

# Comprendre l'infiniment grand: cosmology and large scales in the Universe



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 **COSMOSTAT**

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# Further references

Popular article from 2010:

<http://www.technologyreview.com/view/420529/fine-structure-constant-varies-with-direction-in-space-says-new-data/>

Webb paper on PRL: Varying fine-structure constant: <http://arxiv.org/abs/1008.3907>

Oklo paper: <http://arxiv.org/abs/1008.3957>

More recent, from observations: <http://arxiv.org/abs/1501.00560>

<http://www.nature.com/scientificamerican/journal/v23/n4s/full/scientificamericantime1114-70.html>

Scientific American 23, 70 - 77 (2014)

Published online: 23 October 2014 | doi:10.1038/scientificamericantime1114-70

Inconstant Constants

John D. Barrow & John K. Webb

Uzan, LRR Varying constants, Gravitation and Cosmology : <http://relativity.livingreviews.org/Articles/lrr-2011-2/download/lrr-2011-2Color.pdf>

Proponent of a varying fine structure constant: Bachall et al 2004

<http://www.sns.ias.edu/~jnb/Papers/Preprints/Finestructure/alpha.pdf>

# Overview of standard cosmology

Cosmological principle, isotropy and homogeneity

Distances: Hubble law and expansion of the Universe

Abundances of light elements

Background Cosmology in General Relativity

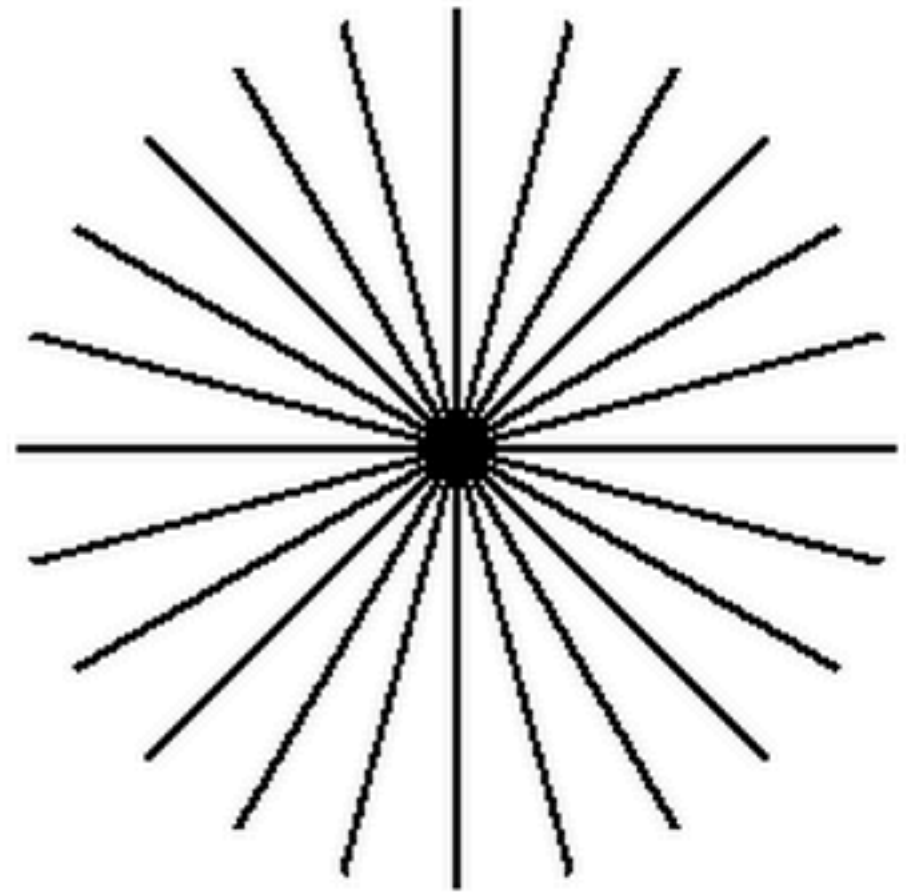
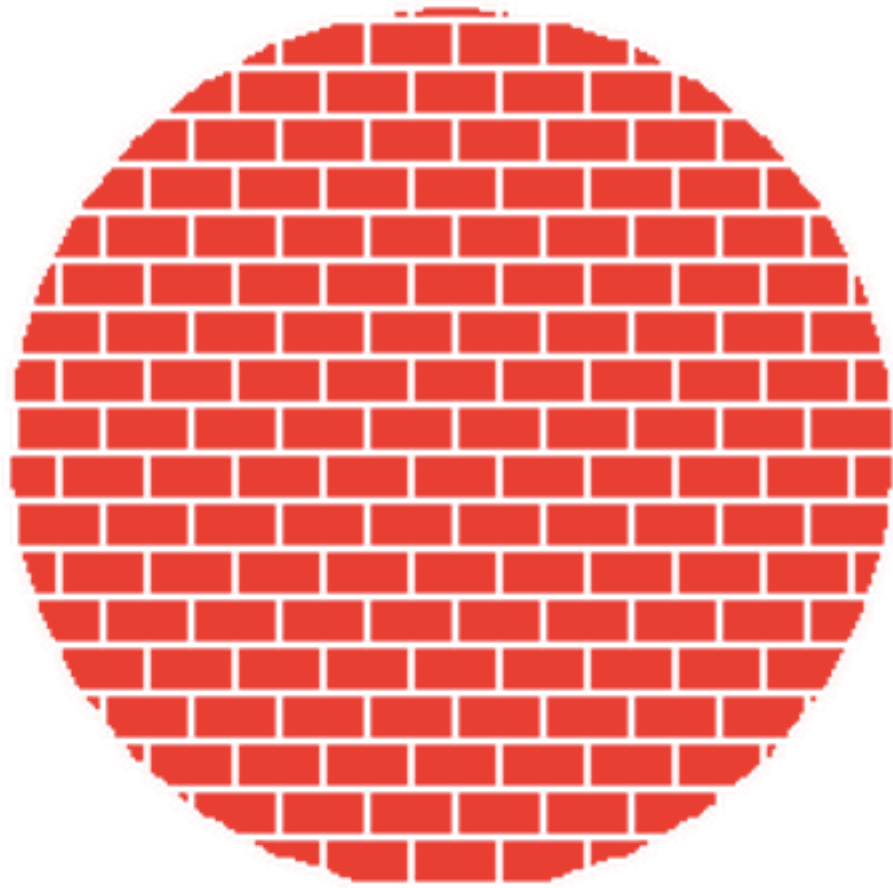
Supernovae and Cosmic acceleration

Cosmic Microwave Background

Structure formation

The Dark Universe

Start from a homogeneous and isotropic Universe



# Friedmann Robertson Walker (FRW) metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$ds^2 = -dt^2 + a(t)^2 \left( \frac{1}{1 - Kr^2} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$$

a(t) scale factor

K curvature: K > 0 (closed, at r = 1/sqrt(K), infinite distance)

K < 0 (open, no singularity)

K = 0 Flat

One can always change the overall normalization of a and r.

