Grand Unification with and without supersymmetry

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GUTs and neutrino mass

$SO(10)$: fermions in 16-dimensional (spinor) representation

$SU(5)$: fermions in 5 and 10 representations

$\Rightarrow \nu_R$ is a singlet

- adding a singlet to the theory implies a lot new parameters
- $SU(5)$ breaks directly to $SU(3) \times SU(2) \times U(1)$
  - no intermediate scales

m_ν can be related to an intermediate scale

The $B - L$ breaking scale

Best idea for small m_ν: the see-saw mechanism

give neutrino a mass by breaking

$B - L$

at a large scale M_R

Neutrino masses suppressed by the large scale:

$$m_\nu = \propto \frac{M_W^2}{M_R} \quad \text{(omitting Yukawa couplings...)}$$

$$\begin{aligned} m_\nu &\sim 0.01 eV \\ M_R &\sim 10^{13} GeV \end{aligned}$$

... An intermediate scale would be convenient

Intermediate scales in SO(10)

$$SO(10)$$

$$M_x \Downarrow \langle S \rangle$$

$$SU(4)_C \times SU(2)_L \times SU(2)_R$$

$$M_c \Downarrow \langle A \rangle$$

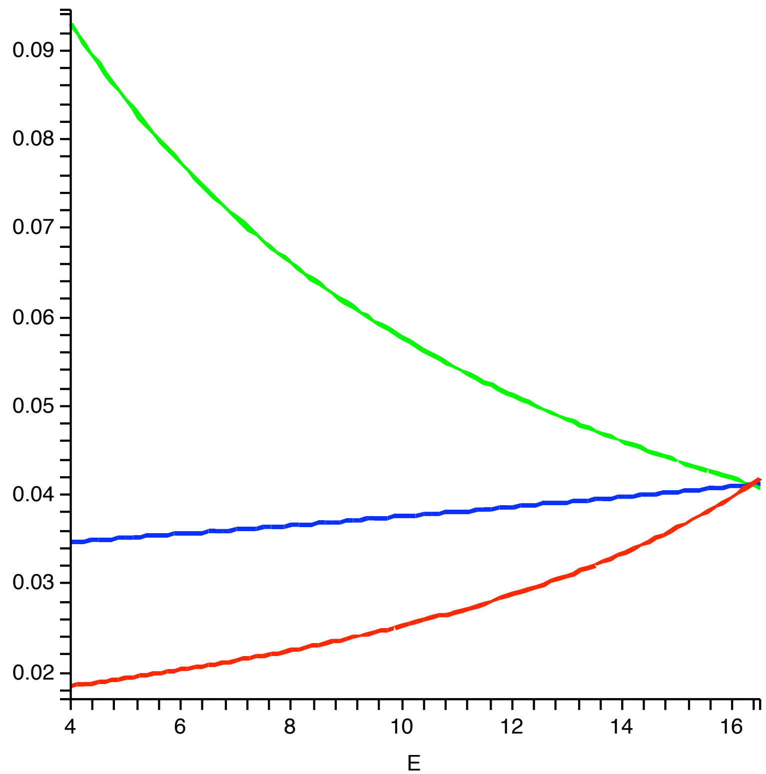
$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$M_R \Downarrow \langle \Delta^c \rangle$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

