Effective theory for few-body systems near unitarity

H.-W. Hammer

Helmholtz-Institut für Strahlen- und Kernphysik and Bethe Center for Theoretical Physics

Universität Bonn

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Agenda

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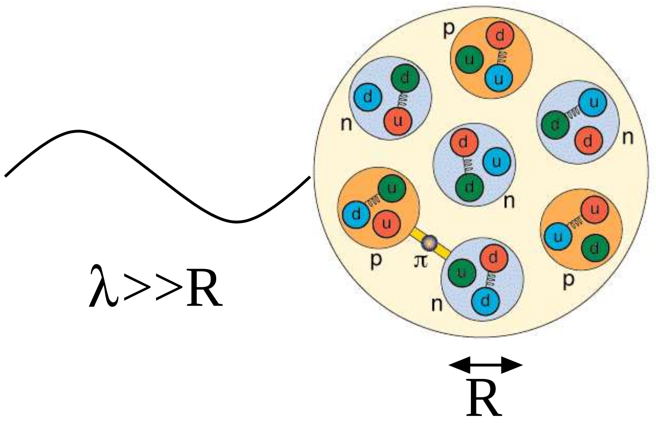
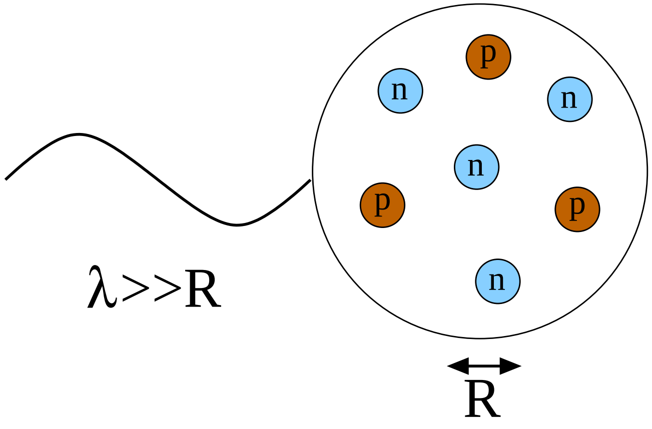
- Introduction
- Few-body Physics near Unitarity
- Effective Field Theory Framework
- Applications
  - Ultracold atoms
  - Nuclear physics
  - Hadronic molecules
- Summary and Outlook

Effective Theory

- Separation of scales:
  $1/k = \lambda \gg R$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of kR



Effective Theory

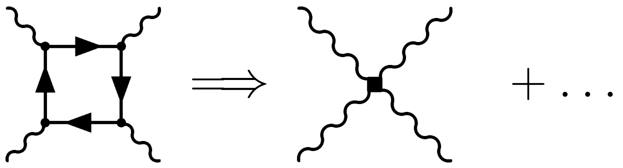


- Separation of scales:
  $1/k = \lambda \gg R$
- Limited resolution at low energy:
  $\longrightarrow$ expand in powers of kR

- Short-distance physics not resolved
  $\longrightarrow$ capture in low-energy constants using renormalization
  $\longrightarrow$ include long-range physics explicitly
- Systematic, model independent $\rightarrow$ universal properties
- Classic example: light-light-scattering (Euler, Heisenberg, 1936)

  Simpler theory for $\omega \ll m_e$:

  $\mathcal{L}_{QED}[\psi, \bar{\psi}, A_\mu] \rightarrow \mathcal{L}_{eff}[A_\mu]$



Few-body Physics near Unitarity

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- Unitary limit: $a \to \infty$, $\ell \to 0$ (cf. Bertsch problem, 2000)

$$T_2(k) = [k \cot \delta - ik]^{-1} \quad \Longrightarrow \quad i/k$$

- Scattering amplitude saturates unitarity bound
- Interesting many-body physics: BEC/BCS crossover, perfect liquid, ...
- Use as starting point for EFT description of few-body systems with resonant interactions
  - Large scattering length: $|a| \gg \ell \sim r_e, l_{vdW}, \ldots$
  - Natural expansion parameter: $\ell/|a|$, $k\ell$,...
  - Universal properties

$$a > 0 \quad \Longrightarrow \quad B_d = \frac{1}{2\mu a^2} + \mathcal{O}(\ell/a)$$

Few-body Systems near Unitarity

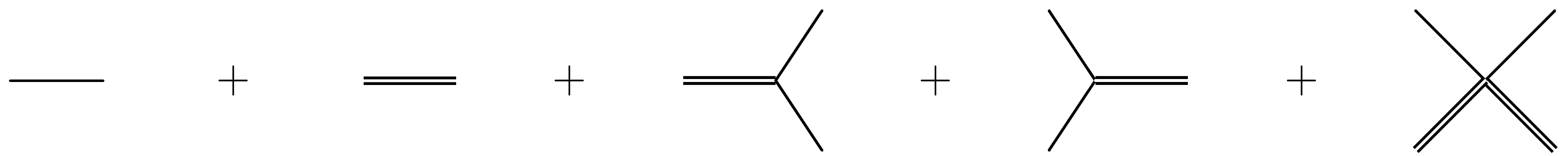
- Many systems are close to unitarity
- Atomic physics:
  - ^{4}He: $a \approx 104$ Å $\gg r_e \approx 7$ Å $\sim l_{vdW} \longrightarrow B_d \approx 100$ neV
  - Feshbach resonances $\Longrightarrow$ scattering length can be tuned using external magnetic field
- Nuclear physics: S-wave NN-scattering, halo nuclei,...
  - $^1S_0$, $^3S_1$: $|a| \gg r_e \sim 1/m_\pi \longrightarrow B_d \approx 2.2$ MeV
  - ^{6}He $\Rightarrow \alpha nn$: $2n$ separation energy ≈ 1 MeV
- Particle physics:
  - $X(3872)$ as a $D^0\bar{D}^{0*}$ molecule? ($J^{PC} = 1^{++}$)

$$B_X = m_{D^0} + m_{D^{0*}} - m_X = (0.3 \pm 0.4) \text{ MeV}$$

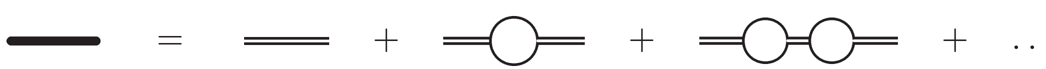
Two-Body System in EFT

- Effective Lagrangian (Kaplan, 1997; Bedaque, HWH, van Kolck, 1999)

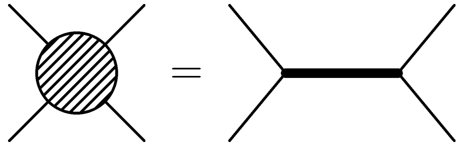
$$\mathcal{L}_d = \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right)\psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4}(d^\dagger \psi^2 + (\psi^\dagger)^2 d) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi + ..$$