r,  $\theta$ ,  $\phi$  are the spherical coordinates

Comoving coordinates (at rest with respect to the expansion):


r,  $\theta$ ,  $\phi$  are constant

## Cartesian coordinates:

$$ds^2 = -dt^2 + a(t)^2 \left( \frac{1}{1 - Kr^2} (dx^2 + dy^2 + dz^2) \right)$$

We can factorize the expansion completely by defining the **conformal time**:

$$d\tau = \frac{dt}{a(t)} ; \quad ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$g_{\mu\nu} \equiv a^2 \cdot \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & (1 - Kr^2)^{-1} & 0 & 0 \\ 0 & 0 & (1 - Kr^2)^{-1} & 0 \\ 0 & 0 & 0 & (1 - Kr^2)^{-1} \end{pmatrix} \equiv a^2 \gamma_{\mu\nu}$$


## Expansion rate:

$$H = \frac{\dot{a}_t}{a} , \quad \mathcal{H} = \frac{\dot{a}}{a} \quad \mathcal{H} = aH$$

Comoving metric

# Stress energy tensor

Energy momentum tensor for a perfect relativistic fluid,  
homogeneous and isotropic

$$T_{\mu}^{\nu} \equiv \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu}$$

Trace  $T = T_{\mu}^{\mu} = -\rho + 3p$

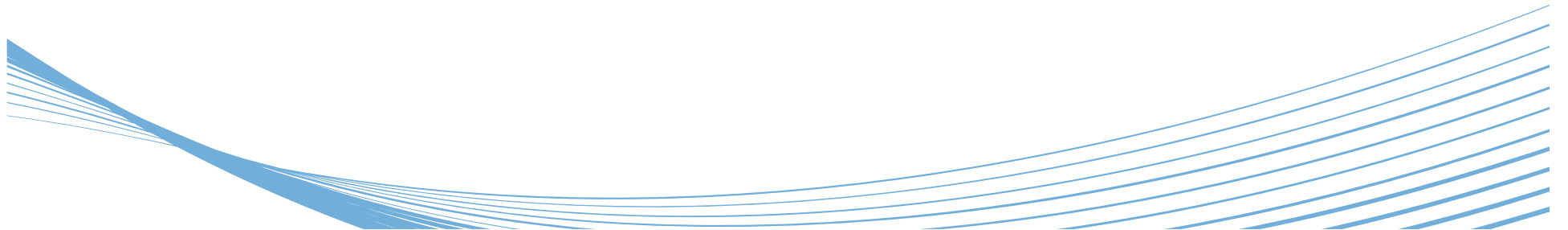


# Einstein equations

$$G_{\mu\nu} = 8\pi GT_{\mu\nu}$$

Left hand side: Einstein tensor, function of the metric and its derivatives, i.e. it depends on the geometry of the space time

Right hand side: stress energy tensor, function of the content of the Universe, i.e. matter, radiation, baryons, ecc.



# Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Friedmann equations for a multi-component Universe:

$$H^2(t) \equiv \left( \frac{1}{a} \frac{da}{dt} \right)^2 = \frac{8\pi G}{3} \sum_i \rho_i(t) - \frac{k}{a^2(t)}$$

$k = 0, +1, -1$  for zero, positive, and negative curvature,

Expansion  $\longleftrightarrow$  Matter content and geometry

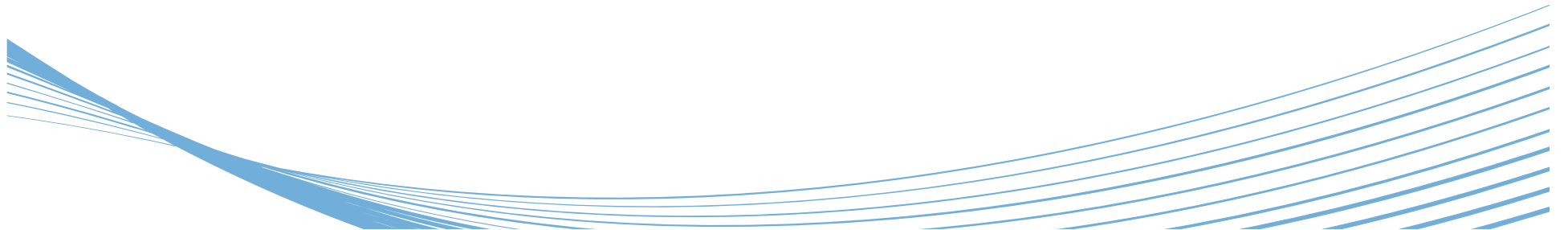
Acceleration equation:

$$\frac{1}{a} \frac{d^2 a}{dt^2} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3p_i)$$

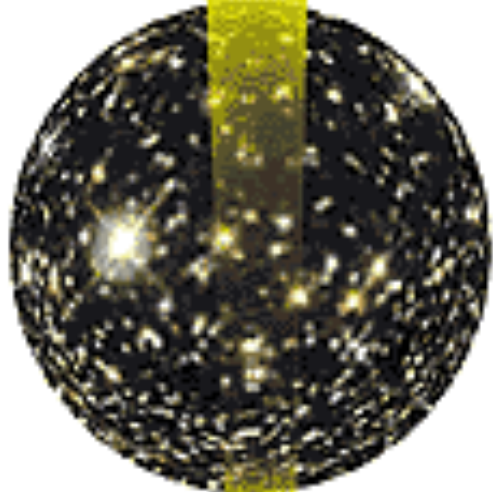
# Density ratios and critical density

Define the critical density:  $\rho_{\text{crit}} = 3H_0^2/8\pi G$   
 $\approx 1.9h^2 \times 10^{-29} \text{ g/cm}^3$

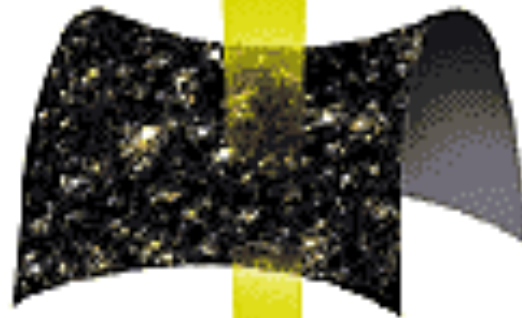
and the density parameter:  $\Omega = \rho/\rho_{\text{crit}}$



