Hydrodynamic modes in neutron star crust

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A neutron star is characterised by:

- A radius: $R \simeq 10 - 15Km$
- A mass: $M \simeq 1 - 2M_{\odot}$
- Compacity: $\Xi = \dfrac{GM}{Rc^2} \simeq 0.2$
- Average density: $\rho \simeq 3.10^{14} g.cm^{-3}$
- Temperature: $T \simeq 10^6 - 10^9 K$
- Period of rotation $P \simeq 0.001 - 10s$
- Magnetic field: $B \simeq 10^7 - 10^{15} G$

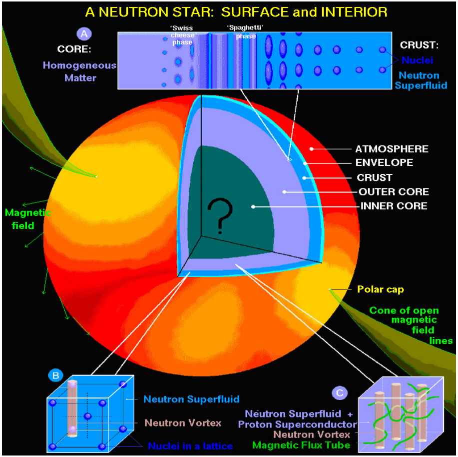

Figure: Neutron star structure, Dany Page

One of the neutron star observables is the surface temperature which can give constraints on the thermal evolution estimating its age.

- Specific heat is one of the elements to study the thermal evolution of neutron star
- Specific heat is a sum over different contributions from the different excitations (nuclei, phonons, electrons,...)
- Shortly after the birth the core contains still a lot of energy which escapes through the crust $\Longrightarrow$ I will study thermal properties of the crust.

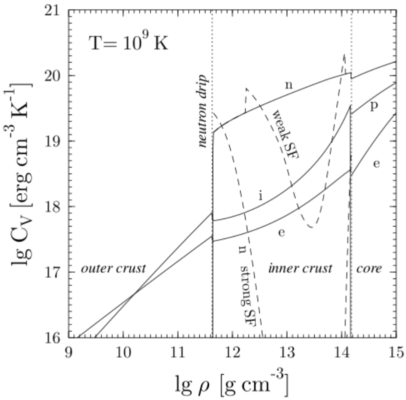

Figure: Specific heat contribution as a function of the density at $T = 10^9$K, Gnedin et al. 2001

We will be interested in the inner crust which contains the structure called "pasta phase".

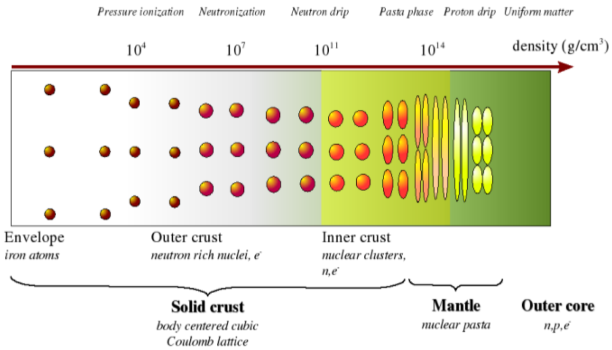

Figure: Neutron star crust

- This part of the crust is characterised by the transition from homogeneous matter to the lattice of atomic nuclei.
- Pasta phase = very deformed nuclei.

What are the different contributions to thermal properties?

- Paired nucleons: contribution strongly suppressed due to pairing gap.
- Contribution of ions, electrons and free neutrons to specific heat
- But superfluidity $\Longrightarrow$ low energy collective excitations called hydrodynamic modes.
- These modes are first order perturbations in density and propagate at sound velocity.

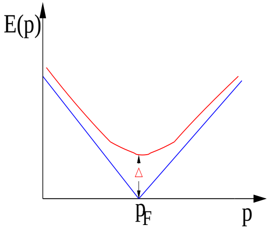

Figure: Energy gap of pairing.

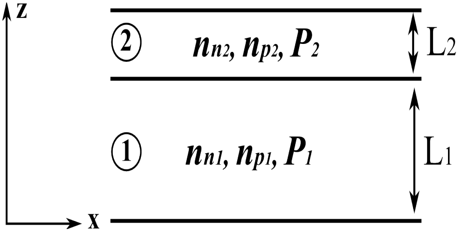

Figure: Representation of "lasagna"

I take the condition of:

- Lasagna: periodic alternance of two slabs("gazeous" and "liquid") with different proton and neutron densities $\Longrightarrow$ different thermodynamical properties
- Zero temperature approximation $\Longrightarrow$ neutrons and protons are treated as superfluids.
- Superfluid hydrodynamics approximation in each slab
- Non-relativistic approximation.

Two basic equations for deriving superfluid hydrodynamics:

- Conservation of particle number: $\partial_\mu \ n^\mu = 0$
- Energy-momentum conservation (Euler equation): $\partial_\mu T^{\mu\nu} = 0$ with $T^\mu{}_\nu = P\delta^\mu{}_\nu + \sum_{x=n,p} n_x^\mu \mu_\nu^x$

Characteristics of hydrodynamics with two superfluid components (n,p):

- No viscosity.
- Entrainment between the two fluids: non dissipative interaction which misalign velocities and momentum.

Parameters appearing in the hydrodynamic equations are calculated within a Landau-Fermi liquid model. Relativistic Mean Field interaction with $\sigma - \omega - \rho - \delta$ mesons is employed with density dependent parameters defined in Avancini et al. (2009).

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At $T \sim 10^8$K $\Longrightarrow$ time period of modes $\gg$ characterisic time of β-interaction, relaxation time
$\Longrightarrow$ Fluids are inviscid
$\Longrightarrow$ Contact is maintained
$\Longrightarrow$ Continuity of perpendicular fluid velocities and continuity of chemical potentials

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3. We use the Floquet-Bloch theorem to take into account the periodicity ($U(z+L) = U(z)e^{iqL}$ where L is the periodicity).
   For now we have considered only waves propagating along z axis

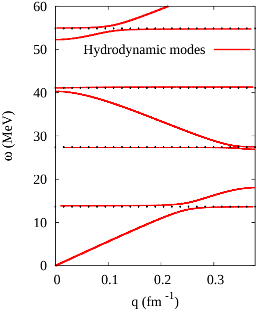

Figure: Baryonic density $n_b = 0.0804 fm^{-3} \sim \frac{\rho_0}{2}$.

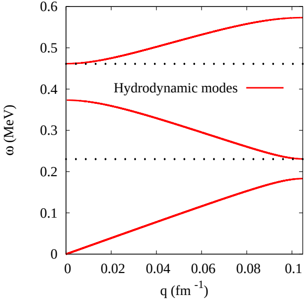

Figure: Baryonic density $n_b = 0.0013 fm^{-3}$.

- Acoustic Branch with linear dispersion law for low momentum
- Optic branches with a cut-off at frequency $\omega = \frac{u_2}{L_2} j\pi$

I have introduced a formalism for wave propagation in "lasagna" taking into account superfluidity and the periodic structure. The dispersion relations show interesting acoustic and optic branches. I expect this kind of excitation may have a significant contribution to thermal properties of the pasta phase (Specific heat...).

I have to develop:

- Consider all directions for hydrodynamic modes in order to calculate the specific heat.
- Resolve the problem for other geometrical structures.
- Non-zero temperature $\Longrightarrow$ addition of a "normal fluid".