- Interacting dimeron propagator $\longrightarrow$ sum bubbles



- Two-body amplitude $\mathcal{T}_2(k,k)$:

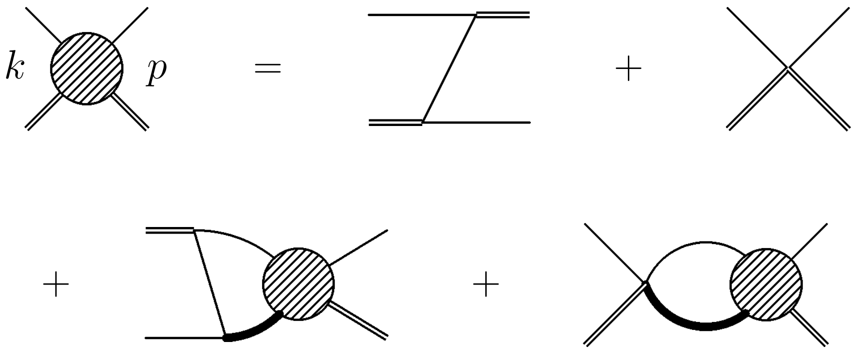


$$\propto \frac{1}{1/a - ik} + \ldots$$

- Matching: $g_2 \longleftarrow a, B_d, \ldots$
- RG fixed points of g_2: $a = 0$ and $a = \infty$ (scale invariance)
- Higher order corrections $\Longrightarrow$ perturbation theory

Three-Body System in EFT

- Three-body equation :

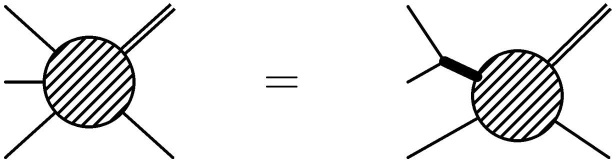


$$\mathcal{T}_3(k,p) = M(k,p) + \frac{4}{\pi}\int_0^{\Lambda} dq\, q^2\, M(q,p) D_d(q)\, \mathcal{T}_3(k,q)$$

with $\quad M(k,p) \quad = \quad \underbrace{F(k,p)}_{\text{1-atom exchange}} \quad \underbrace{-\frac{g_3}{9g_2^2}}_{H(\Lambda)/\Lambda^2}$

($g_3 = 0$, $\Lambda \to \infty \longrightarrow$ Skorniakov, Ter-Martirosian '57)

- Recombination, break-up:



Renormalization

- Observables are independent of regulator/cutoff Λ

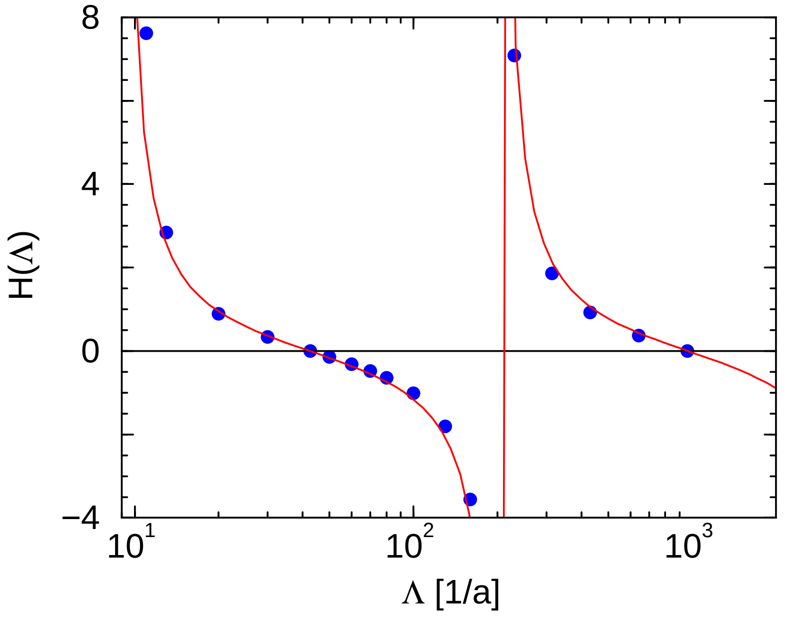
$\Rightarrow$ Running coupling $H(\Lambda)$

- $H(\Lambda)$ periodic: limit cycle

$$\Lambda \to \Lambda\, e^{n\pi/s_0} \approx \Lambda(22.7)^n$$

(cf. Wilson, 1971)

- Full scale invariance broken to discrete subgroup



$$H(\Lambda) = \frac{\cos(s_0 \ln(\Lambda/\Lambda_*) + \arctan(s_0))}{\cos(s_0 \ln(\Lambda/\Lambda_*) - \arctan(s_0))}\,, \qquad s_0 \approx 1.00624$$

- Limit cycle $\Longleftrightarrow$ Discrete scale invariance
- Matching: $\Lambda_* \longleftarrow B_t,\, K_3, \ldots \longrightarrow \kappa_*, a_*, a'_*$

Limit Cycle: Efimov Effect

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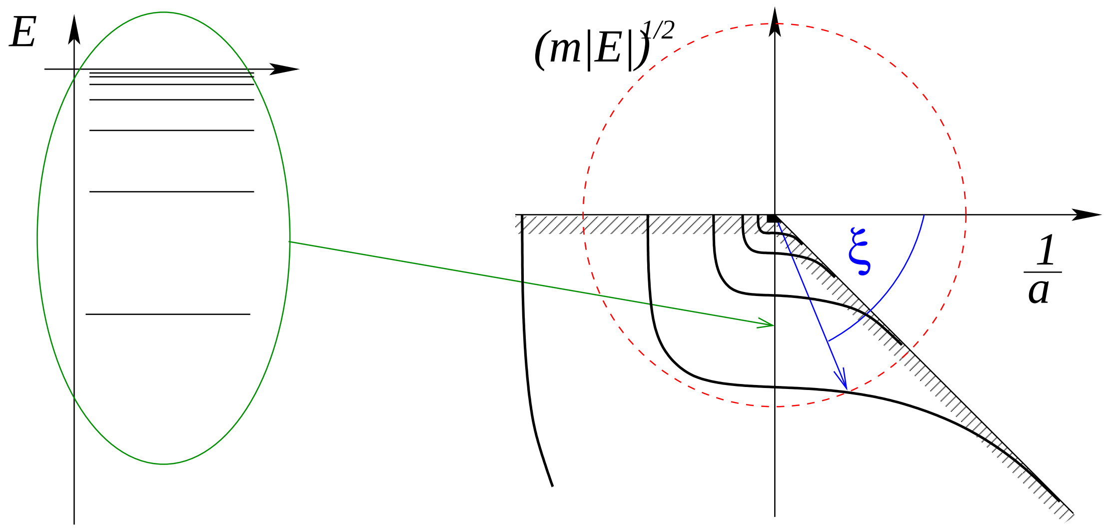
- Universal spectrum of three-body states