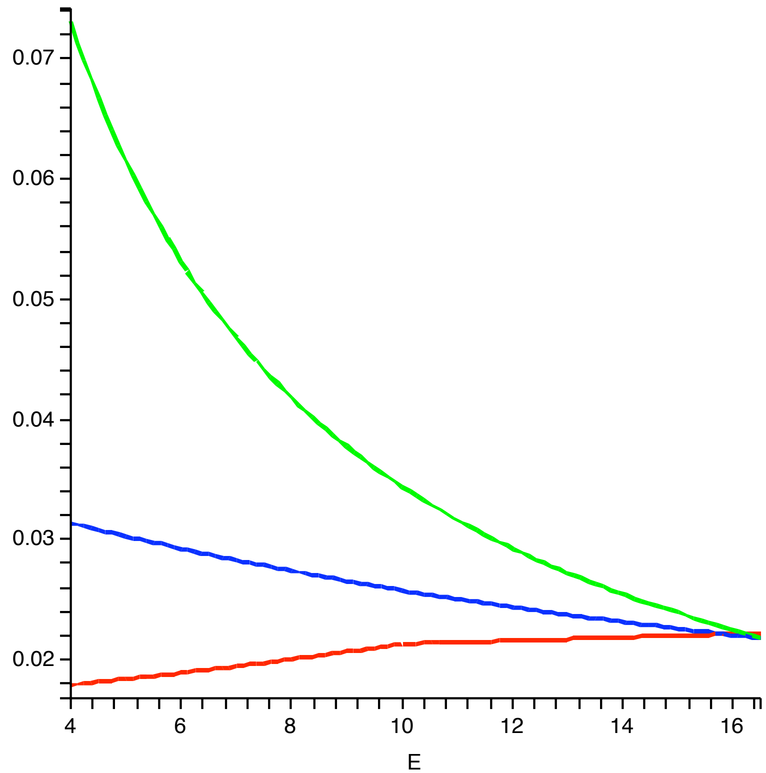
SUSY: one-step unification

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_U} - \frac{b_i}{2\pi} \ln \frac{M_U}{M_W}$$



Non-SUSY: intermediate scales

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_U} - \frac{b_i}{2\pi} \ln \frac{M_R}{M_W} - \frac{b_i'}{2\pi} \ln \frac{M_U}{M_R}$$



Supersymmetry and GUTs (a historical note)

Einhorn, Jones, 1982

Marciano, Senjanović, 1982

Supersymmetry at a scale $\sim M_W - TeV \Rightarrow$ Unification

But with: $sin^2\theta_W(M_W) = [0.23 - 0.26]$

At the time: $\rho \simeq 0.99$ with $m_t \sim 20GeV$

$\Rightarrow \sin^2\theta_W(M_W) = 0.215 \pm 0.014$

smaller than required for SUSY unification

Marciano, Senjanović:
"A very large top quark mass would increase ρ..."

See-saw: 3 types

- Type I: add a fermionic singlet ν^c

  $\langle\Delta^c\rangle \Rightarrow \nu^c$ gets a Majorana mass $\sim M_R$

  EW breaking: Dirac mass m_D

$$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \quad \rightarrow \quad m_\nu \sim \frac{m_D^2}{M_R} \sim \frac{M_W^2}{M_R}$$

- Type II: add a left-handed triplet of Higgs

  from $M\Phi^T\Delta\Phi + M_\Delta^2\Delta^\dagger\Delta$

  with $M \sim M_\Delta \sim M_R$:

$$\langle\Delta\rangle \sim \frac{\langle\Phi\rangle^2 M}{M_\Delta^2} \sim \frac{M_W^2}{M_R}$$

Mass for ν from $L^T\tau_2\langle\Delta\rangle L$

- Type III: add a fermionic triplet N^c

  Works for non-SUSY SU(5) unification

  Bajc, Senjanovic 2006 - 2007

  Dorsner, Fileviez-Perez, 2006-2007

  In $SO(10)$: fields for type I and type II are in the spectrum if the breaking goes through a group containing $U(1)_{B-L}$

Yukawa sector in SO(10)

Pati-Salam fourth color

$$U = \begin{pmatrix} u \\ u \\ u \\ \nu \end{pmatrix} \quad D = \begin{pmatrix} d \\ d \\ d \\ e \end{pmatrix} \quad \ldots$$

$SO(10)$:

$$\Psi = \begin{pmatrix} U \\ D \\ D^c \\ U^c \end{pmatrix}$$

- All fermions in one (spinorial) representation
- Couplings

$$\begin{array}{rcl} \Psi C \Gamma^a \Psi H_a & 10 & \\ \Psi C \Gamma^a \Gamma^b \Gamma^c \Psi D_{abc} & 120 & (antisym.) \\ \Psi C \Gamma^a \Gamma^b \Gamma^c \Gamma^d \Gamma^e \Psi \overline{\Sigma}_{abcd} & \overline{126} & \end{array}$$

