Update on the Mixing and Time-Dependent CPV Chapter

Adrian Bevan

Queen Mary
University of London

- **Draft of chapter available for a while**

- **To-do list:**
  - Discuss with Vertexing Chapter folks about the split on Δt resolution.
  - Cross reference the tagging performance reported in Chapter 5.3 (tagging) against the discussion of BFlav parameter extraction [Re: section 7.5].
  - Add a brief mention about the effect of a data driven $\Delta\Gamma$ constraint on a TDCPV measurement (1 or 2 sentences to reference recent paper by AB, G. Inguglia and B. Meadows: arXiv:1106.5075).
  - Recent set of comments posted on the HN to address.

Contents

Chapter 7
Mixing and time-dependent analyses

Editors:
Adrian Bevan (BABAR)

Section Writers:
Owen Long

This Chapter focuses on introducing the formalism for neutral meson mixing, and the principles underlying time-dependent analysis in *B* meson decays. A detailed discussion of experimental concerns for a time-dependent analysis follows on from a theoretical introduction of mixing and time-dependent formalism. The experimental aspects discussed include the use of flavour tagging methods introduced in Section 5.3 and the inevitable dilution of information when the tagging assignment is incorrect, interference effects of the *B* meson used for the flavour tag assignment, and the impact of resolution effects on the measurement of physical observables. The final part of this Chapter discusses how parameters required to describe mixing and time-evolution of *B* mesons in the detector can be extracted from the data.

Mixing in the neutral *B* meson system was discovered by the ARGUS Collaboration, Albrecht et al. (1987), and Chapter 14.5 summarizes the measurements of *B* mixing performed by *BABAR* and Belle. An understanding of mixing in *B* mesons is one of the crucial ingredients in the study of time-dependent *CP* asymmetries, and tests of *CPT* using *B* meson decays at the $\Upsilon(4S)$. In particular this is crucial for the measurement of the angles of the Unitarity Triangle introduced in Chapter 13 and discussion of measurements of the angles can be found in Chapters 14.6 through 14.8. Tests of the *CPT* symmetry using neutral *B* mesons discussed in Chapter 14.9 also rely on a good understanding of mixing. Neutral meson mixing in charm decays was discovered at the *B* factories, this is discussed in Section 16.2.

7.1 Neutral meson mixing

Meson mixing is a phenomenon that only occurs for neutral K, D and B mesons. Collectively we can refer to these mesons as P when describing formalism common to all three systems. The effective Hamiltonian describing neutral meson mixing is given by

$$\mathcal{H}_{eff} = \mathbf{M} - \frac{i\mathbf{\Gamma}}{2}, \tag{1}$$

where $\mathbf{M}$ and $\mathbf{\Gamma}$ are two-by-two Hermitian matrices describing the mass and decay rate components of $\mathcal{H}_{eff}$,

respectively. Thus neutral meson mixing is described by

$$\begin{pmatrix} |P_1\rangle \\ |P_2\rangle \end{pmatrix} = \left[\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} \right] \begin{pmatrix} |P^0\rangle \\ |\overline{P}^0\rangle \end{pmatrix}, \tag{2}$$

where $|P^0\rangle$ and $|\overline{P}^0\rangle$ are strong eigenstates of neutral B, D, or K mesons, and $|P_{1,2}\rangle$ are the corresponding weak eigenstates. *CP* or *CPT* symmetry imposes that the matrix elements in Eq. 2 satisfy $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. In the limit of *CP* or *T* invariance, $\Gamma_{12}/M_{12} = \Gamma_{21}/M_{21}$ is real.

Weak eigenstates can be represented as an admixture of the strong eigenstates via

$$|P_{1,2}\rangle = p|P^0\rangle \pm q|\overline{P}^0\rangle, \tag{3}$$

where $q^2 + p^2 = 1$ to normalize the wave function, and

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}, \tag{4}$$

so the magnitude of p/q is very nearly one in the Standard Model (SM). If one considers the weak eigenstates under the *CP* operator, it follows that $|P_1\rangle$ is *CP* even (with an eigenvalue of $+1$), and $|P_2\rangle$ is *CP* odd (with an eigenvalue of -1).