  (V. Efimov, Phys. Lett. **33B** (1970) 563)



  - Discrete scale invariance for fixed angle ξ
  - Geometrical spectrum für $1/a \rightarrow 0$

$$B_3^{(n)}/B_3^{(n+1)} \quad \xrightarrow{1/a\to 0} \quad 515.035\ldots$$

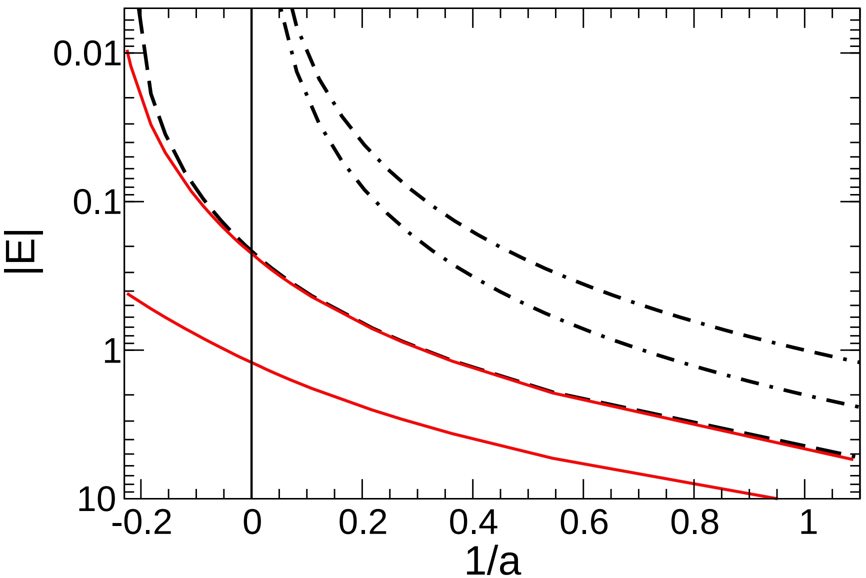
- Ultracold atoms $\Longrightarrow$ variable scattering length

Four-Body System

- No four-body parameter at LO (Platter, HWH, Meißner, 2004)
- Universal properties of 4-body system with large a
  - Bound state spectrum, scattering observables, ...
- "Efimov-plot": 4-body bound state spectrum as function of $1/a$

$$B_4^{(0)} = 5 B_3^{(0)} \qquad (1/a \equiv 0)$$
$$B_4^{(1)} = 1.01 B_3^{(0)}$$

(Platter, HWH, EPJA **32** (2007) 113)



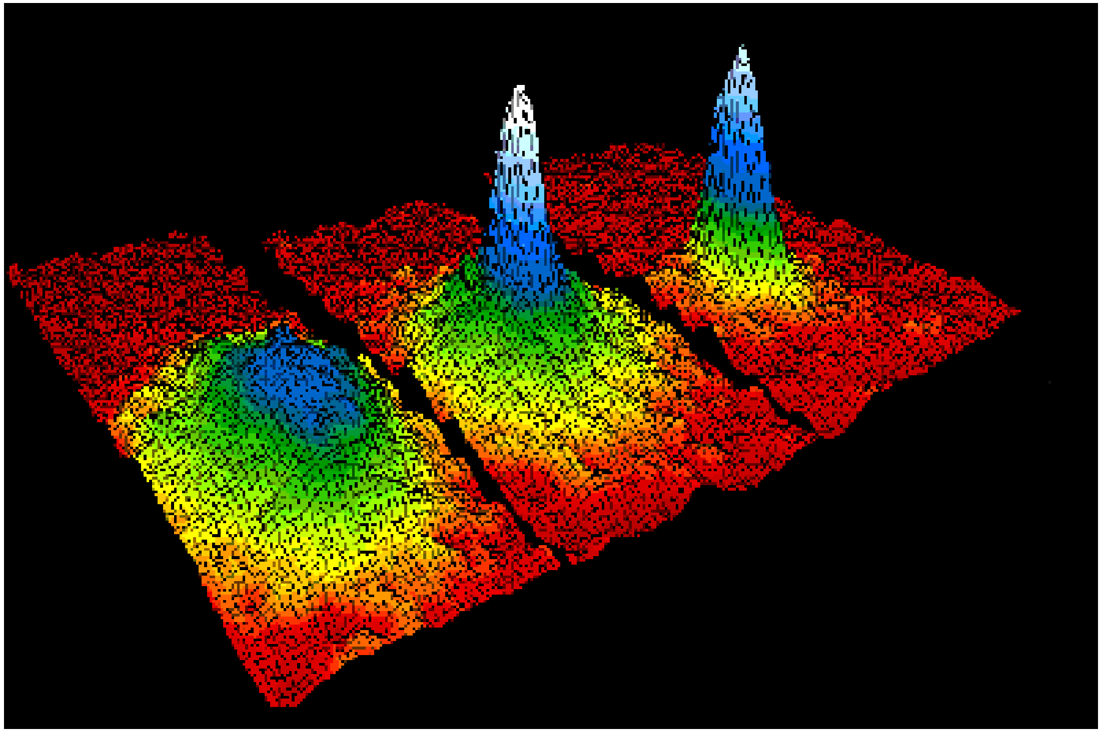
- Improved theoretical decription and signature in Cs loss data

von Stecher, D'Incao, Greene, Nature Physics **5** (2009) 417

Ferlaino, Knoop, Berninger, Harm, D'Incao, Nägerl, Grimm, PRL **102** (2009) 140401

Efimov Physics in Cold Atoms

- Velocity distribution ($T = 400$ nK, 200 nK, 50 nK)



(Source: http://jilawww.colorado.edu/bec/)

- Few-body loss rates provide window on Efimov physics
- Variable scattering length via Feshbach resonances

Three-Body Recombination

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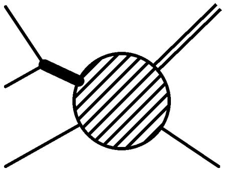


- Three-body recombination:

  3 atoms $\rightarrow$ dimer + atom $\Rightarrow$ loss of atoms

- Recombination constant: $\dot{n}_A = -K_3\, n_A^3$
- K_3 has log-periodic dependence on scattering length

  (Nielsen, Macek, 1999; Esry, Greene, Burke, 1999; Bedaque, Braaten, HWH, 2000)

- Resonant enhancement for $a < 0$
- Universal line shape of recombination resonance

  (Braaten, HWH, 2004)