$SU(4)_C \times SU(2)_L \times SU(2)_R$ decomposition

$$
\begin{aligned}
H_{10} &= (6,1,1) + (1,2,2) \\
D_{120} &= (\overline{10},1,1) + (10,1,1) + (6,3,1) + (6,1,3) \\
&\quad +(1,2,2) + (15,2,2) \\
\overline{\Sigma}_{\overline{126}} &= (10,1,3) + (\overline{10},3,1) + (6,1,1) + (15,2,2)
\end{aligned}
$$

- $\overline{126}$ can give see-saw of type I and type II
- $(15,2,2)$ in $\overline{126}$ contains a SM Higgs doublet

is $\overline{126}$ enough for all fermion masses?

no...

Fermion mass relations

One doublet is not enough:

Lazarides, Shafi, Wetterich 1981

Clark, Kuo, Nakagawa 1982

$$
\begin{aligned}
M_U &= y_{10}\langle 1,2,2,\rangle^u_{10} + y_{126}\langle 15,2,2,\rangle^u_{126} \\
M_D &= y_{10}\langle 1,2,2,\rangle^d_{10} + y_{126}\langle 15,2,2,\rangle^d_{126} \\
M_E &= y_{10}\langle 1,2,2,\rangle^d_{10} - 3y_{126}\langle 15,2,2,\rangle^d_{126}
\end{aligned}
$$

- only 10: $m_d = m_\ell$ at the GUT scale, for all generations
- only 126: $3m_d = m_\ell$
- 126 is required for neutrino mass –but what else ?
  - is there a difference between choosing 10 or 120 ?

Non SUSY: 126 + 10

Bajc, A.M., Senjanović, Vissani, 2005

$$
\begin{aligned}
M_U &= y_{10}\langle 1,2,2,\rangle^u_{10} + y_{126}\langle 15,2,2,\rangle^u_{126} \\
M_D &= y_{10}\langle 1,2,2,\rangle^d_{10} + y_{126}\langle 15,2,2,\rangle^d_{126} \\
M_E &= y_{10}\langle 1,2,2,\rangle^d_{10} - 3y_{126}\langle 15,2,2,\rangle^d_{126} \\
M_{\nu_D} &= y_{10}\langle 1,2,2,\rangle^u_{10} - 3y_{126}\langle 15,2,2,\rangle^u_{126}
\end{aligned}
$$

take 2nd and 3rd generations only, approx. $\theta_q = V_{cb} = 0$

$$\frac{\langle 1,2,2,\rangle^u_{10}}{\langle 1,2,2,\rangle^d_{10}} = \frac{m_c(m_\tau - m_b) - m_t(m_\mu - m_s)}{m_s m_\tau - m_\mu m_b} \sim \frac{m_t}{m_b}$$

- real 10: $m_t = m_b$
- need a complex 10 –PQ symmetry $\rightarrow$ axion as Dark matter

SUSY or not: 126 + 10

take 2nd. and 3rd. generations only with $\theta_D = 0$, $m_s = m_\mu = 0$

$$M_N \propto \begin{pmatrix} 0 & 0 \\ 0 & m_b - m_\tau \end{pmatrix}$$

unless $m_b = m_\tau$, neutrino mixing vanishes

large $\theta_{atm} \leftrightarrow b - \tau$ unification

Bajc, Vissani, Senjanović 2002

Add more generations, detailed analysis:

- result on θ_{atm} still true
- large 1-3 leptonic mixing angle

Matsuda,Koide,Fukuyama,Nishiura,2002

Goh,Mohapatra,Ng,2003

Non-SUSY: 126 + 120

$$
\begin{aligned}
M_U &= y_{120}\langle 1,2,2,\rangle^u_{120} + y_{120}\langle 15,2,2,\rangle^u_{120} + y_{126}\langle 15,2,2,\rangle^u_{126} \\
M_D &= y_{120}\langle 1,2,2,\rangle^d_{120} + y_{120}\langle 15,2,2,\rangle^d_{120} + y_{126}\langle 15,2,2,\rangle^d_{126} \\
M_E &= y_{120}\langle 1,2,2,\rangle^d_{120} - 3y_{120}\langle 15,2,2,\rangle^d_{120} - 3y_{126}\langle 15,2,2,\rangle^d_{126} \\
M_{\nu_D} &= y_{120}\langle 1,2,2,\rangle^u_{10} - 3y_{120}\langle 15,2,2,\rangle^u_{10} - 3y_{126}\langle 15,2,2,\rangle^u_{126}
\end{aligned}
$$