A detailed discussion of the measurement of neutral *B* meson mixing is given in Section 14.5, and the extensions to these measurements allowing for possible *CPT* violation are presented in Section 14.9. Section 16.2 is a review of *D* mixing.

7.2 Time-dependent evolution

Neutral *B* mesons are produced via $e^+e^- \to \Upsilon(4S) \to B^0\overline{B}^0$ at *BABAR* and Belle. The wave function for the final state *B* meson pair is prepared in a coherent P-wave ($L = 1$) state. The *B* mesons remain in a coherent state until one of them decays. When the first *B* meson decays, the wave function collapses into a decoherent state. The remaining un-decayed *B* meson will continue to propagate through space-time and mix with a characteristic frequency Δm_d, until this decays.

If one of the *B* mesons decays into a final state that can be used to unambiguously determine the flavour of the *B* at the time it decayed, we refer to that as a B_{tag}. The set of decay modes of interest as a B_{tag} candidate are referred to as flavour specific final states. An example of a flavour specific decay is $B^0 \to D^-\pi^+$, with a subsequent $D^- \to K^+\pi^-\pi^-$ decay. The *CP* conjugate process has a K^- in the final state. The charge of the final state kaon is used to identified the flavor of the B_{tag} with a B^0 ($\overline{B}^0$) tag originating from a decay with a K^+ (K^-). Similarly if the other *B* decays into a *CP*-eigenstate or admixture we refer to that as the B_{rec}. Events with one B_{tag} and one B_{rec} are of interest in the study of time-dependent *CP* violation. This sequence of events is illustrated in Fig. 1 as seen from the laboratory frame of reference. The center

of mass frame is boosted forward in the direction of the electron beam in the laboratory frame of reference.

Having identified the flavor of B_{tag}, one can infer the flavour of B_{rec} at the instant it decays using the time evolution of the B^0-$\overline{B}^0$ system. The detailed study of this system leads to the measurement of so-called time-dependent asymmetries.

Assuming a negligible difference between the decay rates of B^0 and $\overline{B}^0$ mesons (i.e. $\Delta\Gamma = 0$), the B_{rec} decay rate distribution for a B^0 ($\overline{B}^0$) tagged event is given by f_+ (f_-) where

$$f_\pm(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} \left[1 \pm \frac{2Im\lambda}{1 + |\lambda|^2} \sin(\Delta m_d \Delta t) \right. $$
$$\left. \mp \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \cos(\Delta m_d \Delta t) \right], \tag{5}$$

where τ_{B^0} is the B^0 meson lifetime and λ is given by

$$\lambda = \frac{q}{p} \frac{\overline{A}}{A}, \tag{6}$$

which is related to the ratio of coefficients of B^0 and $\overline{B}^0$ contribution to the mixing amplitude in Eq. 3, and $\overline{A}/A$ is the ratio of amplitudes of a $\overline{B}^0$ decay to a final state and the *CP* conjugate process. The current values of the *B* meson lifetime, and mixing frequency are $\tau_{B^0} = 1.525 \pm 0.009$ ps, and $\Delta m_d = 0.507 \pm 0.005$ ps^{-1}, respectively (Nakamura, 2010).

The coefficients of the sine and cosine terms in Eq. 5 are often referred to in terms of the parameters S and C by the *BABAR* experiment, and in terms of S and $-A$ by Belle. Thus

$$S = \frac{2Im\lambda}{1 + |\lambda|^2}, \tag{7}$$

$$C = -A = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}. \tag{8}$$

For brevity, we use the notation S and C to refer to these coefficients in the remainder of this section.