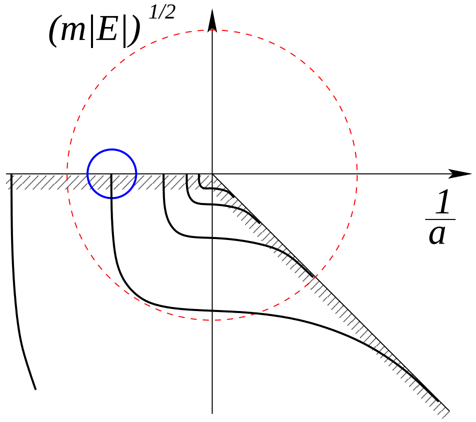
$$K_3^{deep} = \frac{(4677 \pm 2)\, \sinh 2\eta_*}{\sin^2\left[s_0 \ln(a/a'_*)\right] + \sinh^2 \eta_*}\, \frac{\hbar\, a^4}{m}\,, \qquad s_0 \approx 1.00624..$$

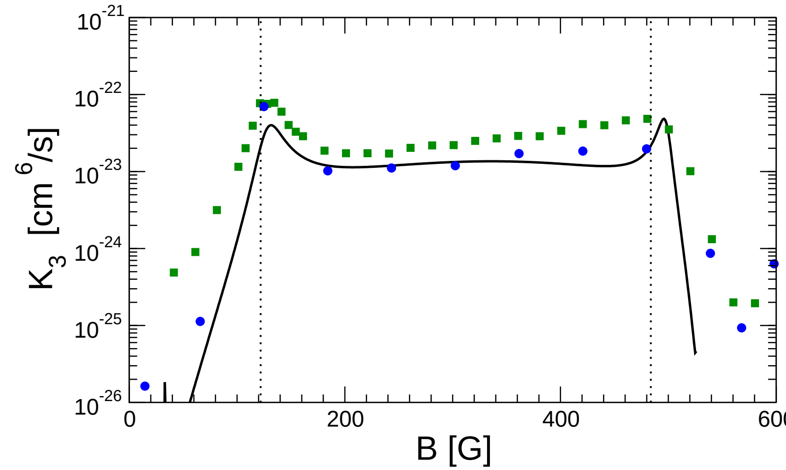
- Evidence for Efimov trimers in ^{133}Cs (... ^{6}Li, ^{7}Li, ^{39}K, ^{41}K/^{87}Rb)

  (Kraemer et al. (Innsbruck), Nature **440** (2006) 315)

Efimov Physics with Fermions

- Efimov effect for fermions $\Rightarrow \geq 3$ spin states ($|1\rangle, |2\rangle, |3\rangle, ..$)
- Experimental evidence for Efimov states in ^{6}Li
  - ■ Ottenstein et al. (Heidelberg), Phys. Rev. Lett. **101** (2008) 203202
  - ● Huckans et al. (Penn State), Phys. Rev. Lett. **102** (2009) 165302





Braaten, HWH, Kang, Platter, Phys. Rev. Lett. **103** (2009) 073202

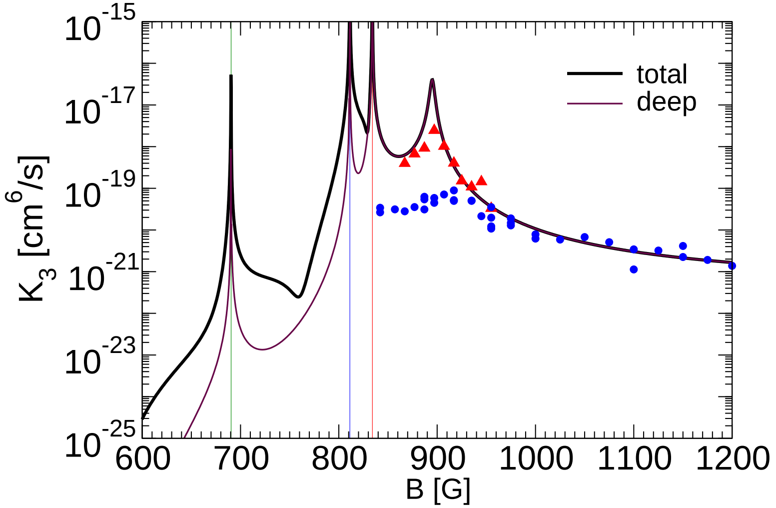
- Systematic normalization error: 70-90%
- Related work: Naidon, Ueda; Schmidt, Floerchinger, Wetterich (2009)

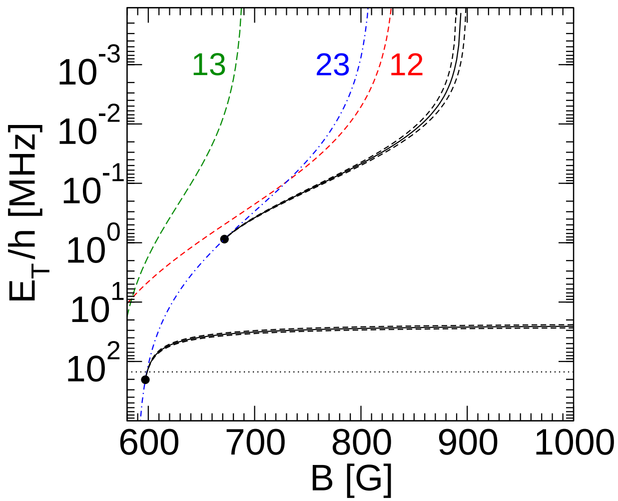
Efimov Physics in ^{6}Li

- Recombination resonances in high field region ($|a| \gtrsim 30\,\ell_{vdW}$)

  Williams et al. (Penn State), Phys. Rev. Lett. **103** (2009) 130404

- Recombination and bound state spectrum





  Braaten, HWH, Kang, Platter, Phys. Rev. A **81** (2010) 013605

- Prediction: atom-dimer relaxation resonance around 680 G ($1-23$)

