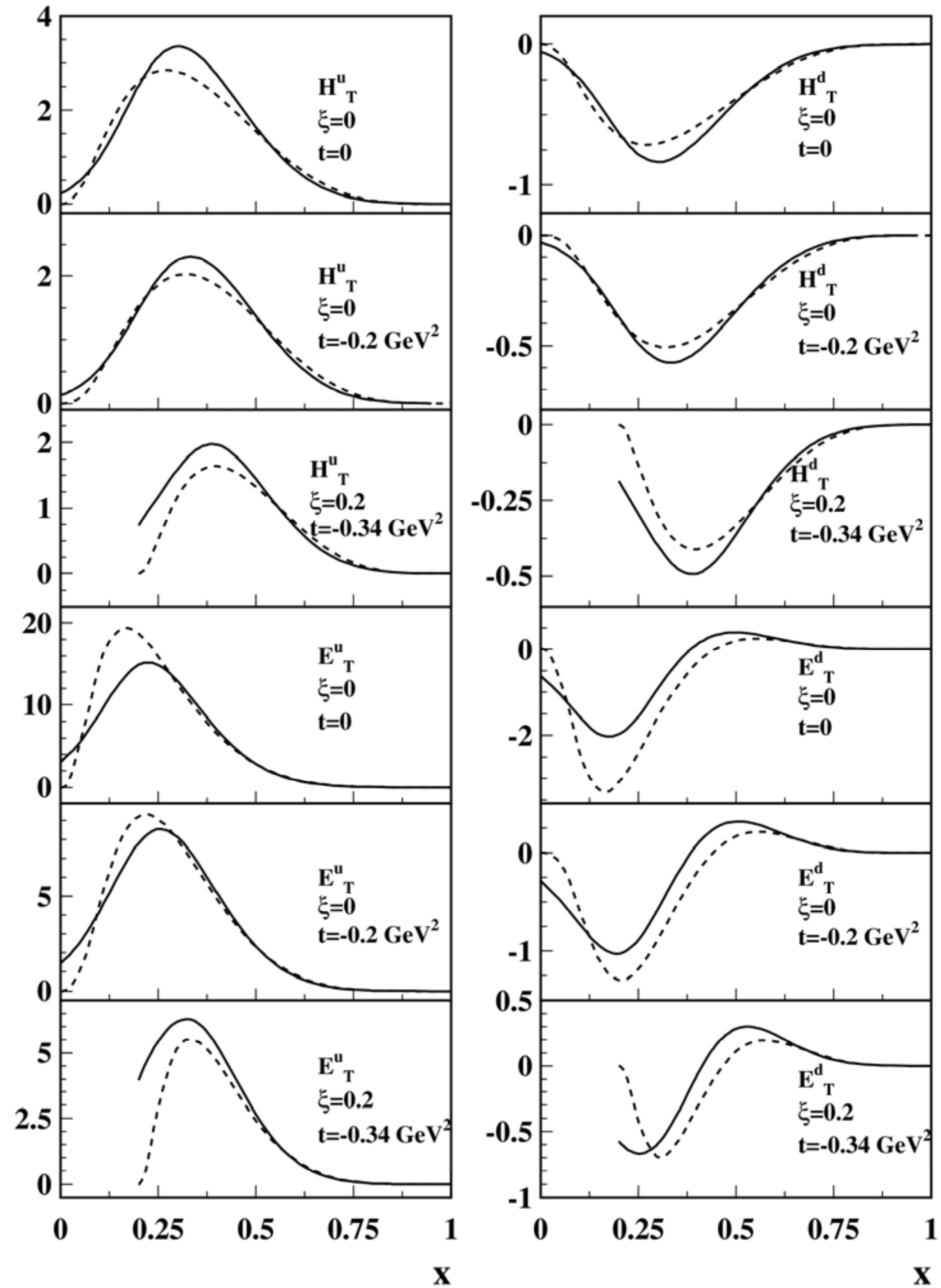
H^u
ξ=0
t=0
H^d
ξ=0
t=0
H^u
ξ=0
t=-0.2 GeV^2
H^d
ξ=0
t=-0.2 GeV^2
H^u
ξ=0.2
t=-0.34 GeV^2
H^d
ξ=0.2
t=-0.34 GeV^2
E^u
ξ=0
t=0
E^d
ξ=0
t=0
E^u
ξ=0
t=-0.2 GeV^2
E^d
ξ=0
t=-0.2 GeV^2
E^u
ξ=0.2
t=-0.2 GeV^2
E^d
ξ=0.2
t=-0.2 GeV^2
0.2
0.4
0.6
0.8
1
x

