

SUSY@ILC

*Un sujet dont je n'ai pas encore parlé dans une
reunion SOCLE?*

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**Un petit résumé du chapitre de physique du ILC Reference Design
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1. Introduction: motivations for low-energy SUSY

If SUSY is to solve some of the most severe problems of the SM:

We need light SUSY particles: $M_S \lesssim 1 \text{ TeV}$.

- The hierarchy problem: radiative corrections to the Higgs masses

$$\Delta M_H^2 = \frac{\lambda_f^2 N_f}{4\pi^2} \left[(m_f^2 - M_S^2) \log\left(\frac{\Lambda}{M_S}\right) + 3m_f^2 \log\left(\frac{M_S}{m_f}\right) \right] + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$$

- The unification problem: the slopes of the α_i SM gauge couplings need to be fixed early enough to meet at $M_{\text{GUT}} \sim 2 \times 10^{16} \text{ GeV}$.
- The dark matter problem: the electrically neutral, weakly interacting, stable LSP should have a mass $\lesssim \mathcal{O}(1 \text{ TeV})$ for Ωh^2 to match WMAP.

In this case, sparticles are accessible at future machines.

- We expect great discoveries at the LHC.
- We will have a great deal of exciting physics to do at the ILC.

1. Introduction: SUSY models

Focus mainly on the Minimal Supersymmetric Standard Model (MSSM):

- minimal gauge group: $SU(3) \times SU(2) \times U(1)$,
- minimal particle content: 3 fermion families and 2 Φ doublets,
- $R = (-1)^{(2s+L+3B)}$ parity is conserved,
- minimal set of terms (masses, couplings) breaking “softly” SUSY.

To reduce the number of the (too many in general) free parameters:

- impose phenomenological constraints: $\mathcal{O}(20)$ free parameters,
- unified models, $\mathcal{O}(5)$ parameters (mSUGRA: $m_0, m_{\frac{1}{2}}, A_0, \tan \beta, \epsilon_\mu$),

In this talk, I will concentrate on the MSSM with gravity mediated breaking.

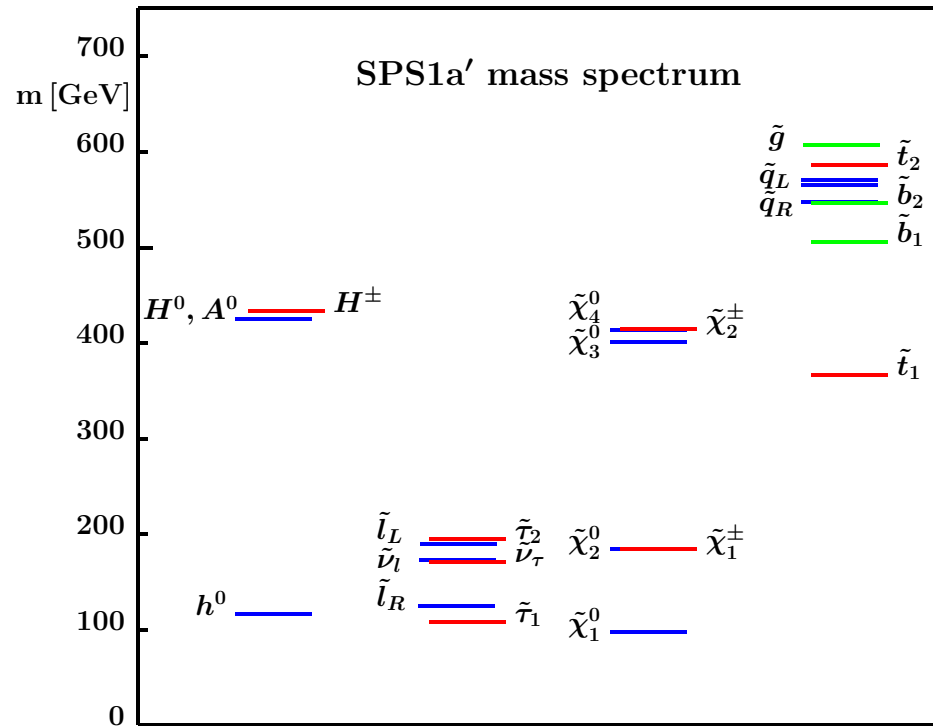
But, one should not forget that:

- other possibilities are models with GMSB/AMSB....
- the impact of relaxing some MSSM basic assumptions can be large
- other scenarios are possible (strings, right-handed neutrinos,...)

There is a need for model independent analyses...

1. Introduction: example of SUSY spectrum

SPS1a': $m_{1/2} = 250\text{GeV}$, $m_0 = 70\text{GeV}$, $A_0 = -300\text{GeV}$, $\tan \beta = 10$, $\mu >$



\tilde{p}/mass	χ_1^0	χ_2^0	χ_1^\pm	\tilde{e}_1	\tilde{e}_2	$\tilde{\nu}_e$	$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\nu}_\tau$	\tilde{t}_1	\tilde{b}_1
SPS1a'	98	184	184	125	190	172	108	195	170	366	506
SPS1a	96	177	176	143	202	186	133	206	185	379	492

1. Introduction: probing SUSY

All these particles will be produced at the LHC (direct/cascades)...