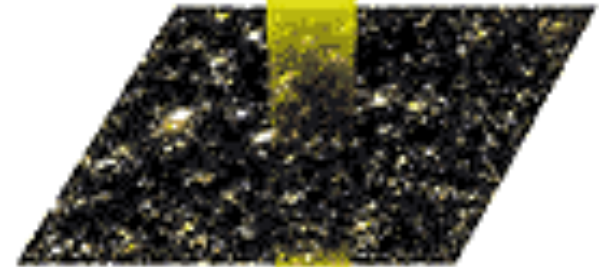
# Geometry of the spatial 3d universe



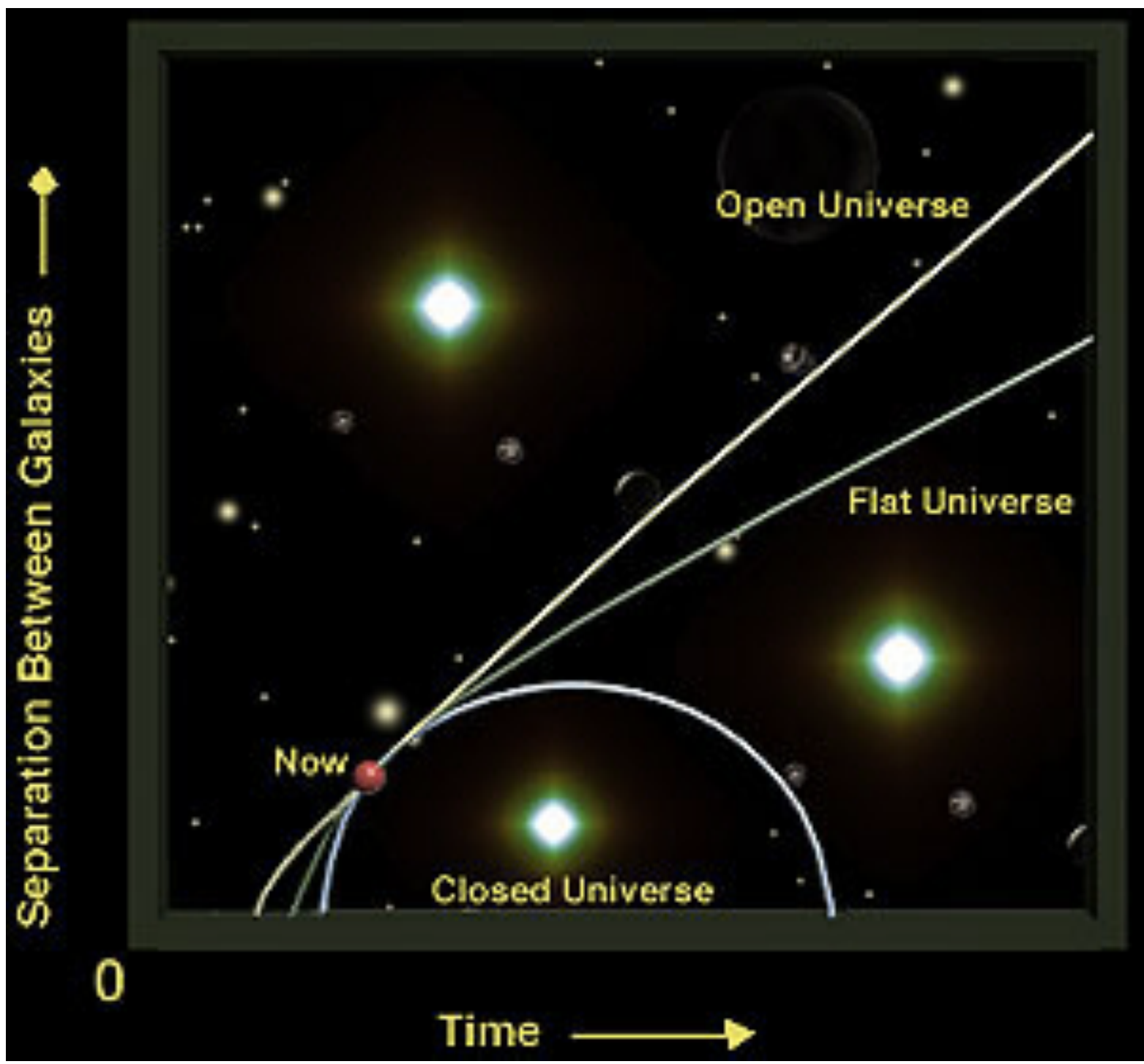
Closed



Open



Flat

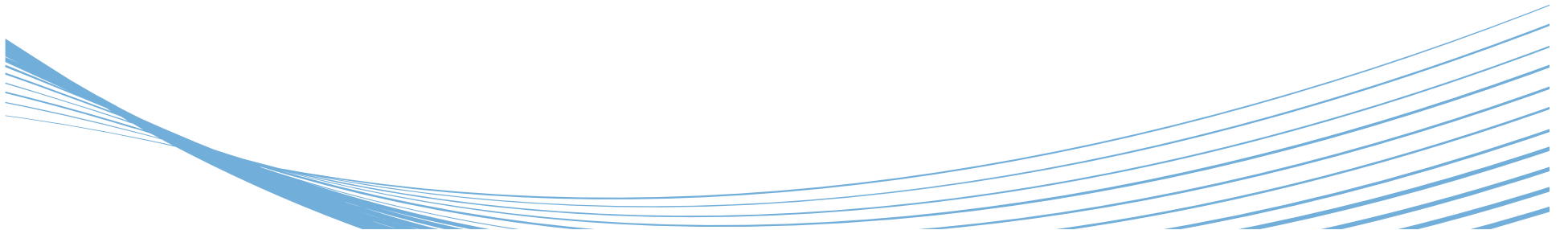


# More on Friedmann equation

It can be also written in terms of  $H_0$  as:

$$H^2(z) = H_0^2 \left( \Omega_m(1+z)^3 + \Omega_K(1+z)^2 + \Omega_r(1+z)^4 + \Omega_\Lambda + \dots \right)$$

$$\Omega_m + \Omega_K + \Omega_r + \Omega_\Lambda + \dots = 1$$



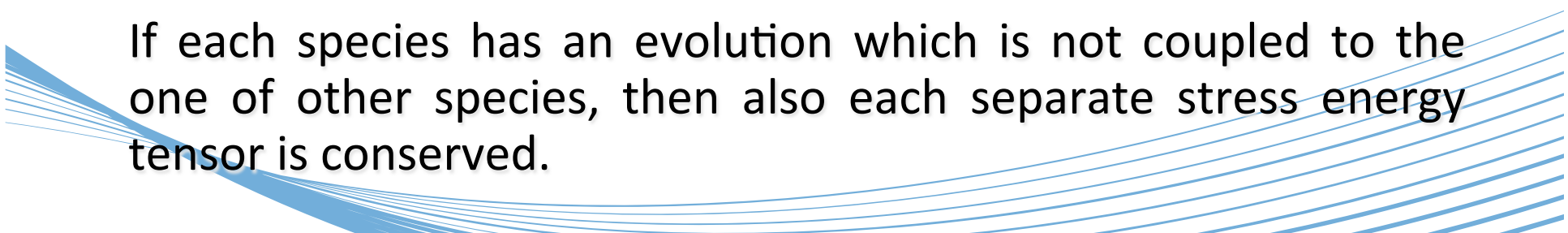
# Conservation equations

$$T_{\mu\nu}^{;\nu} = 0$$

The (covariant) derivative of the stress energy tensor (which depends on energy density and pressure of each fluid) is zero, i.e. energy is conserved.

Valid for the total stress energy tensor (summed up over all components).

If each species has an evolution which is not coupled to the one of other species, then also each separate stress energy tensor is conserved.