GPDs



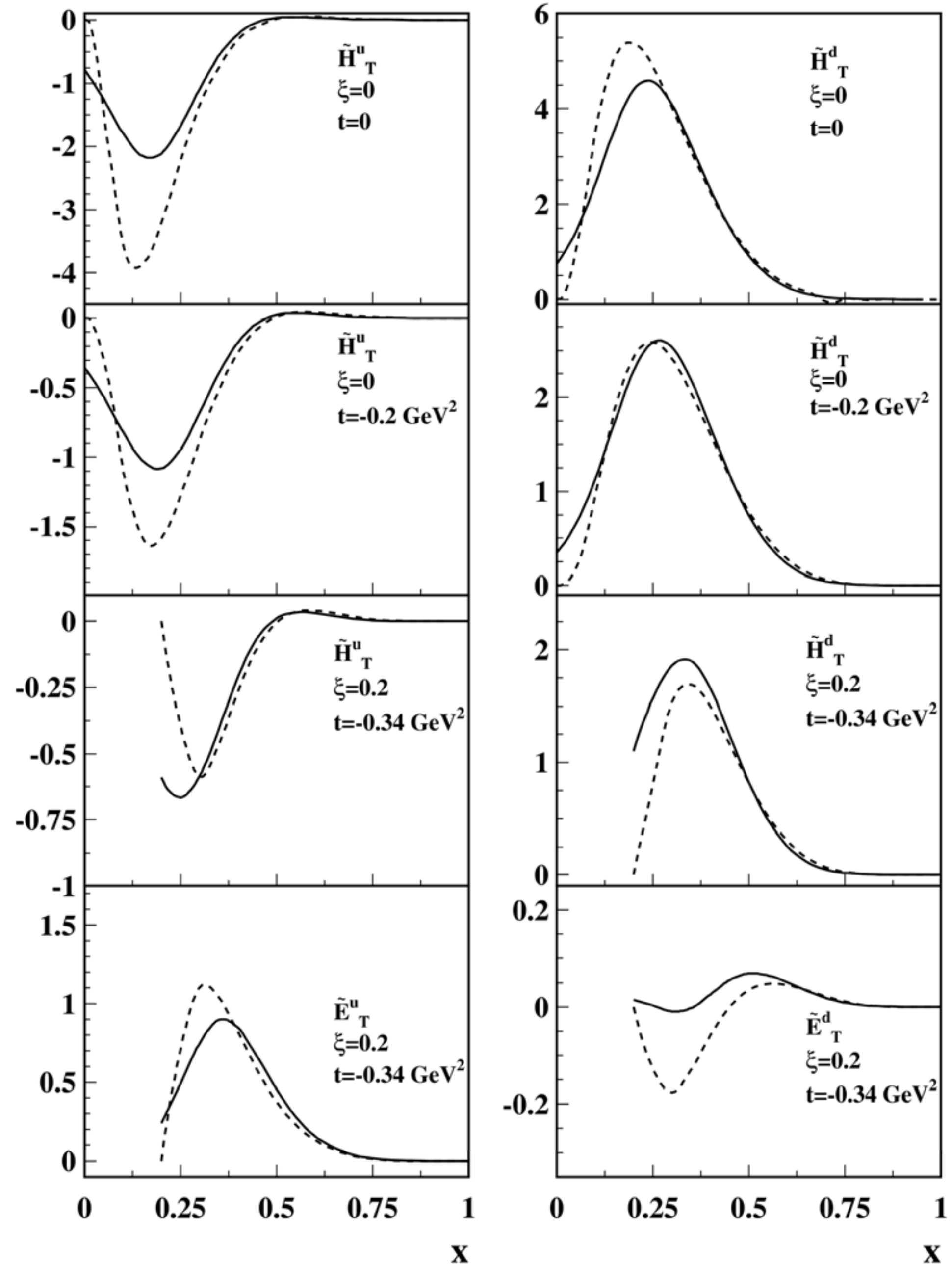
$\tilde{H}^u$ ξ=0 t=0
$\tilde{H}^d$ ξ=0 t=0
$\tilde{H}^u$ ξ=0 t=-0.2 GeV^2
$\tilde{H}^d$ ξ=0 t=-0.2 GeV^2
$\tilde{H}^u$ ξ=0.2 t=-0.34 GeV^2
$\tilde{H}^d$ ξ=0.2 t=-0.34 GeV^2
$\tilde{E}^u$ ξ=0. t=0.
$\tilde{E}^d$ ξ=0. t=0.
$\tilde{E}^u$ ξ=0. t=-0.2 GeV^2
$\tilde{E}^d$ ξ=0. t=-0.2 GeV^2
$\tilde{E}^u$ ξ=0.2 t=-0.34 GeV^2
$\tilde{E}^d$ ξ=0.2 t=-0.34 GeV^2
0 0.25 0.5 0.75 1
x