- real 120: again $m_t = m_b$
- complex 120: interesting relations between masses and mixings

SUSY or not: 126 + 120

Bajc, A.M., Senjanović, Vissani, 2005

Defining some small ratios: $\epsilon_f = m_2^f / m_3^f$, predictions are

- neutrino masses

$$\frac{m_3^2 - m_2^2}{m_3^2 + m_2^2} = \frac{cos2\theta_A}{1 - sin^2 2\theta_A / 2}$$

  – large θ_A gives degenerate neutirnos

- quark masses relation at the GUT scale $m_\tau \sim 3 m_b + O(\epsilon)$

  – wrong for SUSY

- quark mixing $|V_{cb}| \sim \cos 2\theta_A \frac{m_s}{m_b} + O(\epsilon^2)$

  – large neutrino mixing implies small quark mixing

Choice in SUSY theories

- include the $10 + 126$ combination
- get a connection θ_A with $b - \tau$ unification at GUT scale
- get θ_{13} close to experimental limit

but the light Higgs must be a combination of these two fields

Enter 210

How to have both $\langle(1,2,2)_{10}\rangle$ and $\langle(15,2,2)_{126}\rangle \neq 0$?

$$\begin{aligned}\Phi_{210} \quad = \quad & (15,1,1)+(1,1,1)+(15,1,3)+(15,3,1) \\ & +(6,2,2)+(10,2,2)+(\overline{10},2,2)\end{aligned}$$

Allows for:

$$W = .. + \overline{\Sigma}_{126}\, H_{10}\, \Phi_{210} + ...$$

$\langle(15,1,1)\rangle$ breaks P-S symmetry and mixes doublets

$$(15,2,2)\,(1,2,2)\,\langle(15,1,1)\rangle$$

$\rightarrow$ light doublets are combinations of those in Σ_{126} and H_{10}.

Babu, Mohapatra,1993

But in addition, Φ_{210} can

- induce $\langle\Delta\rangle$ via couplings $(10,3,1)_{126}\,(1,2,2)_{10}\,(\bar{1}0,2,2)_{210}$
- break $SO(10) \longrightarrow$ P-S with parity-odd singlet $(1,1,1)_{210}$
- break P-S $\longrightarrow$ L-R with $(15,1,1)_{210}$

$\Sigma, \overline{\Sigma}$ alone are not sufficient to break SO(10) – too simple superpotential

$$W = M\Sigma\overline{\Sigma}$$

Need extra fields: Φ_{210} is best candidate !

Clark, Kuo, Nakagawa, 1982 ; Aulakh, Mohapatra, 1983

Aulakh, Bajc,A.M., Senjanović, Vissani 2003

Other possibilities:

54 + 45 rep: need both - and cannot give vev to $(15,2,2)_{\overline{\Sigma}}$

Non-renormalizable terms with 16 rep: no R-parity conservation

Minimal Model

$$\Psi_{16}, H_{10}, \Sigma_{126}, \overline{\Sigma}_{\overline{126}}, \Phi_{210}$$

$$\begin{aligned} W_H &= m_\Phi \Phi^2 + m_\Sigma \Sigma\overline{\Sigma} + \lambda\Phi^3 + \eta\Phi\Sigma\overline{\Sigma} + m_H H^2 + \Phi H(\alpha\Sigma + \bar{\alpha}\overline{\Sigma}) \\ &+ y_{10}\Psi C\Gamma\Psi H + y_{126}\Psi C\Gamma^5\Psi\overline{\Sigma} \end{aligned}$$

- 26 real parameters: same as MSSM
- rich enough Yukawa structure for realistic fermion spectrum
- both type I and type II see-saw
  - possibility of connecting large θ_A with small quark mixings
  - symmetry broken to the MSM + R-parity
    - stable LSP