An asymmetry between the quantities $f_+(\Delta t)$ and $f_-(\Delta t)$ is constructed in order to probe for possible *CP* violation. Neglecting experimental effects, this time-dependent decay-rate asymmetry is given by

$$\mathcal{A}(\Delta t) = \frac{f_+(\Delta t) - f_-(\Delta t)}{f_+(\Delta t) + f_-(\Delta t)}, \tag{9}$$

which reduces to the form

$$\mathcal{A}(\Delta t) = S \sin(\Delta m_d \Delta t) - C \cos(\Delta m_d \Delta t). \tag{10}$$

In certain modes, the fitted parameters S and C are related to fundamental parameters of the SM, the angles of the Unitarity Triangle. As discussed in Chapter 13, two notations are used in the literature for these angles. The *BABAR* experiment uses α, β, and γ to denote the angles, whereas the Belle experiment reports results in terms of ϕ_1, ϕ_2, and ϕ_3. In this section we use the second notation

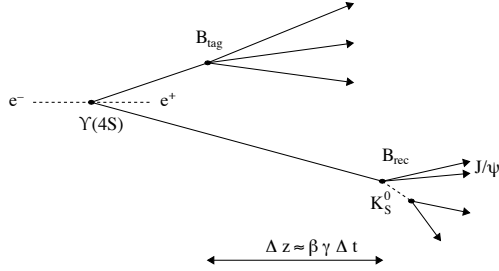** Insert brief comment or footnote about ΔΓ=0 assumption here.*

Fig. 1. An illustration of a B meson pair decaying in the laboratory frame of reference. On the left hand side of the figure, the initial e^+e^- pair collide producing an $\Upsilon(4S)$. This subsequently decays into two B mesons, one of these decays into a B_{tag} final state, and the other into a B_{rec} final state. The spatial distance Δz between the decay vertices of the B_{tag} and B_{rec} is related to the proper time difference Δt between the decays of these particles. In this example the B_{rec} final state is $J/\psi K_S^0$.

7.3 Use of flavour tagging

The purpose of flavor tagging is to classify the B_{tag} either as a B^0 or as a $\overline{B}^0$ (See Section 5.3). The remainder of this section discusses several issues that need to be considered when performing a time-dependent analysis of the data, namely that of dilution, and of tag-side interference. Both of these effects need to be taken into account if one aims to extract the correct values of S and C from data.

The *BABAR* experiment classifies events according to the information content used in determining the flavor of the B_{tag} meson. These categories of event are ranked in order of decreasing contribution to the total tagging efficiency Q (see Eq. **??**). Thus the *BABAR* classification is effectively one based on the B_{tag} decay mode. The Belle experiment uses the same information, but instead of having distinct categories of event, that algorithm computes a continuous variable that assigns a tagging efficiency of an event.

Taking dilution into account, the time dependence of the physical states given by Eq. (12) becomes

Dilution

The algorithm for assigning a flavour tag to an event, thus categorizing the tag-side B meson as a B^0 or as a $\overline{B}^0$ is not perfect. There is a finite probability to incorrectly tag an event and thus dilute measurements being made using that information. The mistag probability is denoted by ω, the value of which depends on the B_{tag} final state used to classify an event, and the dilution is given by

$$D = 1 - 2\omega. \tag{11}$$

The time-dependent formalism developed in Section 7.2 needs to be modified to account for the dilution, and in general one should also account for possible differences in ω between B^0 and $\overline{B}^0$ tagged events, denoted by $\Delta\omega = \omega - \overline{\omega}$. On allowing for dilution effects, the rates of tagged B^0 and $\overline{B}^0$ events are given by

$$f_+^{Phys} = (1-\omega)f_+ + \overline{\omega}f_-,$$
$$f_-^{Phys} = (1-\overline{\omega})f_- + \omega f_+. \tag{12}$$

$$f_\pm^{Phys}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}\left[\,1 \mp \Delta\omega \pm (D+\Delta\omega)S\sin(\Delta m_d \Delta t) \mp (D+\Delta\omega)C\cos(\Delta m_d \Delta t)\right]. \tag{13}$$

The observed amplitudes of sine and cosine terms in Eq. (13) are suppressed by the dilution factor $D+\Delta\omega$. As $\Delta\omega$ is small, this factor is sometimes omitted for analyses with a low number of signal events. The analog of the asymmetry given by Eq. (9) is

$$\mathcal{A}(\Delta t) = \frac{f_+^{Phys}(\Delta t) - f_-^{Phys}(\Delta t)}{f_+^{Phys}(\Delta t) + f_-^{Phys}(\Delta t)} = -\Delta\omega + (D+\Delta\omega)[S\sin(\Delta m_d \Delta t) - C\cos(\Delta m_d \Delta t)]. \tag{14}$$