GPDs



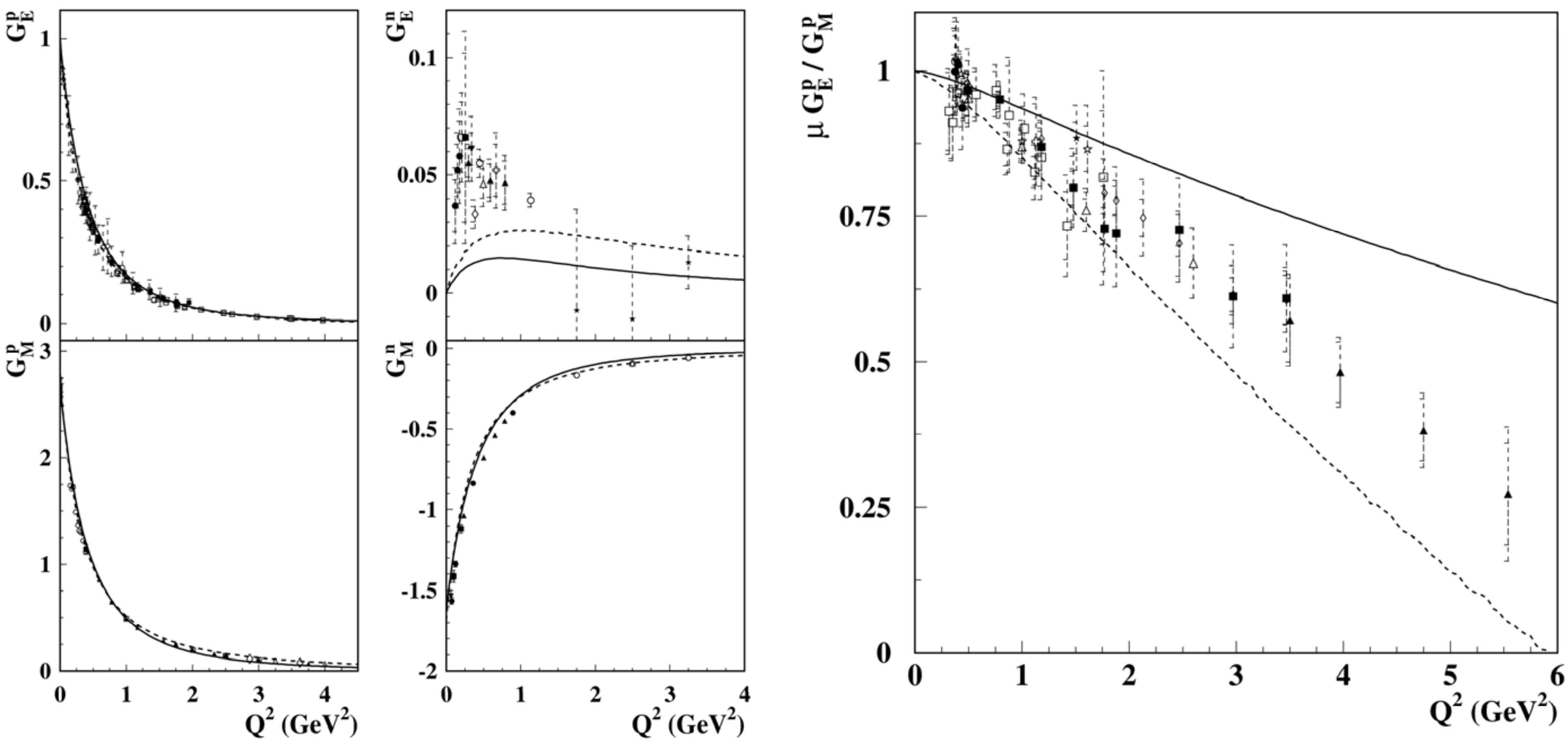
4
2
0
H^u_T
ξ=0
t=0
2
0
H^u_T
ξ=0
t=-0.2 GeV²
2
1
0
H^u_T
ξ=0.2
t=-0.34 GeV²
20
10
0
E^u_T
ξ=0
t=0
5
0
E^u_T
ξ=0
t=-0.2 GeV²
5
2.5
0
E^u_T
ξ=0.2
t=-0.34 GeV²
0
0.25
0.5
0.75
1
x
0
-1
H^d_T
ξ=0
t=0
0
-0.5
H^d_T
ξ=0
t=-0.2 GeV²
0
-0.25
-0.5
H^d_T
ξ=0.2
t=-0.34 GeV²
0
-2
E^d_T
ξ=0
t=0
0
-1
E^d_T
ξ=0
t=-0.2 GeV²
0.5
0
-0.5
-1
E^d_T
ξ=0.2
t=-0.34 GeV²
0
0.25
0.5
0.75
1
x