# Conservation equations

$$\frac{d\rho_i}{dt} + 3(\rho_i + p_i)H = 0$$

$$p_i = w_i \rho_i$$

$$\rho_i \propto \exp \left[ 3 \int_0^z [1 + w_i(z')] d \ln(1 + z') \right]$$



Exercise 1: derive the evolution of matter in MDE and radiation in RDE in a flat universe as a function of the scale factor and of redshift. Assume matter is cold (zero pressure) and radiation has  $w = 1/3$

Exercise 2: using the Friedmann equations, show whether the universe is decelerating or accelerating in MDE and RDE.

Exercise 3: using the Friedmann equations, calculate the evolution of the Hubble parameter  $H$  as a function of  $a$  and  $t$ .

# Conservation equations

$$\frac{d\rho_i}{dt} + 3(\rho_i + p_i)H = 0$$

$$p_i = w_i \rho_i$$

$$\rho_i \propto \exp \left[ 3 \int_0^z [1 + w_i(z')] d \ln(1 + z') \right]$$

Matter

$$w = 0$$

$$\rho_m \sim a^{-3}$$

Radiation

$$w = 1/3$$

$$\rho_r \sim a^{-4}$$

$$\ddot{a} < 0$$

deceleration

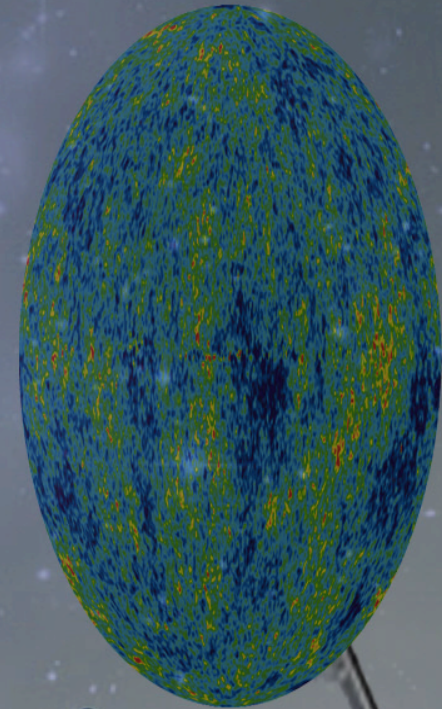
# Clues of the Universe

Supernovae

Cosmic Microwave Background (CMB)

Large scale structures

Galaxy rotation curves, clusters and X-rays, gravitational lensing, baryonic acoustic oscillations, age of the universe, ...



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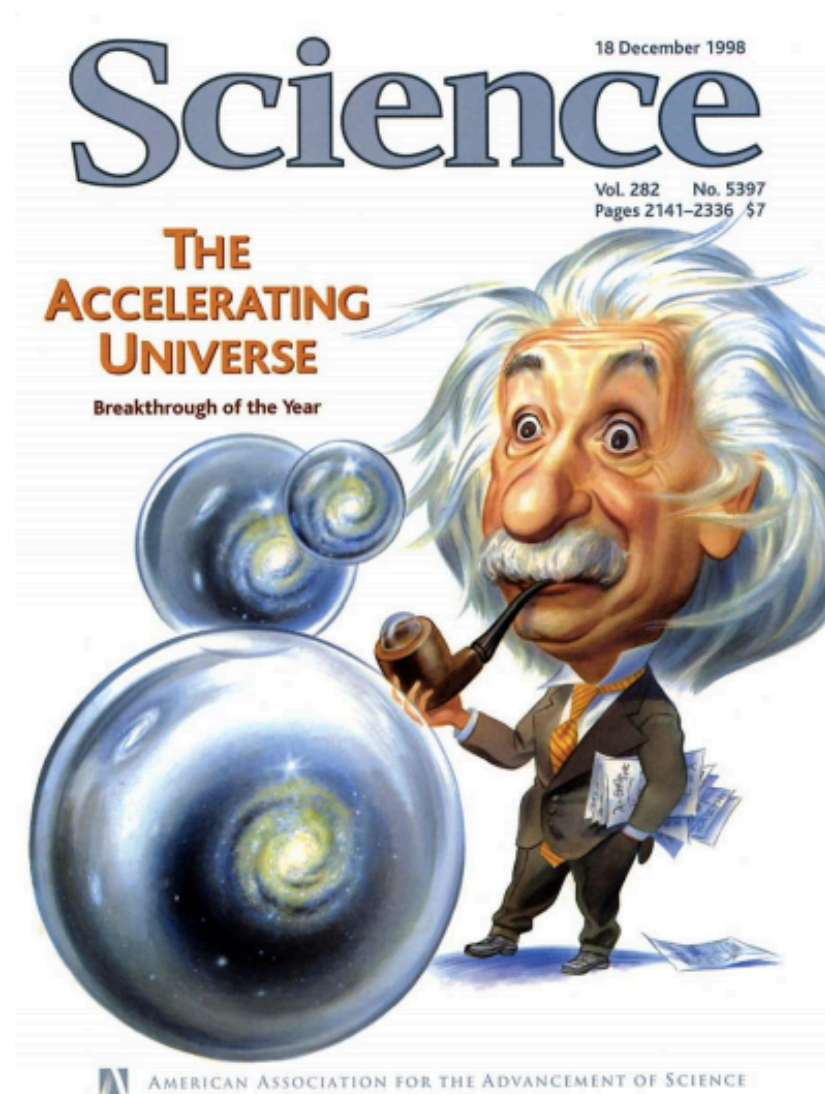
Structure formation

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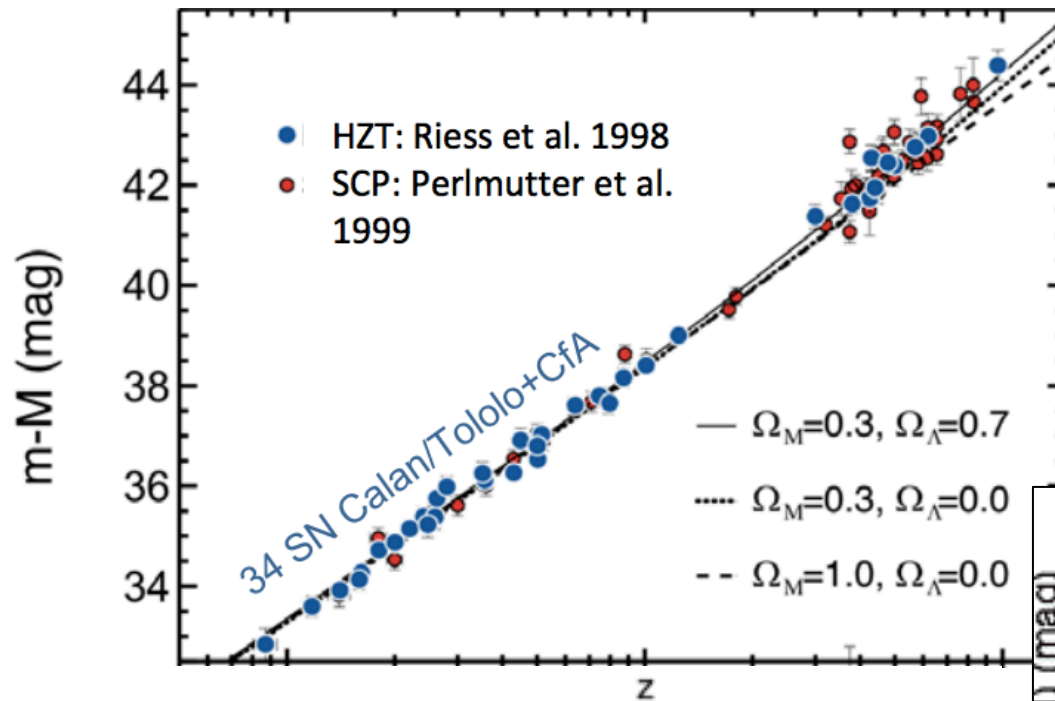
# Supernovae