Figure 2 shows the distribution of $\mathcal{A}(\Delta t)$ for $S = 0.7$, $C = 0.0$, and $\Delta\omega = 0.0$. The amplitude of the sinusoidal oscillation is given by magnitude of S in the case of a perfectly tagged asymmetry. In reality dilution effects reduce the measured amplitude relative to the physical one, as illustrated in the figure with the case of $\omega = 0.2$.
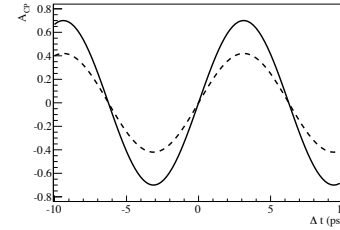

Fig. 2. Distributions of the time-dependent CP asymmetry with $S = 0.7$, $C = 0$, and $\Delta\omega = 0$ for (solid) perfect tagging, and (dashed) the corresponding distributions after taking into account dilution with $\omega = 0.2$.

Tag-side interference

In order for a decay channel to have non-zero CP asymmetry, it must have more than one interfering amplitude with more than one weak phase. This is a necessary condition, but it is not sufficient to guarantee that there will be an observable CP violation effect in that final state. The discussion so far has focused on interfering amplitudes on the B_{rec} side of the event. However it was pointed out by Long, Baak, Cahn, and Kirkby (2003) that in addition to interference on the B_{rec} side, one has to consider possible effects of interference on the B_{tag} side where more than one amplitude contributes to the final state. If neglected, interference effects on the B_{tag} side of the event could result in an undesired contribution to the measured CP asymmetry for the B_{rec}. As there are different physical states reconstructed on the tag-side of the decay, there are potentially different contributions from this so-called tag-side interference effect.

As discussed in Section 5.3, the dominant contributions to the tagging efficiency come from semi-leptonic decays with final state leptons, and hadronic decays such as $B \to D^{(*)-}\pi^+$. The semi-leptonic decays proceed via a single amplitude in the SM, hence the use of semi-leptonic tagged decays is free from tag-side interference. However possible interference effects need to be considered when performing a time-dependent analysis as hadronic decays can proceed by more than one amplitude.

If one considers the decay $B \to D^-\pi^+$, with subsequent $D^- \to K^+\pi^-\pi^-$ decay as an example, the final state can be reached via a CKM allowed $b \to c\overline{u}d$ transition. The charge on the kaon can be used to identify the flavour of the decaying B_{tag}, so that a K^+ (K^-) is associated with a decaying $\overline{B}^0$ (B^0) meson. The same final state can also be reached via a B^0 decay through a doubly-CKM suppressed $\overline{b} \to \overline{u}c\overline{d}$ transition. The ratio of these two amplitudes is given approximately by the ratio of CKM matrix elements $|(V_{ub}^* V_{cd})/(V_{cb} V_{ud}^*)| \simeq 0.02$. The strength of the amplitude of the doubly-CKM suppressed relative to the allowed decay can be parameterized as

$$\frac{\overline{A}_f}{A_f} = \frac{re^{-i\phi_3 + i\delta}}{1}, \tag{15}$$

where r is the ratio of suppressed to favored decays, and δ is the relative phase difference between the B^0 and $\overline{B}^0$ decay proceeding via $\overline{b} \to \overline{c}u\overline{d}$ and $b \to u\overline{c}d$ transitions, respectively. In practice a number of modes are summed over on the tag-side of the event, and we replace r and δ with primed variants to represent the effective ratio of amplitudes and phase difference of an ensemble of modes.

On a mode-by-mode, or an ensemble, basis it is possible to compute factors corresponding to correction on the time-dependent asymmetry parameters S and C measured from the use of hadronic modes for tagging. These factors are a function of Δt and have the effect of slightly reducing the amplitude and broadening the time distribution, or increasing the amplitude and narrowing the distribution as discussed in Section VI of Long, Baak, Cahn, and Kirkby (2003). Thus one can expect the measured values of S and C in a time-dependent analysis to differ from the true values for hadronically tagged events.