GPDs



H̃ᵘ_T
ξ=0
t=0
H̃ᵈ_T
ξ=0
t=0
H̃ᵘ_T
ξ=0
t=-0.2 GeV²
H̃ᵈ_T
ξ=0
t=-0.2 GeV²
H̃ᵘ_T
ξ=0.2
t=-0.34 GeV²
H̃ᵈ_T
ξ=0.2
t=-0.34 GeV²
Ẽᵘ_T
ξ=0.2
t=-0.34 GeV²
Ẽᵈ_T
ξ=0.2
t=-0.34 GeV²
x
x

FFs

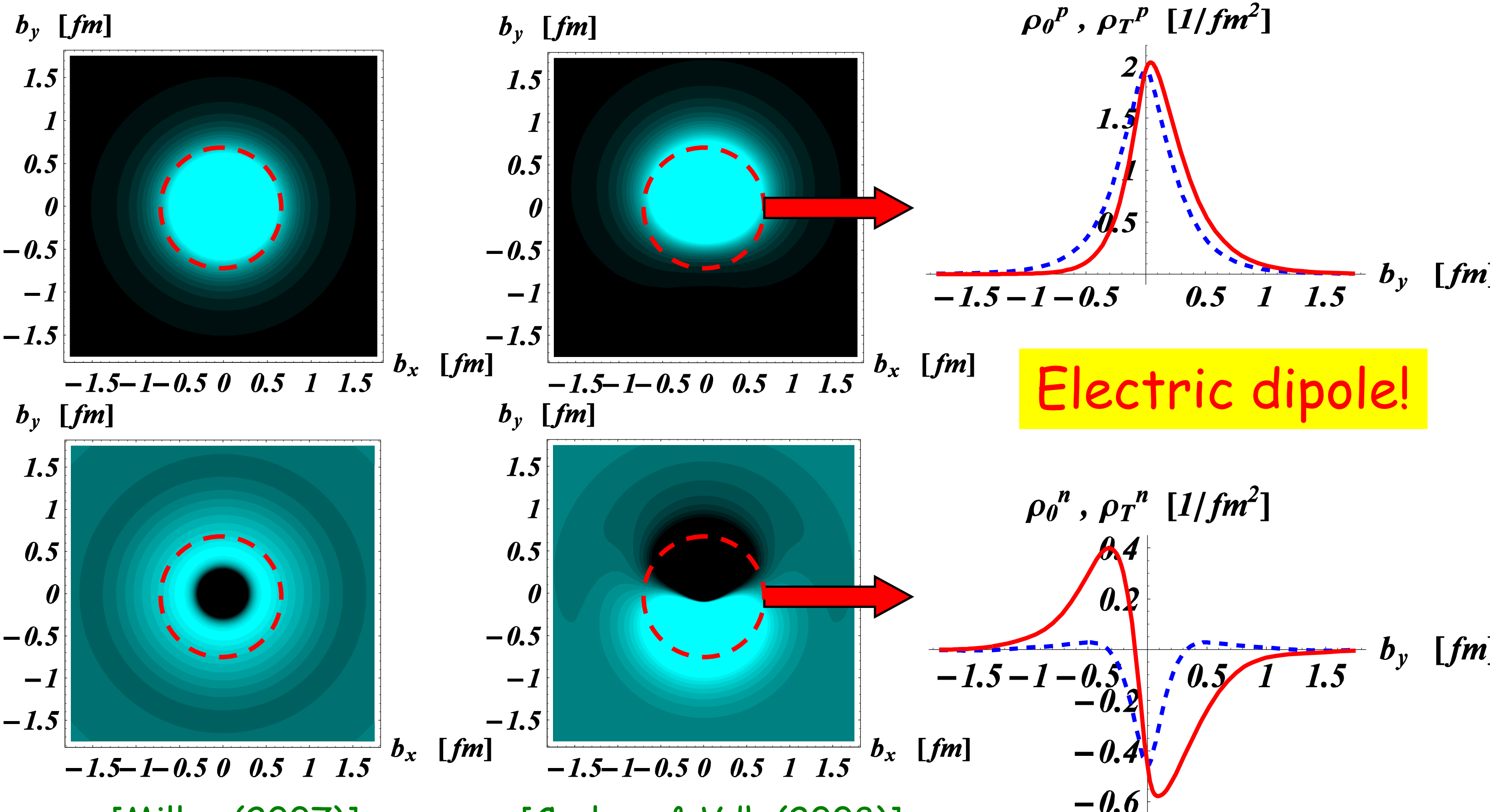


Transverse Charge Densities

Long. pol. Transv. pol.

p

n



[Miller (2007)]

[Carlson & Vdh (2008)]

Transverse Charge Densities

Transversity basis $|s_\perp = \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\rangle + |-\frac{1}{2}\rangle\right)$

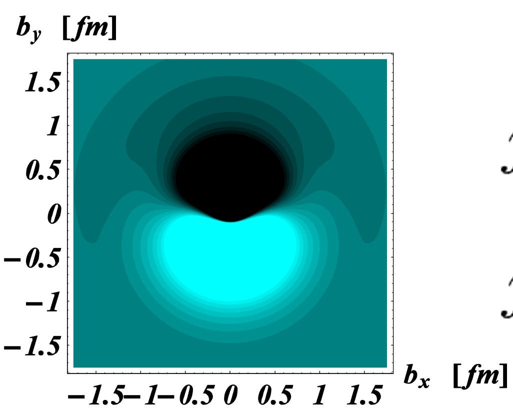
$$\rho^N_{T\frac{1}{2}}(\vec{b}) = \boxed{\int_0^\infty \frac{dQ}{2\pi} Q\, J_0(bQ)\, F_1(Q^2)} + \boxed{\sin\phi_b \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M} J_1(bQ)\, F_2(Q^2)}$$

Monopole

Dipole Anomalous!