R-parity in SO(10)

R-parity $\equiv$ Matter parity $= (-1)^{3(B-L)}$

Mohapatra, 1986

SO(10) has a Z_4 center:

$$\mathbf{16} \to i\mathbf{16}, \quad \mathbf{10} \to -\mathbf{10},$$

$$\mathbf{210} \to \mathbf{210}, \quad \mathbf{126} \to -\mathbf{126}, \quad \overline{\mathbf{126}} \to -\overline{\mathbf{126}}$$

Under M, **16** is odd, rest even

$M \in Z_4 \Rightarrow$ R-parity is in SO(10)

Can be shown: R-parity exact at all energies – survives SUSY breaking

Aulakh, A.M., Rašin, Senjanović, 1998

see-saw + SUSY $\Rightarrow$R-parity

Breaking SO(10)

$$
\begin{aligned}
\Phi \equiv \mathbf{210} \quad &= \quad (15,1,1) + (1,1,1) + (15,1,3) \\
&+ \quad (15,3,1) + (6,2,2) + (10,2,2) + (\overline{10},2,2) \\
\Sigma \equiv \mathbf{126} \quad &= \quad (\overline{10},1,3) + (10,3,1) + (6,1,1) + (15,2,2) \\
\overline{\Sigma} \equiv \overline{\mathbf{126}} \quad &= \quad (10,1,3) + (\overline{10},3,1) + (6,1,1) + (15,2,2)
\end{aligned}
$$

SM singlets are allowed to get a vev

- Find the symmetry breaking conditions
- Calculate masses for all states
- Find the composition of the Higgs doublet

Fukuyama et. al 2004

Aulakh, Girdhar, 2004

Bajc, A.M., Senjanović, Vissani, 2004

An overconstrained model

Fine tune m_H: only 8 parameters left in the Higgs sector:

$$m,\ \alpha,\ \overline{\alpha},\ |\lambda|,\ |\eta|,\ \phi = \arg(\lambda) = -\arg(\eta),\ x = \mathrm{Re}(x) + i\mathrm{Im}(x)$$