## Supernova Cosmology Project and the High-z Team

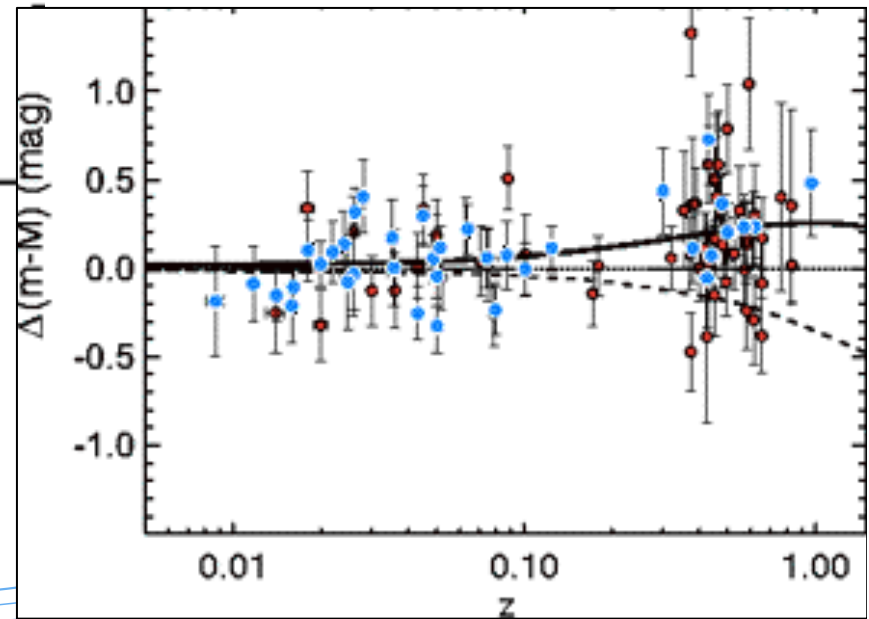
In 1994 the High-z Team was formed:  
“To Measure **the Cosmic Deceleration**  
of the  
Universe with Type Ia Supernovae”



# Supernovae



Distant SNe are 0.25 mag dimmer than they would be in a decelerating Universe

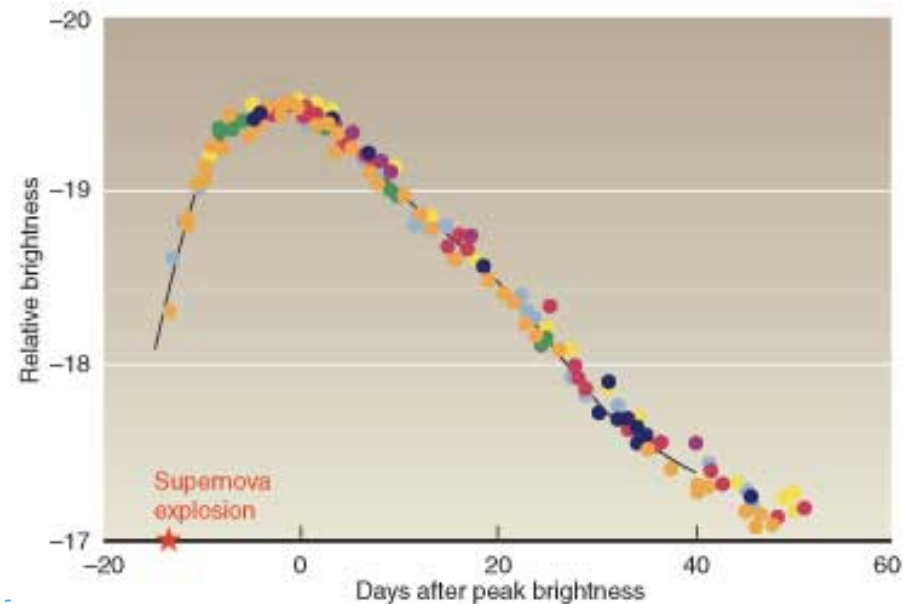
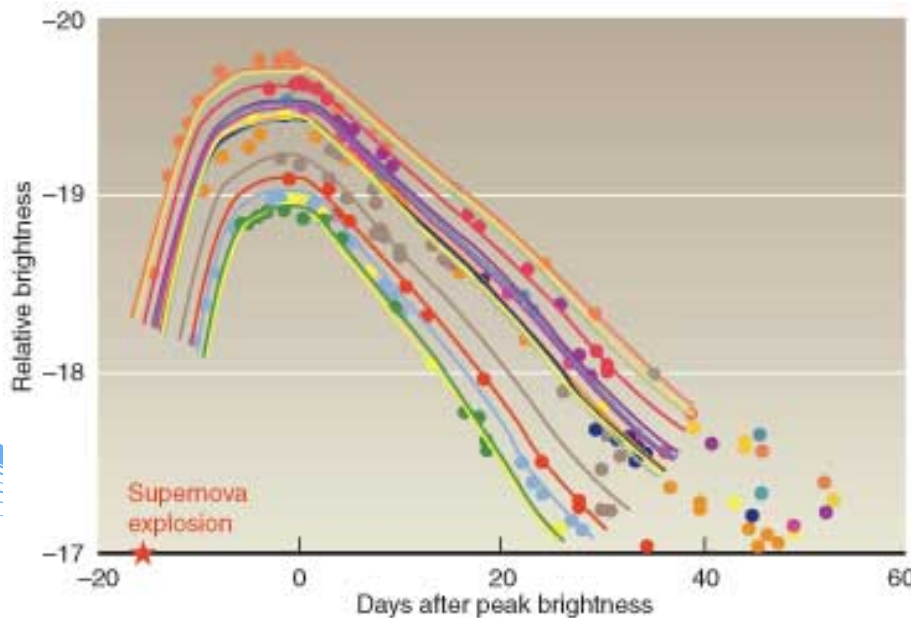


# Supernovae

SNIa not standard candles, 1 sigma spread of 0.3 mag in B-band luminosity  
Wells in 1990's establishes an empirical correlation between SN Ia peak brightness and the rate at which the luminosity declines with time after the peak:  
Intrinsically brighter SNe decline more slowly  
-> standardizable candles with a dispersion of 15% in peak brightness.