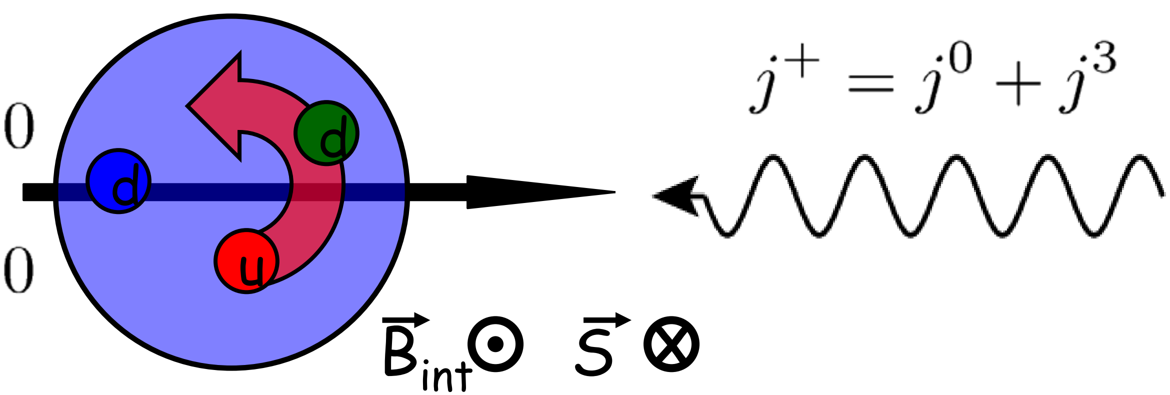
Induced EDM $d_y = F_2(0)\,\frac{e}{2m}$

Interpretation:



$j^3 < 0$

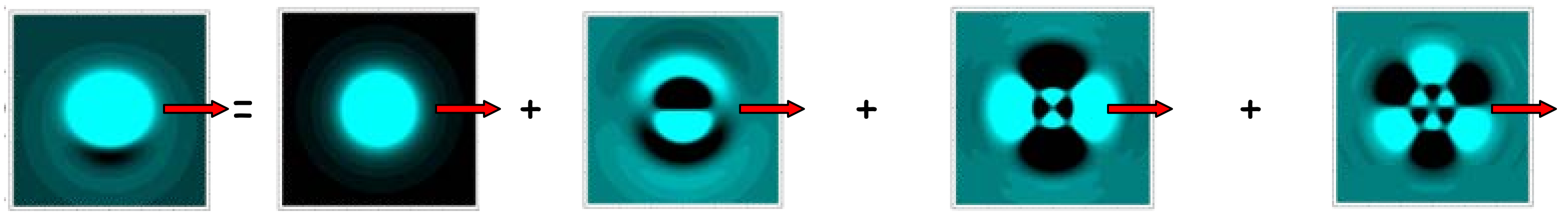
$j^3 > 0$



$j^+ = j^0 + j^3$

Transverse Charge Densities

Similarly for higher spins 2j+1 circular multipoles!



monopole dipole quadrupole octupole

Δ^+

[Alexandrou *et al.* (2008,2009)]

$$\rho_{T s_\perp}(\vec{b}) = \int_0^\infty \frac{\mathrm{d}Q}{2\pi}\left[G_{TE0}(b,Q) + \sin\phi_b\, G_{TE1}(b,Q) + \cos 2\phi_b\, G_{TE2}(b,Q) + \sin 3\phi_b\, G_{TE3}(b,Q) + \cdots\right]$$

[Lorcé (2009)] $A_{\lambda,\lambda}$ $A_{\lambda,\lambda-1}$ $A_{\lambda,\lambda-2}$ $A_{\lambda,\lambda-3}$

Claim: Distortions of transverse charge densities due to anomalous values of EM moments

Higher Spins

2j+1 circular multipoles!

j	$G_{E0}(0)$ (e)	$G_{M1}(0)$ $(e/2M)$	$G_{E2}(0)$ (e/M^2)	$G_{M3}(0)$ $(e/2M^3)$	$G_{E4}(0)$ (e/M^4)	$G_{M5}(0)$ $(e/2M^5)$
0	1					
$1/2$	1	1				
1	1	2	-1			
$3/2$	1	3	-3	-1		
2	1	4	-6	-4	1	
$\vdots$						
j	C^0_{2j}	C^1_{2j}	$-C^2_{2j}$	$-C^3_{2j}$	C^4_{2j}	C^5_{2j}

Dirac
EW
} SM

Supergravity

[Lorcé (2009)]

Charge normalization

Universal g=2 factor

$$G_{M1}(0) = 2j$$