  HWH, Kang, Platter, Phys. Rev. A **82** (2010) 022715

- Experiment: resonance at 685 G $\Rightarrow$ higher order corrections

  Lompe et al. (Heidelberg), Phys. Rev. Lett. **105** (2010) 103201

Efimov Physics in ^{6}Li

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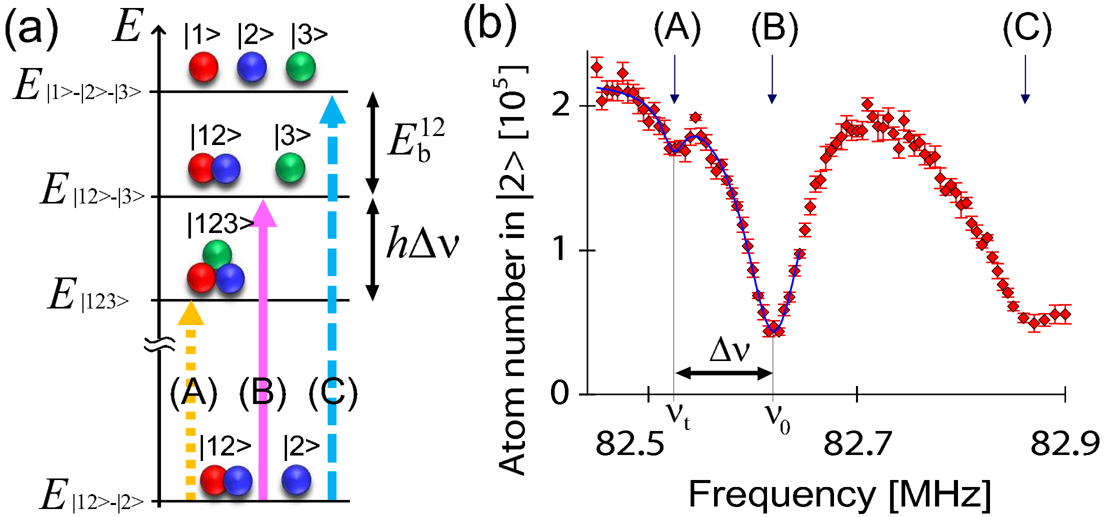
- Direct observation of Efimov trimers through radio frequency association in ^{6}Li

  Lompe et al. (Heidelberg), arXiv:1006.2241

  Nakajima et al. (Tokyo), arXiv:1010.1954



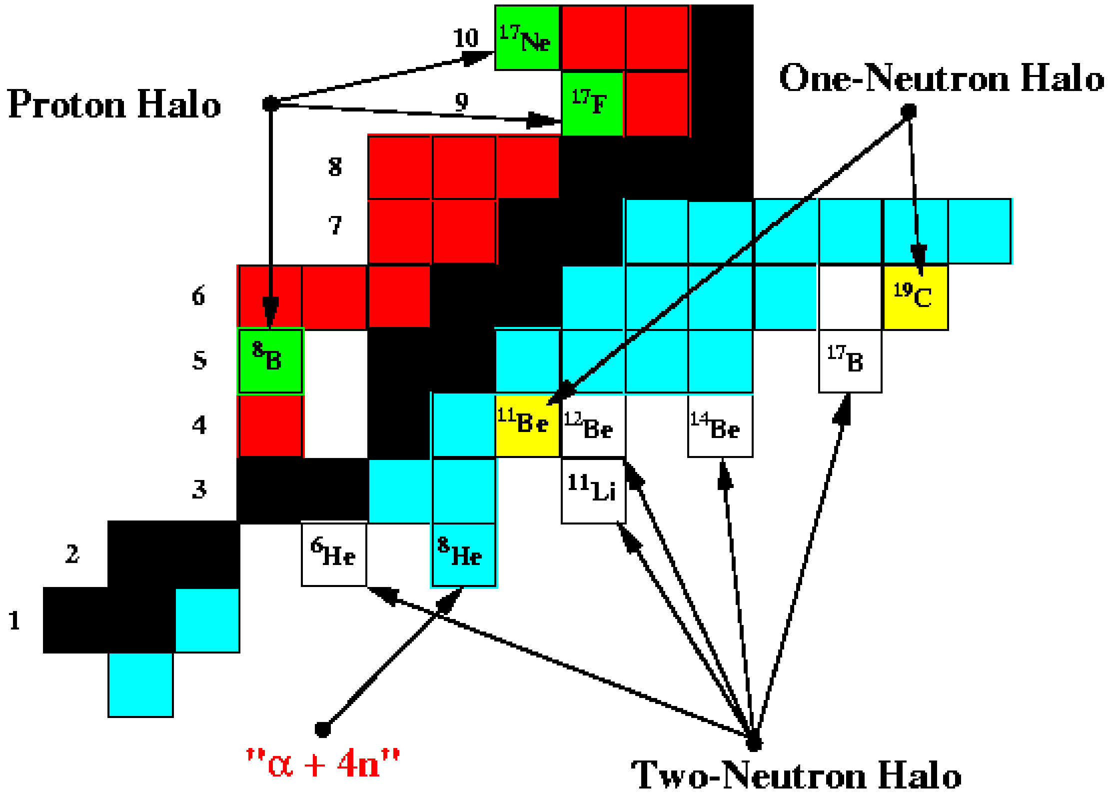
- Higher order corrections are required for quantitative description
- Phenomenological model with energy-dependent scattering length and three-body parameter can describe data

  Nakajima et al. (Tokyo), arXiv:1010.1954

Halo Nuclei

- Low separation energy of valence nucleons: $B_{valence} \ll B_{core}, E_{ex}$

  $\longrightarrow$ close to "nucleon drip line" $\longrightarrow$ scale separation $\longrightarrow$ EFT