Shape and color corrections plus a third parameter describing variation of SNe luminosity with host galaxy mass

## Different light-curve fitters



# Luminosity distance

$$f = \frac{Ld\Omega}{4\pi} = \frac{L}{4\pi a_0^2 r^2 (1+z)^2} \equiv \frac{L}{4\pi d_L^2}$$

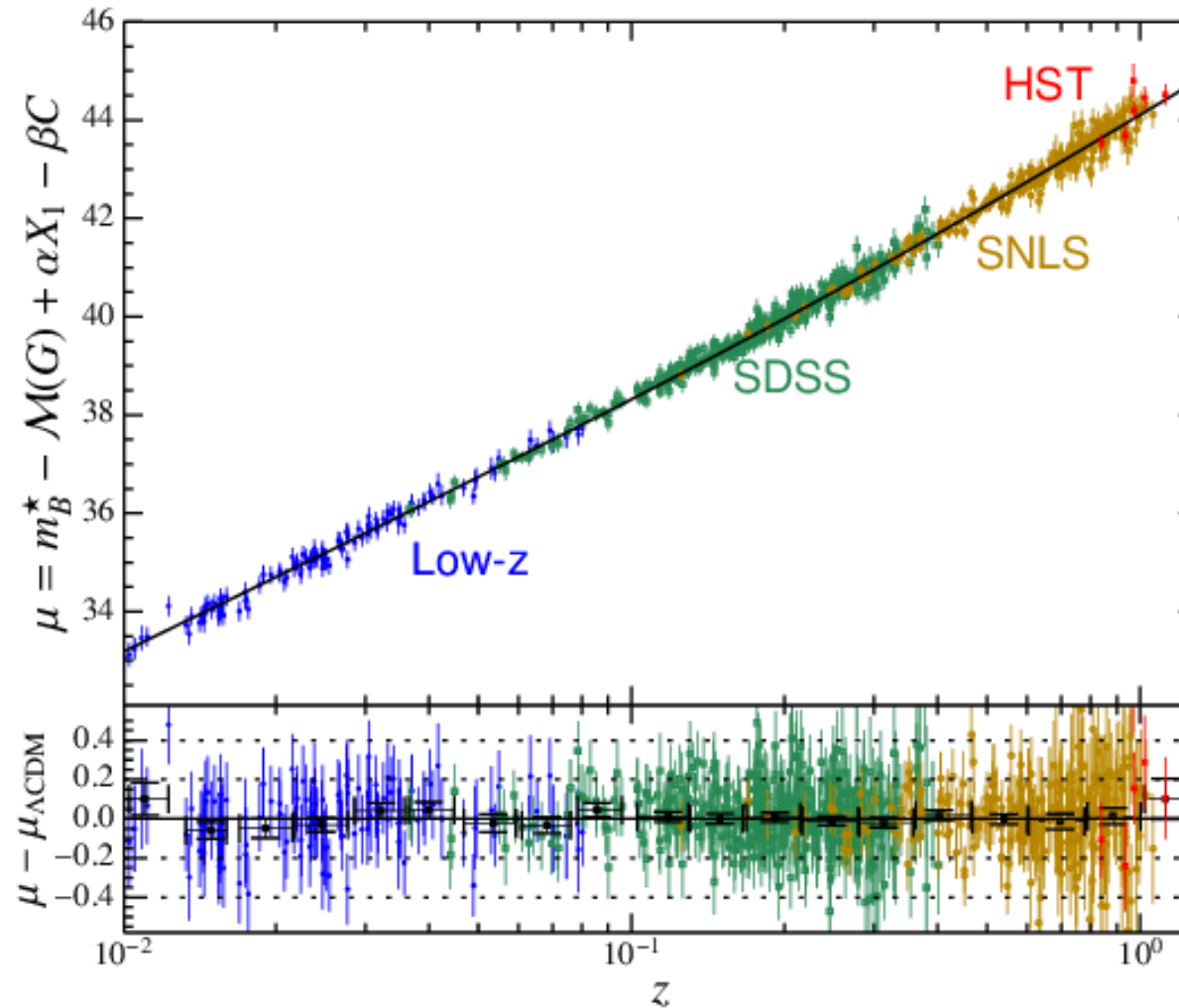
$$d_L = r(1+z) = c(1+z)|\Omega_k|^{-1/2} S_k \left( \int |\Omega_k|^{1/2} \frac{da}{H_0 a^2 (H(a)/H_0)} \right)$$

↗  
Calculate from  
flux and intrinsic  
luminosity

Given a model, it can  
be written in terms of  
the various  $\Omega_\alpha$



# Hubble Diagram



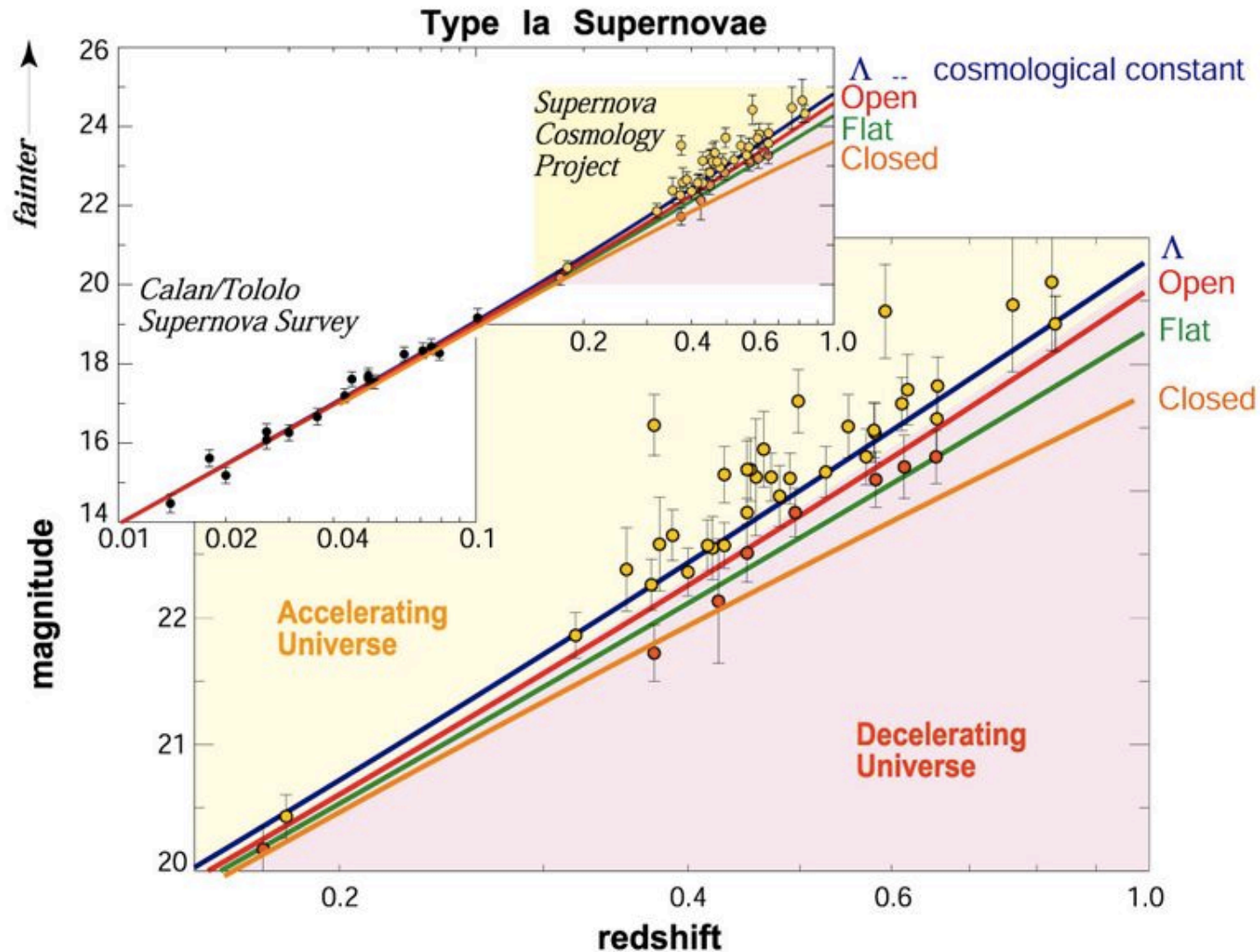
Hubble (Leavitt) diagramme, JLA analysis