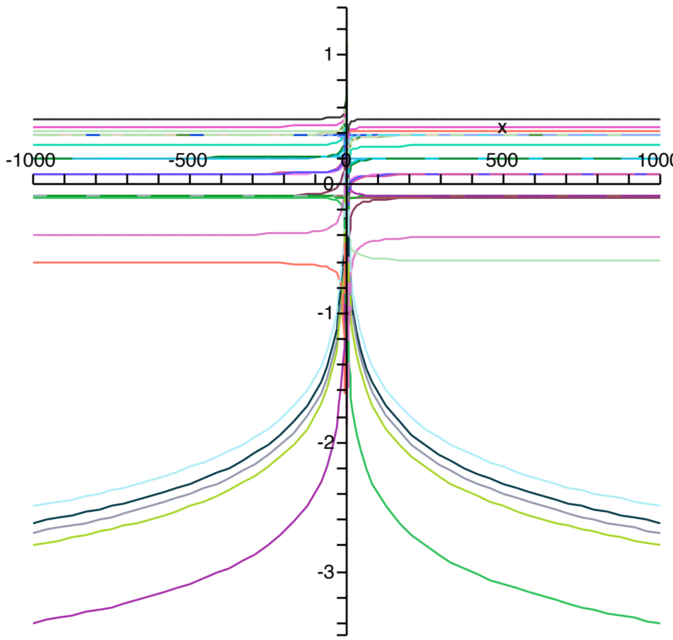
Vevs and masses of all states are

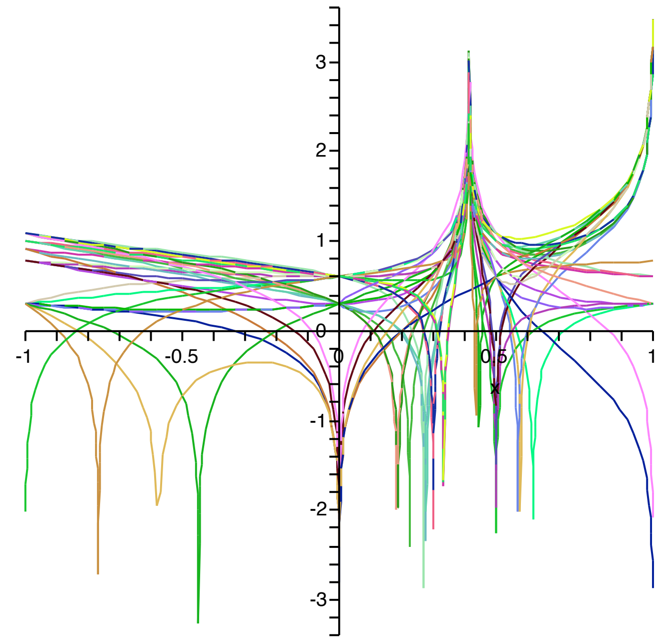
$$\sim \frac{m}{\lambda} f(x)$$
$$\frac{m}{\sqrt{\lambda\eta}} f(x)$$

– variation with parameters quite smooth, with x non trivial

see Aulakh, 2005

Limit $x \to \infty$





Masses $Log[M_i/10^{16}]$ of all states for large and small x

Fermion mass fitting

- The light Higgs is a combination no longer arbitrary

$$H_{u,d} = r^{\mathbf{10}}_{u,d} H^{\mathbf{10}}_{u,d} + r^{\overline{\mathbf{126}}}_{u,d} H^{\overline{\mathbf{126}}}_{u,d} + r^{\mathbf{126}}_{u,d} H^{\mathbf{126}}_{u,d} + r^{\mathbf{210}}_{u,d} H^{\mathbf{210}}_{u,d}$$

with $r^{\mathbf{I}}_{u,d} = v \sin\beta \, N_{u,d} \xi^{\mathbf{I}}_{u,d}$ known functions of the parameters.

- Assume for example type II see-saw

$$m_\nu = y_{126} v_\Delta \quad v_\Delta = \frac{(\alpha r^{\mathbf{10}}_u + \sqrt{6}\eta r^{\overline{\mathbf{126}}}_u) r^{\mathbf{210}}_u}{m_\Delta}$$

– neutrino mass depends on the same parameters

Type II in trouble

Some relations among fermion masses depend only on x

$$M_u = \frac{N_u}{N_d} \tan\beta \times [M_d + \xi(x)(M_d - M_e)]$$

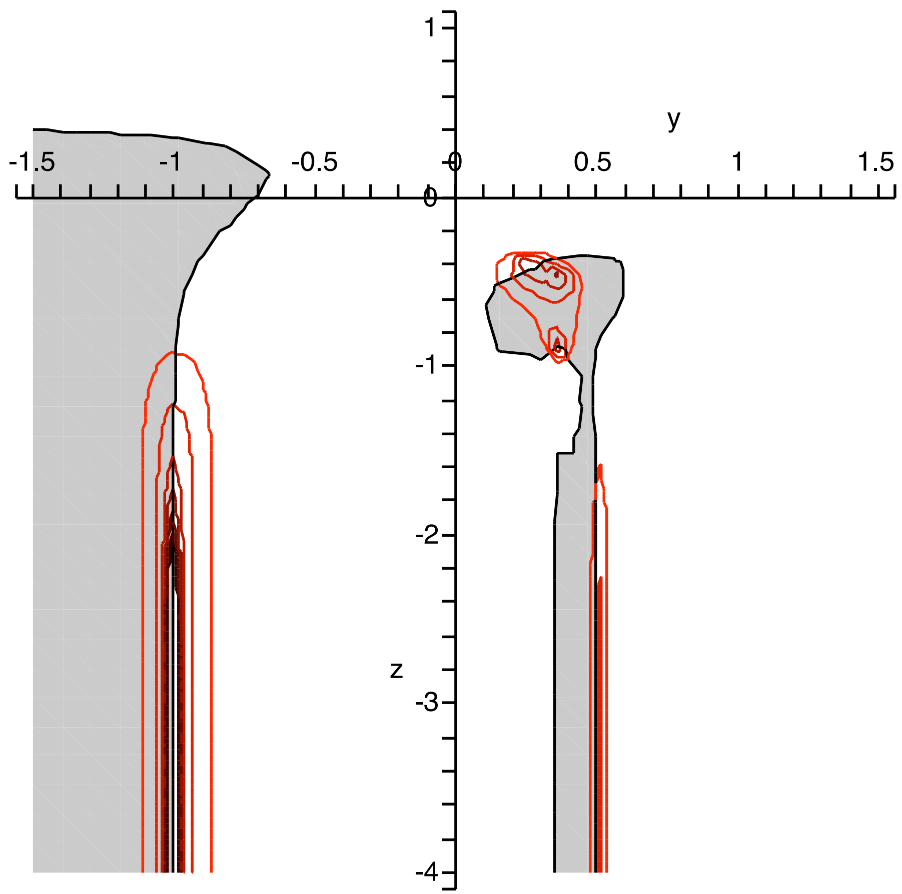
Define the ratio $R(x) = |1 + 1/\xi(x)|$