http://www.nupecc.org

- EFT for halo nuclei $\Longleftrightarrow$ cluster models

3-Body Halos

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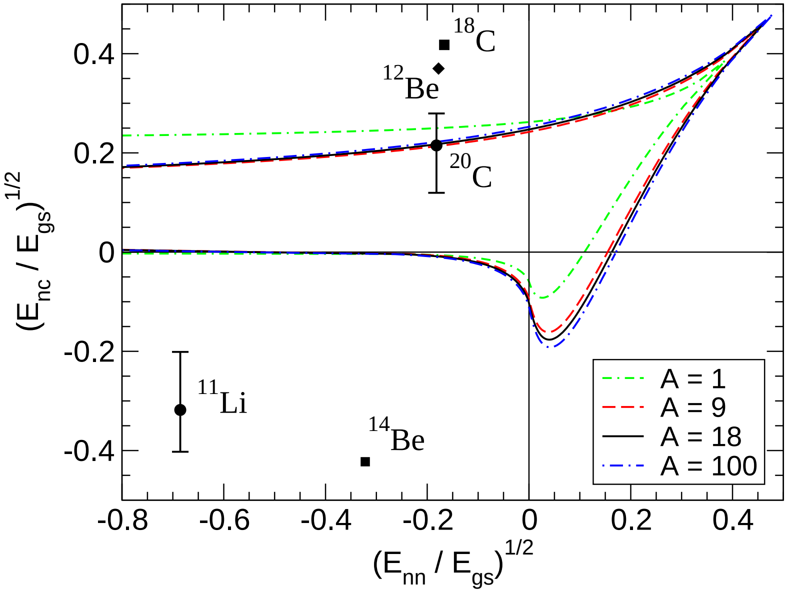
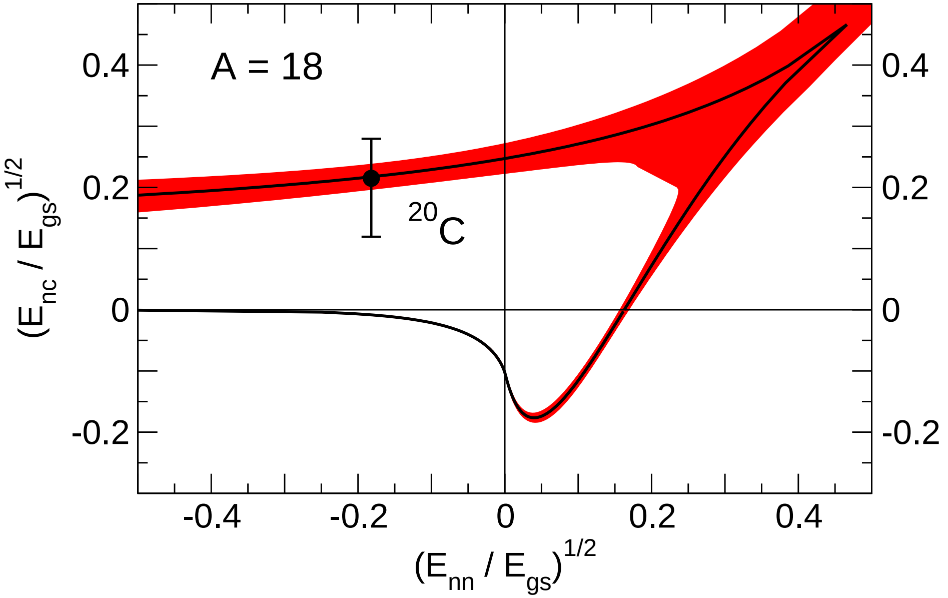
- Examples: $^{14}\text{Be} \longleftrightarrow {}^{12}\text{Be} + n + n$, $\quad {}^{20}\text{C} \longleftrightarrow {}^{18}\text{C} + n + n$
- "Effective" 3-body system: separation energy of valence nucleons small compared to binding energy of "core"
- Efimov effect in halo nuclei? $\Rightarrow$ excited states





Canham, HWH, Eur. Phys. J. A **37** (2008) 367 (cf. Amorim, Frederico, Tomio, 1997)

- Unchanged by NLO range corrections (Canham, HWH, 2010)

Form Factors and Radii

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- Structure of halo nuclei $\rightarrow$ matter form factors, radii

nucleus	B_{nnc} [keV]	B_{nc} [keV]	$\sqrt{\langle r_{nn}^2 \rangle}$ [fm]	$\sqrt{\langle r_{nc}^2 \rangle}$ [fm]
^{14}Be	1120	-200.0	4.1 $\pm$ 0.5	3.5 $\pm$ 0.5
^{20}C	3506	162	2.8 $\pm$ 0.3	2.4 $\pm$ 0.3
	3506	60	2.8 $\pm$ 0.2	2.3 $\pm$ 0.2
^{20}C*	65 $\pm$ 6.8	60	42 $\pm$ 3	38 $\pm$ 3

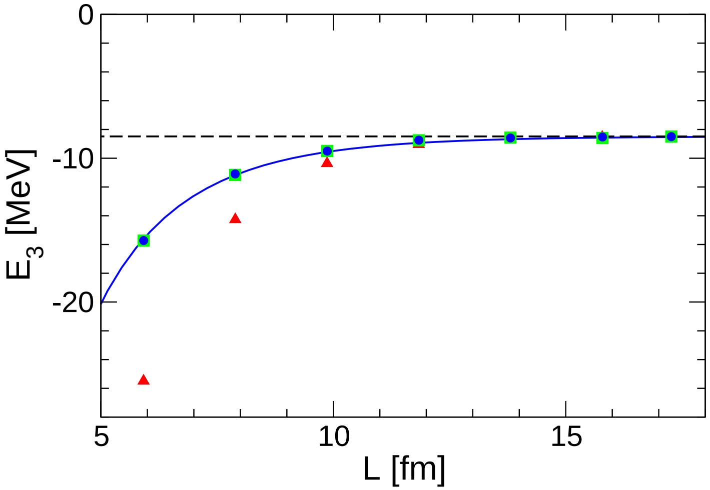
Canham, HWH, Eur. Phys. J. A **37** (2008) 367

(cf. Yamashita, Tomio, Frederico, 2004)

- Input: TUNL Nuclear data evaluation project, ...
- Experiment: ^{14}Be $\rightarrow \sqrt{\langle r_{nn}^2 \rangle} = (5.4 \pm 1.0)$ fm
  (Marques et al., Phys. Rev. C **64** (2001) 061301)

Triton in a Finite Volume

- Light nuclei can also be described in an expansion around the unitary limit (Efimov 1981) $\rightarrow$ pionless effective field theory
- Can be used to calculate volume dependence of triton binding energy
  $\Rightarrow$ Lattice QCD calculations of light nuclei (e.g. NPLQCD collaboration)
- Modification of spectrum by cubic box ($V = L^3$)
  - Box provides infrared cutoff $1/L$
    $\Rightarrow$ calculable in EFT
  - Box breaks rotational invariance
    $\Rightarrow$ partial wave mixing
  - Momenta quantized $\vec{p} = \vec{n}\,(2\pi/L)$
    $\Rightarrow$ 3d sum equation



Kreuzer, HWH, arXiv:1008.4499

Hadronic Molecules

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- Many new $c\bar{c}$-mesons at B-factories: X, Y, Z
  - Challenge for understanding of QCD
  - Large scattering length physics important
- Example: $X(3872)$ (Belle, CDF, BaBar, D0)

$$m_X = (3871.55 \pm 0.20)\ \text{MeV} \qquad \Gamma < 2.3\ \text{MeV} \qquad J^{PC} = 1^{++}$$

- No ordinary $c\bar{c}$-state
  - Decays violate isospin
  - Measured mass depends on decay channel
- Nature of $X(3872)$?
  - $D^0 D^{0*}$-molecule? (cf. Tornquist, 1991)
  - Tetraquark
  - Charmonium Hybrid
  - ...

Nature of $X(3872)$

- Nature of $X(3872)$ not finally resolved
- Assumption: $X(3872)$ is weakly-bound D^0-$\bar{D}^{0*}$-molecule

  $\Longrightarrow |X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0 D^{0*}\rangle)/\sqrt{2}\,, \qquad B_X = (0.26 \pm 0.41)$ MeV

  $\Longrightarrow$ universal properties (cf. Braaten et al., 2003-2008, ...)

  - Explains isospin violation in decays of $X(3872) \Rightarrow$ superposition of $I=1$ and $I=0$
  - Different masses due to different line shapes in decay channels
- Large scattering length to LO determines interaction of $X(3872)$ with D^0 and D^{0*}
- Higher orders: EFT with perturbative pions

  (Fleming, Kusunoki, Mehen, van Kolck, 2007; Fleming, Mehen, 2008)

  (Braaten, HWH, Mehen, 2010)

Interactions of $X(3872)$

- Large scattering length determines interaction of $X(3872)$ with D^0 and D^{0*}
- Efimov effect?