Betoule et al. 2014

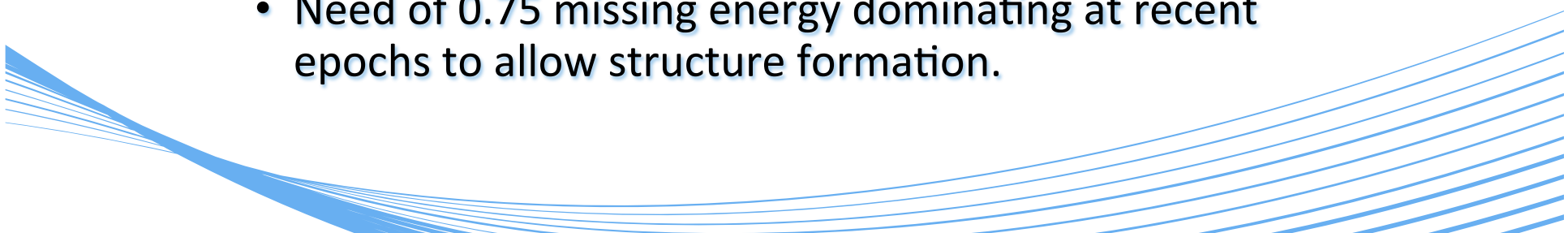
# Older results, compared to different geometries



- Two groups, the Supernova Cosmology Project and the Hi-Z Team, find evidence for an accelerating Universe.



# Surprise?

- 1998: Supernovae
  - 1988: already a theoretical framework for dark energy.
    - Inflation predicts a flat Universe  $\Omega_0 = 1$
    - Observations on DM were pointing towards  $\Omega_m \simeq 0.25$
    - Ages of globular clusters and H, required accelerated expansion
  - Need of 0.75 missing energy dominating at recent epochs to allow structure formation.
- 

# Expansion and acceleration

It started accelerating about 5 billion years ago

Most of the universe is not made of ordinary matter!

## Expansion

Expansion  $\rightarrow$   $\ddot{a}/a = -\frac{1}{6}(\rho + 3p)$   $\leftarrow$  Density and pressure