$\Rightarrow$ then $R(x) > 1$ from trace identities

Write type II mass as:

$$m_{II} = \frac{v^2}{M_x} \times \frac{\sin^2\beta}{\cos\beta} \times \alpha \sqrt{\frac{|\lambda|}{|\eta|}} \times \frac{M_d - M_e}{v} \times \frac{N_u^2}{N_d} \xi(x)$$

$\Rightarrow$ then $\xi(x)$ must give a $10^2 - 10^3$ factor



Maxima of $\frac{N_u^2}{N_d}\xi_{II}(x)$ (red contours) and regions with $R > 1$ (white).

General analysis (type I and II)

Aulakh,Garg,Ghirdaar,2005-2006

Bertolini, Frigerio, Malinsky,2005-2006

Mohapatra, Goh, Ng, Dutta Mimura...

Babu, Macesanu

Wang, Yang

- Do the compete fit with all the fermion masses and all parameters
- Parameter space for type I and type II getting smaller
- Include unification constraints, threshold effects
  - even worse

too small neutrino mass

What to do

Aulakh, 2005-2007

Use the maximal Yukawa sector: add a 120

$$D_{120} = (\overline{10}, 1, 1) + (10, 1, 1) + (6, 3, 1) + (6, 1, 3) + (1, 2, 2) + (15, 2, 2)$$

(another 10 or 126 cannot help

- No SM singlets: symmetry breaking is the same
- Antisymmetric: only 3 complex Yukawa couplings more
- Two doublets mix through:

$$c_1 D_{120} H_{10} \Phi_{210} + c_2 D_{120} \Sigma_{126} \Phi_{210} + c_3 D_{120} \overline{\Sigma}_{126} \Phi_{210}$$

- More parameters in the superpotential: $26 + 15 = 41$

  $m_D, \lambda_D, c_1, c_2, c_3,\ y_{120}$

Or: change the Higgs sector

Alternative model: S = 54 and A = 45 instead of 210

Aulakh,Bajc,Melfo, Rasin, Senjanović,2001

$$
\begin{aligned}
W &= m_H H^2 + m_S S^2 + m_A A^2 + m_\Sigma \Sigma\overline{\Sigma} + \eta A \Sigma\overline{\Sigma} \\
&+\lambda_H^2 S + \lambda_S S^3 + \lambda_A A^2 S + \lambda_\Sigma \Sigma^2 S + \overline{\lambda}_\Sigma \overline{\Sigma}^2 S
\end{aligned}
$$

- 29 real parametrs
- see-saw of type I and II
- 10 + 126 are there but...
  - they do not mix -light Higgs is only the 10

wrong fermion masses

Yukawa sector has to be maximal in this model

54 + 45 with added 120

$$c_1 D_{120} H_{10} A_{45} + c_2 D_{120} \Sigma_{126} A_{45} + c_3 D_{120} \overline{\Sigma}_{126} A_{45}$$

Compare with the (already not !) minimal model

- once Yukawa sector is maximal, 46 parameters
- smaller representations

$\Rightarrow$ find symmetry breaking and mass spectrum

Ramírez, A.M, in preparation

Type II neutrino masses controlled

RGE in the MSSM at one loop

$$\ln \frac{M_X}{M_W} = \left(\frac{1}{\alpha_j} - \frac{1}{\alpha_i}\right) \frac{2\pi}{b_i - b_j}$$