  $\Rightarrow$ occurs if 2 out of 3 pairs have resonant interactions
- $X(3872)$: only 3 out of 6 pairs have resonant interactions

  $\Rightarrow$ no Efimov effect (Braaten, Kusunoki, 2003)

  $\Rightarrow$ no X-D^0- and X-D^{0*}-molecules

  $\Rightarrow$ no three-body interaction at leading order

Interactions of $X(3872)$

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- $X(3872)$: only 3 out of 6 pairs have resonant interactions

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  $\Rightarrow$ no X-D^0- and X-D^{0*}-molecules

  $\Rightarrow$ no three-body interaction at leading order
- But: parameter-free prediction of X-D^0-, X-D^{0*}-scattering
- Low-energy parameters: $B_X = (0.26 \pm 0.41)$ MeV

  $\Rightarrow$ Scattering length in the X channel: $a = (8.8^{+\infty}_{-3.3})$ fm

X-D- and X-D^*-Scattering

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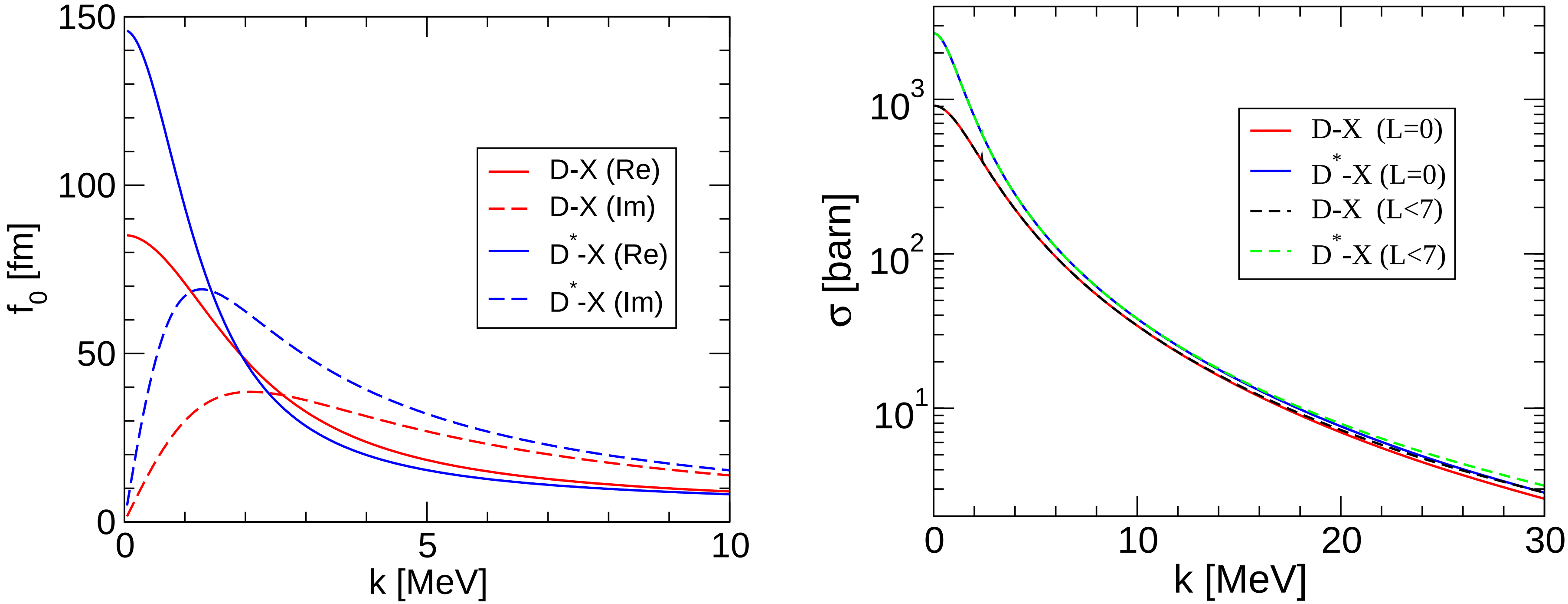
- Predictions for scattering amplitude/cross section



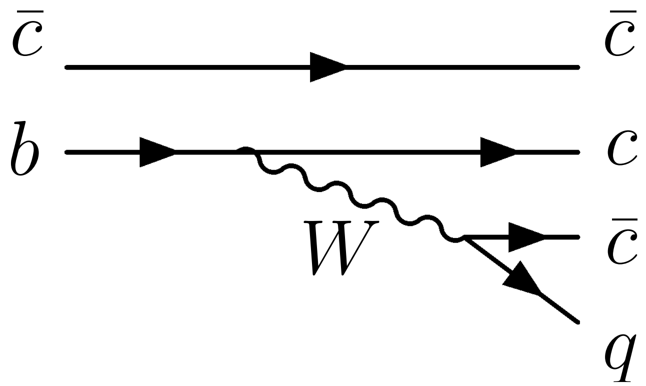
Canham, HWH, Springer, Phys. Rev. D **80**, 014009 (2009)

- Three-body scattering lengths

$$a_{D^0X} = a_{\bar{D}^0X} = -9.7a\,, \quad \text{and} \quad a_{D^{*0}X} = a_{\bar{D}^{*0}X} = -16.6\,a$$

Experimental Observation ?

- Behavior of $X(3872)$ produced in isolation should be distinguishable from its behavior when in the presence of $D^0, D^{*0}, \bar{D}^0, \bar{D}^{*0}$
- Rare events in $B\bar{B}$ production ($B \to X$, $\bar{B} \to D, D^*$)
- Final state interaction of D, D^* mesons in B_c-decays
- Example: quark-level B_c decay yielding three charmed/anticharmed quarks in final state



- Process may be accessible at the LHC

Nature of the $Y(4660)$

- $Y(4660)$ has been intepreted as $f_0(980)\psi'$ bound state
  (Guo, Hanhart, Meißner, Phys. Lett. B **665** (2008) 26, ...)