$p_i = w_i \rho_i$

Matter

$w = 0$

$\rho_m \sim a^{-3}$

Radiation

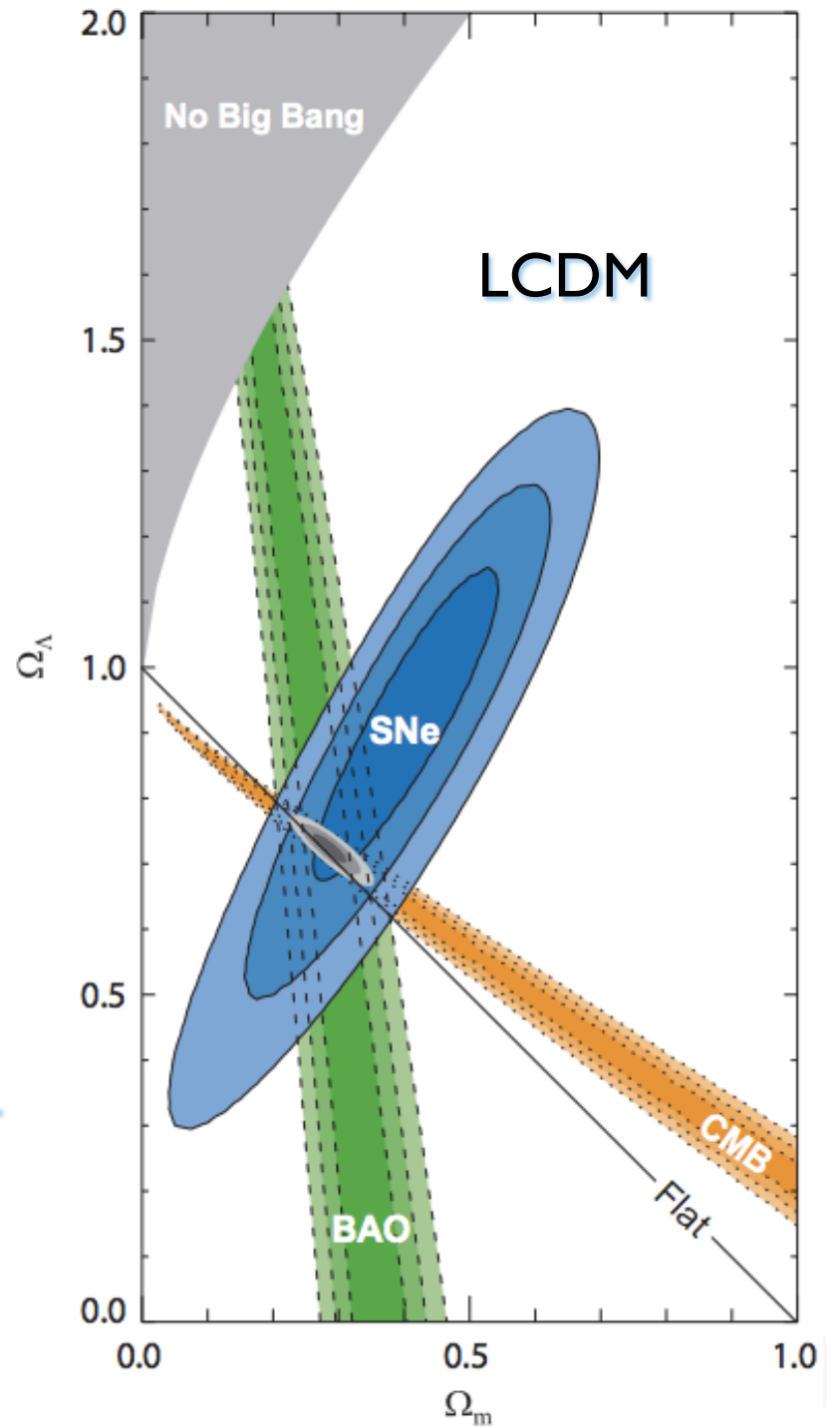
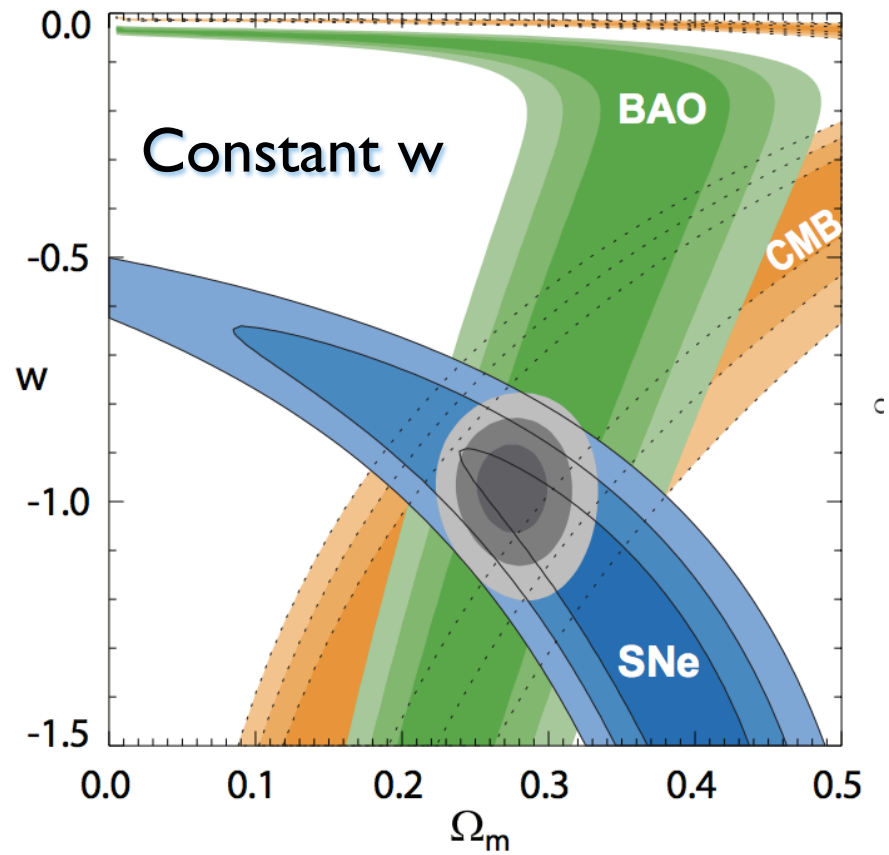
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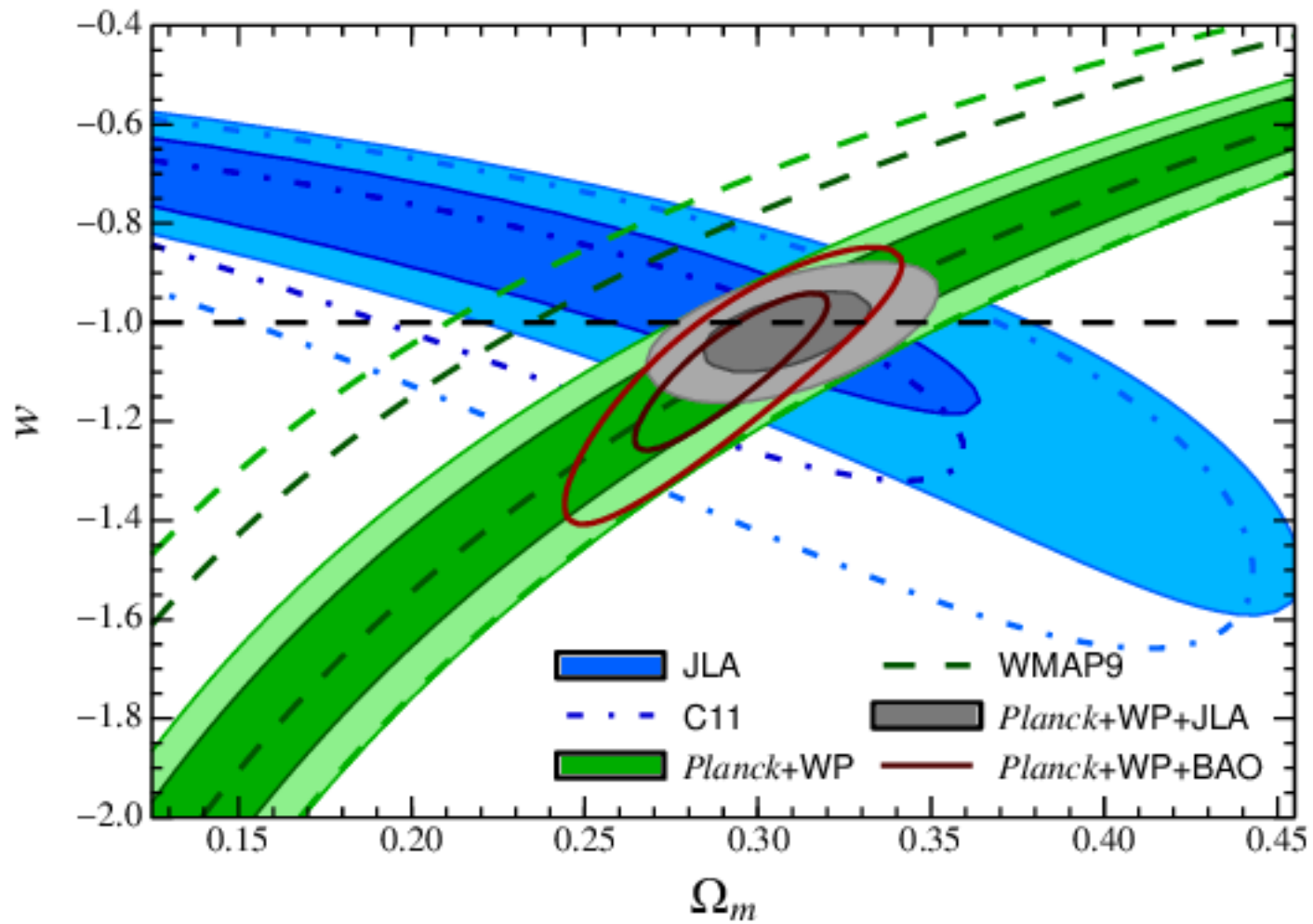
deceleration

Most of the universe is not made of ordinary matter!



Followed by SNLS, ESSENCE, HST, ...





Flat  $w$ -CDM fit parameters, JLA analysis

Betoule et al. 2014