If a time-dependent analysis were limited by systematic uncertainties arising from tag-side interference, there are two possible approaches that may be considered to mitigate this uncertainty: (i) only use semi-leptonic tagged events, thus removing the affected data from the analysis, and (ii) given sufficient data, to measure the ratio of CKM allowed to suppressed decays, and the corresponding phase difference between the amplitudes using control samples. In the following discussion the true values of these time-

Resolution function discussion

dependent asymmetry parameters are represented by S_0 and C_0, whereas the measured values of these observables is denoted by S_{fit} and C_{fit}.

The tree dominated $B^0 \to J/\psi K_s^0$ decay

The prime example of a time-dependent measurement made by the B Factories is that of $B^0 \to J/\psi K_s^0$, which is described in Chapter 14.6. The correction to the true value of measured time-dependent asymmetries in this decay arising from tag-side interference is given by

$$C_{fit} = C_0 \left[1 + 2r' \cos\delta' \{G\cos(2\phi_1 + \phi_3) - S_0\sin(2\phi_1 + \phi_3)\}\right] - 2r'\sin\delta'\{S_0\cos(2\phi_1 + \phi_3) + G\sin(2\phi_1 + \phi_3)\}, \quad (16)$$
$$S_{fit} = S_0 \left[1 + 2r'\cos\delta' G\cos(2\phi_1 + \phi_3)\right] + 2r'\sin\delta' C_0 \cos(2\phi_1 + \phi_3), \quad (17)$$

where the factor G is $2Re\lambda_{CP}/(|\lambda_{CP}|^2 + 1)$, and λ_{CP} is the quantity in Eq. (6) evaluated for the B_{rec} reconstructed in a CP eigenstate.

Using a Monte Carlo simulation based approach, one can estimate the magnitude of the effect on the value of S_{fit} and C_{fit} extracted from data, and hence determine S_0 and C_0. In order to do this one has to determine r' and δ'. The value of r' is given by $|(V_{ub}^* V_{cd})/(V_{cb} V_{ud}^*)|$ and an estimate of the uncertainty on this can be derived from a comparison of allowed to suppressed $D \to K\pi$ transitions. This comparison indicates that the error on r' is about 25%. As there is no knowledge of the phase difference, one can assume that this parameter is uniformly distributed in the simulated pseudo experiments. This approach of evaluating the effect of tag-side interference for $B^0 \to J/\psi K_s^0$ has been broadly applied to $b \to c\bar{c}s$ and $b \to c\bar{c}d$ final states.

The complication of loop amplitudes in $B^0 \to \pi^+\pi^-$

An example of a decay with both tree and loop (penguin) amplitudes used in a time-dependent analysis is $B^0 \to \pi^+\pi^-$ which is discussed further in Chapter 14.7. The decay amplitude for the reconstructed B meson depends on ϕ_3, as does the tag-side. Thus the situation encountered with $B^0 \to \pi^+\pi^-$ is therefore much more complicated than the previous case. The uncertainty from tag-side interference can be as large as $2r'$. This complication for calculating tag-side interference applies not only to $B^0 \to \pi^+\pi^-$ decays, but more generally to the set of $b \to u\bar{u}d$ transitions related to ϕ_2 where there are significant penguin contributions. The least problematic of these decays being $B^0 \to \rho^+\rho^-$, which is known to have a small penguin contribution, relative to other $b \to u\bar{u}d$ transitions.