Suppose the Δ_L triplet has a mass $< M_X$

$$\langle \Delta \rangle \propto \frac{1}{m_\Delta} \, , \qquad m_\nu = y_{126} \langle \Delta \rangle$$

other fields could cancel its contribution to the running

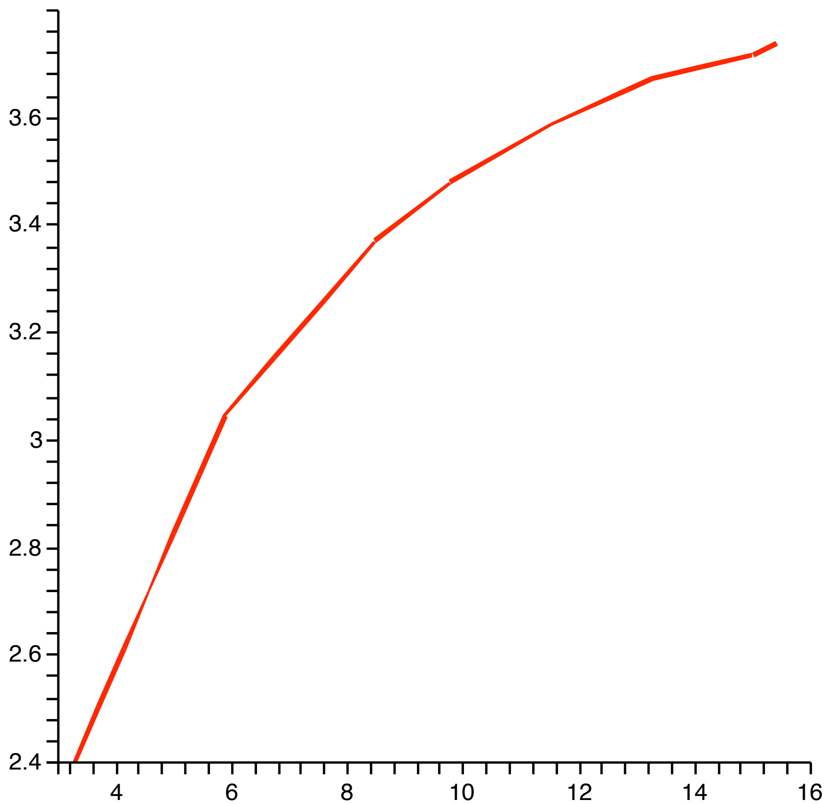
Goh,Mohapatra, Nasri,2004

No need for new fields: already present

$SU(3) \times SU(2) \times U(1)$	δb_1	δb_2	δb_3
$(1,3;\pm 1)$ Δ	$9/5$	2	0
$(6,1;\pm 1/3)$	$2/5$	0	$5/2$
$(1,2;\pm 1/2)$	$3/10$	$1/2$	0
Total	$5/2$	$5/2$	$5/2$

Type II scale undetermined

- enough free parameters to tune their masses at an intermediate scale
- triplet can be as light as desired without afecting one-loop running
- two-loop effects are negligible



Allowed values of $\log(m_{\rm susy}/GeV)$ as a function of $\log(M_\Delta/GeV)$ for two-loop unification. M_Δ is the common mass scale of the left-handed triplet, color sextet and SM-like doublet.

Summary

- $SO(10)$: ideal framework for small neutrino masses
- Models with a non-maximal Yukawa sector can provide connections between fermion masses and mixings
- Minimal SUSY GUT has the smallest number of parameters and
  - * Realistic charged fermion spectrum
  - * R-parity exact at all energies
  - * Small ν mass through type I and type II see-saw
  - * B-S-V connection large $\theta_{atm} \leftrightarrow b - \tau$ unification
  - * Large 1-3 mixing $|U_{13}| \sim .15$ –close to exp. limit
- However lack of intermediate scales gives too small neutrino mass

- Next-to minimal SUSY GUTs do not seem to be predictive
  * but work is in progress...