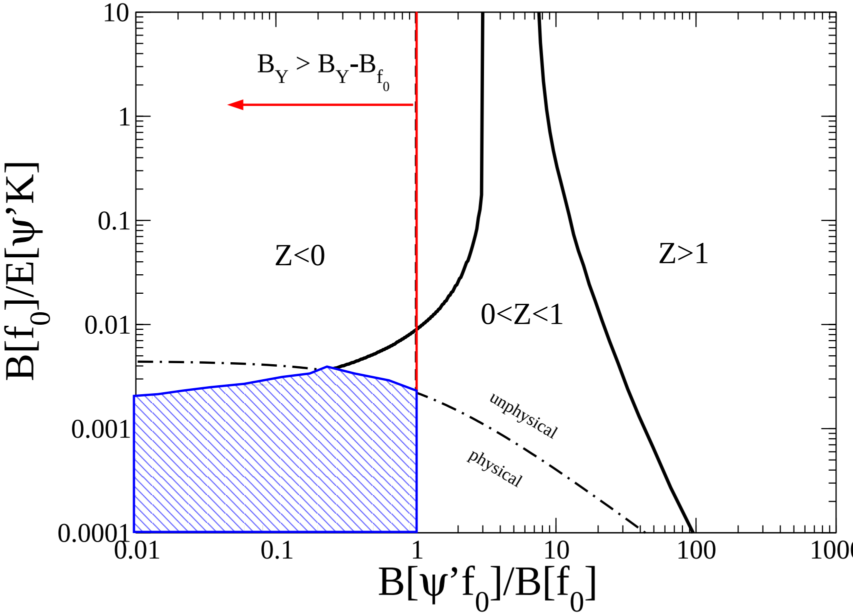
  $\Rightarrow$ predictions for invariant mass distributions, spin partner, ...
  $\Rightarrow$ substructure of the $f_0(980)$?

- Relevant scales: B_Y, B_{f_0}, $a_{K\psi'}$, $B_{\psi' f_0}$

  - Study in 3-body picture:

    $\Rightarrow$ parameter space where 2-body picture applies

$$E_{\psi' K} = 1/(2\mu_{\psi' K} a^2_{\psi' K})$$



Hagen, HWH, Hanhart, arXiv:1007.1126

Summary and Outlook

- Effective field theory for large scattering length
  - Discrete scale invariance, universal correlations,...
- Applications in atomic, nuclear, and particle physics
  - Cold atoms close to Feshbach resonance
  - Few-body nuclei
  - Hadronic molecules: $X(3872)$
- Future directions:
  - Hadronic molecules: universal properties, three-body molecules? (e.g. $Y(4660) \leftrightarrow \psi' f_0(980) \leftrightarrow \psi' K\bar{K}$)
  - Three-nucleon system on the lattice: finite volume corrections, limit cycle in “deformed” QCD?
  - Halo nuclei: reactions, external currents, ...
  - Cold atoms: heteronuclear systems, $N \geq 4$, 2d-systems, ...

Additional Slides

Efimov Physics

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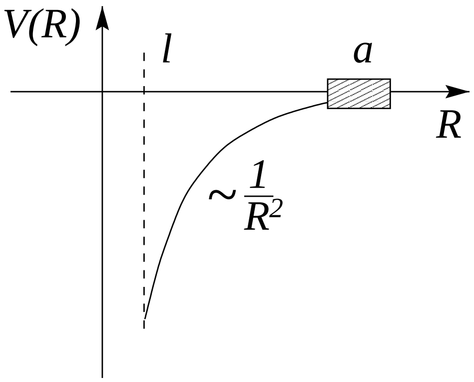
(V. Efimov, Phys. Lett. **33B** (1970) 563)

- Three-body system with large scattering length a
- Hyperspherical coordinates: $R^2 = (r_{12}^2 + r_{13}^2 + r_{23}^2)/3$
- Schrödinger equation simplifies for $|a| \gg R \gg l$:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial R^2} + \frac{s_0^2 + 1/4}{R^2}\right] f(R) = \underbrace{-\frac{\hbar^2\kappa^2}{m}}_{E} f(R)$$



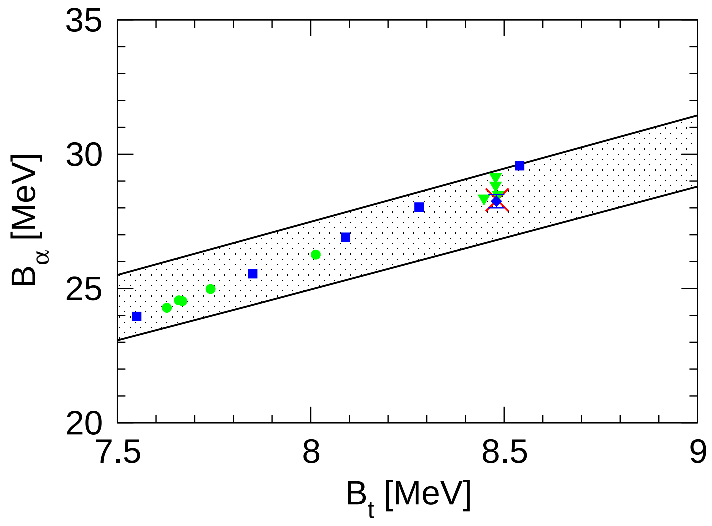
- Singular Potential: renormalization required
- Boundary condition at small R: breaks scale invariance

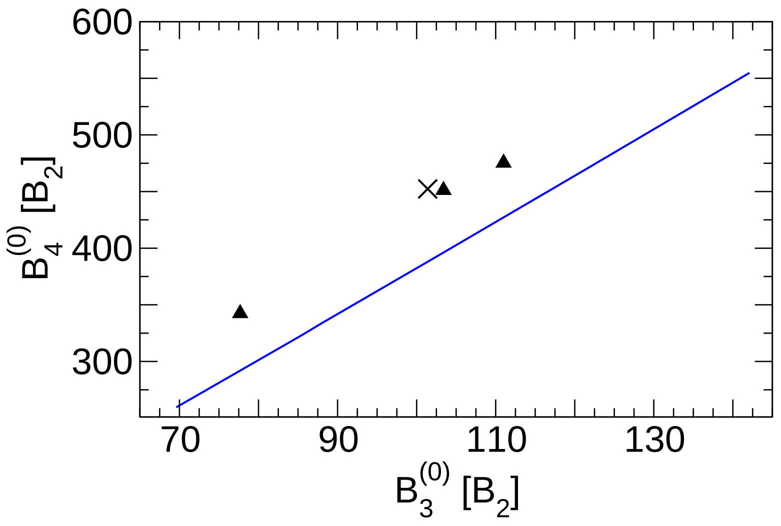
$\Longrightarrow$ dependence of observables on 3-body parameter (and a)

- EFT formulation: boundary condition $\Rightarrow$ 3-body interaction

Universal Correlations

- Two parameters at LO
  $\Rightarrow$ universal correlations (Phillips line, ...)
- RG analysis (Platter, HWH, Meißner, 2004)
  $\Rightarrow$ No four-body parameter at LO
  $\Rightarrow$ 4-body observables are correlated $\Longrightarrow$ Tjon line, ...





- Nuclear physics: Λ dependence of V_{low-k} (Bogner et al., 2004)
- Tjon line also at NLO (Kirscher et al., 2009)