Time-dependent measurement of $\sin(2\phi_1 + \phi_3)$

The measurement of $\sin(2\phi_1 + \phi_3)$ using $B \to D^{*\pm}\pi^\mp$ decays is discussed in Chapter 14.6. The manifestation of tag-side interference in this time-dependent measurement differs from that discussed for the previous two examples as described below. As with the $b \to u\bar{u}d$ transition case the reconstructed B meson depends on ϕ_3, so it is not straightforward to extract an estimate of tag-side interference for $B \to D^{*\pm}\pi^\mp$ decays. Furthermore, the amplitude

of the $\sin(\Delta m \Delta t)$ term in the time-evolution of this decay is $2r\sin(2\phi_1 + \phi_3)$. Here the parameter r is the ratio of doubly-CKM suppressed to allowed decays for the reconstructed B meson (the $B \to D^{*\pm}\pi^\mp$) and has nothing to do with the tag-side of the event[1]. The magnitudes of both r and r' are expected to be comparable and of the order of 0.02, thus there is the potential for tag-side interference to obscure the signal measurement. It is possible to perform an analysis of the time-dependence of $B \to D^{*\pm}\pi^\mp$ explicitly taking into account the effect of tag-side interference while doing so. A scheme for doing this is presented by Long, Baak, Cahn, and Kirkby (2003).

7.4 Resolution on Δt

A number of factors contribute to the resolution on the reconstructed value of Δz, and hence on the computed value of $\Delta t \simeq \Delta z / \beta\gamma$. Experimental resolution $R(\Delta t, \sigma_{\Delta t})$, as a function of Δt and the uncertainty on Δt, $\sigma_{\Delta t}$, can be accounted for when measuring time-dependent CP asymmetry parameters by convoluting this with $f_\pm^{Phys}(\Delta t)$, giving

$$F_\pm^{Phys} = f_\pm^{Phys}(\Delta t) * R(\Delta t, \sigma_{\Delta t}). \quad (18)$$

Therefore one can replace $f_\pm^{Phys}$ with $F_\pm^{Phys}$ in Eqns. (13) and (14) to obtain the corresponding equations that account for both dilution and resolution effects. Factors contributing to the resolution on Δt include

- B_{tag} vertex resolution, which is a superposition of tracking effects and the finite lifetime of D mesons for a sub-sample of B_{tag} mesons.
- B_{rec} vertex resolution, which is a superposition of tracking effects.
- Resolution on the measurement of the boost factor $\beta\gamma$ determined from the energy of the e^+ and e^- beams.

It is important to understand the Δt resolution in detail as this is of a similar magnitude to the average separation between the B_{rec} and B_{tag} proper decay times. Thus the

[1] The parameter r here should not be confused with either the ratio r in Eq. (15), or the effective parameter r' for and ensemble of modes on the tag-side of the event.

---

resolution has a significant effect on the extraction of S and C from a time-dependent analysis.

Different approaches are used to understand resolution effects at the B Factories. BABAR adopts a parametric approach to describe the Δt resolution, whereas Belle characterizes resolution effects according to their physical source. Both approaches work well, and provide a good description of resolution for use in time-dependent analyses.

The BABAR Δt resolution function has a triple Gaussian form, where the mean μ_i and width σ_i of the two central Gaussian components are scaled by $\sigma_{\Delta t}$ on an event-by-event basis. The three Gaussians G_i, where $i = core$, $tail$, and $outlier$, in order of increasing width. The resolution function is given by

$$\mathcal{R}_{sig}(\Delta t, \sigma_{\Delta t}) = f_{core} G_{core}(\Delta t, \mu_{core}\sigma_{\Delta t}, \sigma_{core}\sigma_{\Delta t}) + f_{tail} G_{tail}(\Delta t, \mu_{tail}\sigma_{\Delta t}, \sigma_{tail}\sigma_{\Delta t}) + f_{outlier} G_{outlier}(\Delta t, \mu_{outlier}, \sigma_{outlier}). \quad (19)$$

The parameters σ_{tail}, $\sigma_{outlier}$ and $\mu_{outlier}$ are set to 3.0 ps, 8.0 ps and 0.0 ps, respectively, and the other parameters are determined from reference samples of fully reconstructed B meson decays as described in the section 7.5. As the physical tagging categories for BABAR have different purities and dilutions, the values of μ_i and σ_i for the core Gaussian contribution to the resolution function depend on the flavor category of an event. This difference is taken into account when analyzing data. For early analyses each of the BABAR flavor tagging categories had a separate values for μ_{core} and σ_{core}, in later iterations however the distinction was only made between Lepton and non-Lepton tagging categories.

The Belle Δt resolution function (Tajima et al., 2004) accounts for four different physical effects

- B_{tag} vertex resolution,
- B_{rec} vertex resolution,
- The shift in the B_{tag} vertex position resulting from secondary tracks from charm meson decays,
- The kinematic approximation that the B mesons are at rest in the center of mass frame.

The B_{tag} and B_{rec} vertices are described by (i) a Gaussian resolution function in the case of multi-track vertices, and (ii) a sum of two Gaussians in the case of a single track vertex. The widths of these Gaussians are scaled by the uncertainty on the reconstructed vertex being described. The resolution function resulting from non-prompt tracks associated with a decay in flight of charm mesons is described by the sum of a delta function and exponentials. The kinematic approximation is described by a resolution function dependent on the polar angle of the B_{tag} as reconstructed in the center of mass frame of references. The physical time-dependence $f_\pm^{Phys}$ is convoluted by each of these resolution functions in turn in order to obtain the resultant $F_\pm^{Phys}$.

Figure 3 shows the $f_\pm^{Phys}$ and $F_\pm^{Phys}$ distributions for $S = 0.7$ and $C = 0.0$. The distribution $f_\pm^{Phys}$ is smeared out considerably as a result of experimental resolution

when computing $F_\pm^{Phys}$. The effect of dilution serves to reduce the reconstructed asymmetry between B^0 and $\overline{B}^0$ tagged events. This can be seen as a reduction in the asymmetry between F_+ and F_- in comparison with the true distributions f_+ and f_-.
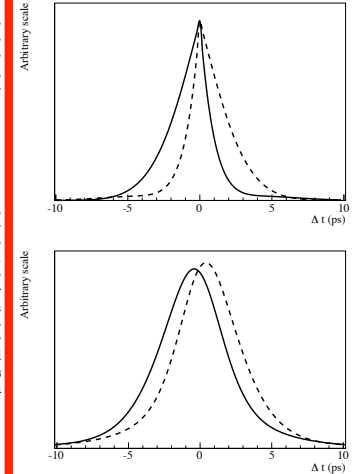

Fig. 3. Distributions of (top) $f_\pm^{Phys}(\Delta t)$ with $S = 0.7$, $C = 0.0$ for (solid) B^0 and (dashed) $\overline{B}^0$ tagged events for perfectly reconstructed decays, and (bottom) the corresponding distributions $F_\pm^{Phys}$ after taking into account dilution and resolution effects.

7.5 Parameter extraction from data

In order to perform a time-dependent analysis one needs to determine the values of ω, $\Delta\omega$, tagging efficiencies, which are collectively referred to as tagging parameters, and the resolution function parameters required to evaluate the convolution of $f_\pm(\Delta t)$ with $R(\Delta t)$. A sample of neutral B mesons decaying into flavour specific final states is used to determine these parameters. The set of modes used by BABAR for this is $B^0 \to D^{(*)-}(\pi^+, \rho^+, a_1^+)$, whereas Belle use $B^0 \to D^{(*)-}\pi^+$, $D^{*-}\rho^+$, $D^{*-}\ell^+\nu$ as well as the charmonium decays $J/\psi K_s^0$, and $J/\psi K^{*0}(892)$. No flavor tag information is used by Belle when extracting parameter using the charmonium decays. BABAR only use the

$B \to D^* \ell^- \nu$ sample to perform a cross-check as there is a larger background in that mode than the other control sample channels. Collectively this ensemble flavor specific decay modes is referred to as a B_{flav} control sample in the following. In addition to determining tagging and resolution function parameters for use in extracting information on CP asymmetries from neutral B_{flav} modes, a set of charged control samples is also used to perform a number of independent validation checks. One of these validations is the determination of S for a sample of charged B decays. As S is physically related to the $B^0 - \overline{B}^0$ mixing amplitude, the fitted value for this parameter in a sample of charged B decays should be consistent with zero. The charged B control sample is formed using $B^+ \to J/\psi K^+$, $J/\psi K^*(892)$, $\psi(2S)K^+$, $\chi_{c1}K^+$, and $\eta_c K^+$ in the case of *BABAR*, where both $B^+ \to J/\psi K^+$ and $\overline{D}^0 \pi^+$ are used by Belle. The corollary of using a set of control modes is that for each mode used to determine the parameters of interest, one introduces additional parameters relating to the shape of distributions of signal and background events, and the purity of each control channel in the signal region. Having determined the purities for each B_{flav} mode, one can use these events to extract estimates of tagging and resolution parameters. This procedure implicitly assumes that there is no significant interference on the tag side of the event, so that the mistag probabilities computed from the B_{flav} sample are the same as those on the B_{rec} side of the event. While this assumption was valid for the B Factories, the precision of measurements at a Super Flavour Factory may require that one accounts for tag-side interference when determining mistag probabilities.

In order to determine tagging efficiencies, one simply needs to determine the fractions of the B_{flav} sample reconstructed in each of the physical categories, and to determine the mistag probabilities and differences, one needs to account for $B^0 - \overline{B}^0$ mixing in the B_{flav} control sample. The time evolution of these decays $h_{\pm}(\Delta t)$, neglecting resolution effects is similar to Eq. 13

$$h_{\pm} = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}}[1 \mp \Delta\omega \pm (D + \Delta\omega)\cos(\Delta m_d \Delta t)] \quad (20)$$

where the $\pm$ index refers to mixed $(-)$ and unmixed $(+)$ events, respectively. One can account for experimental resolution by convoluting $h_{\pm}$ with a resolution function as described in Section 7.4

$$H_{\pm}^{Phys} = h_{\pm}^{Phys}(\Delta t) * R(\Delta t, \sigma_{\Delta t}). \quad (21)$$

Therefore it is possible to not only extract the tagging parameters, but also the resolution function parameters from the B_{flav} sample.

Given the complexity of the situation, the extraction of parameters related to the tagging performance and Δt resolution is done in a two step process. The first step involves extracting the purity of each of the B_{flav} decay modes used. Having done this it is possible to determine the tagging and resolution function parameters from the ensemble of B_{flav} modes. The result of this process is a set of parameters, and the corresponding error matrix which

can be subsequently used as input parameters for time-dependent analyses described in Chapter 14. In a number of cases, the time-dependent asymmetry parameters are extracted from a simultaneous fit to both the B_{rec} and B_{flav} samples so that tagging and resolution parameters are transparently propagated into the CP analysis.

> – **Note: It could be useful to include a full set of parameters to describe the resulting tagging and resolution functions determined from both experiments. The question arises - does that information all go in this section, or has it already been discussed elsewhere in earlier parts of the book...?**

Tagging Parameter Estimation

Bibliography: BaBar Publications

Bibliography: Belle Publications

Bibliography

Albrecht et al. 1987:
  H. Albrecht et al. "Observation of B0 - anti-B0 Mixing". *Phys. Lett.* **B192**, 245 (1987). doi:10.1016/0370-2693(87)91177-4.

Long, Baak, Cahn, and Kirkby 2003:
  O. Long, M. Baak, R. N. Cahn, and D. P. Kirkby. "Impact of tag-side interference on time dependent CP asymmetry measurements using coherent B0 anti-B0 pairs". *Phys. Rev.* **D68**, 034010 (2003). doi:10.1103/PhysRevD.68.034010. `hep-ex/0303030`.

Nakamura 2010:
  K. Nakamura. "Review of particle physics". *J. Phys.* **G37**, 075021 (2010). doi:10.1088/0954-3899/37/7A/075021.

Tajima et al. 2004:
  H. Tajima et al. "Proper-time resolution function for measurement of time evolution of B mesons at the KEK B-factory". *Nucl. Instrum. Meth.* **A533**, 370–386 (2004). doi:10.1016/j.nima.2004.07.199. `hep-ex/0301026`.

This chapter draft has:
  7 Pages of text
  3 Figures
  0 Tables
  21 Equations

Some questions for you

- Is the balance OK?
  - Section related to tagging, vertexing, mixing (B&D) , all TDCPV sections and CKM Chapter.

- Is the level of detail OK?

- Is the level of clarity OK?
  - Road-tested on my grad student (Gianluca) and got useful feedback.

- Any comments welcome (please post these to the chapter hypernews).