These particles can also be produced directly at the ILC...

But producing these new states is not the whole story! We need to:

- measure the masses and mixings of the newly produced particles, their decay widths, branching ratios, production cross sections, etc...;
- verify that there are indeed superpartners and, thus, determine their spin and parity, gauge quantum numbers and their couplings;
- reconstruct the low-energy soft-SUSY breaking parameters with the smallest number of assumptions (model independent way);
- ultimately, unravel the fundamental SUSY breaking mechanism and shed light on the physics at the very high energy scale.
- make the connection to cosmology and predict the relic density.

To achieve this goal, a combination of LHC and ILC is mandatory!

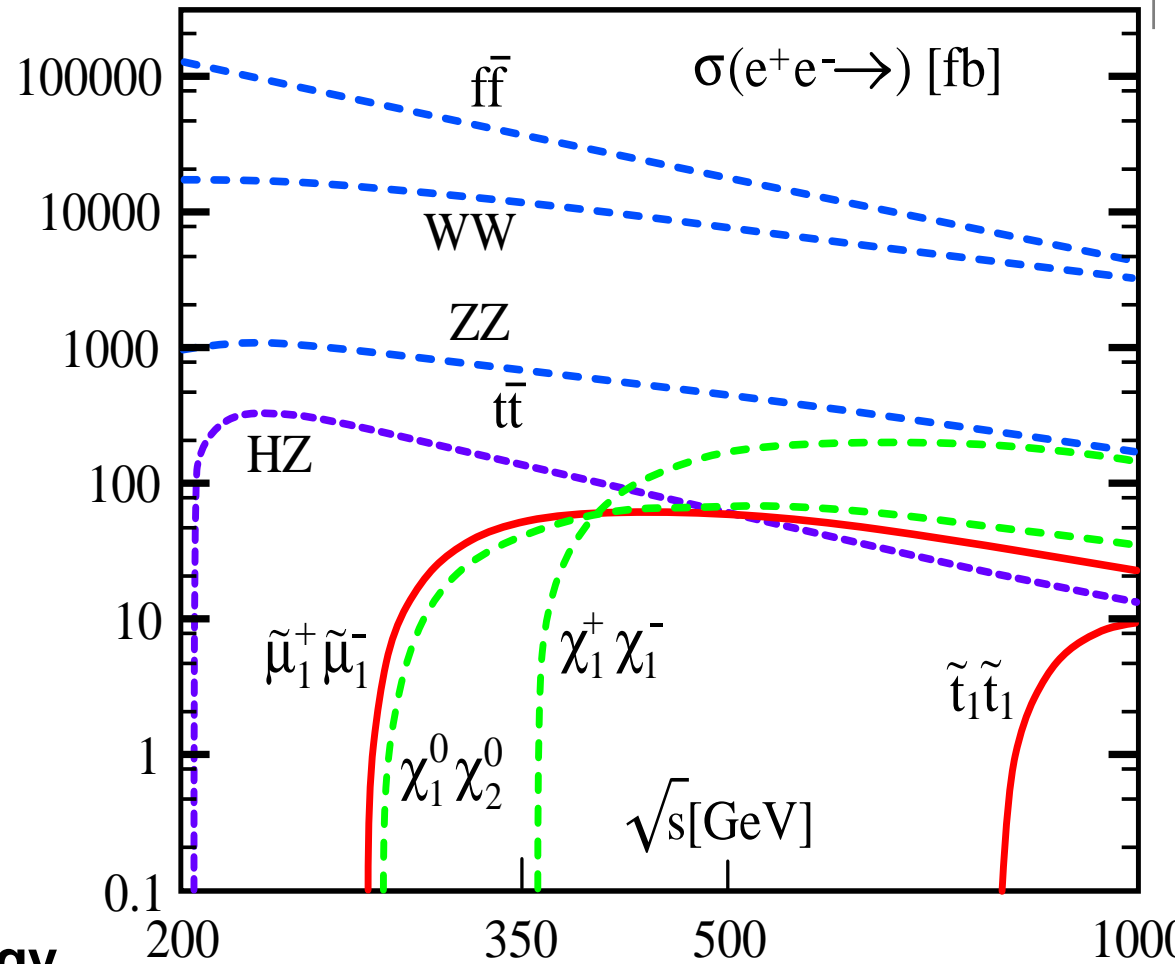
1. Introduction: the role of the ILC

At the LHC:

- copious \tilde{q}/\tilde{g} production
- $\tilde{\ell}/\chi$ from cascades
- complicated topologies
- very large backgrounds
- difficult environment.

At the ILC:

- direct $\tilde{\ell}/\chi$ production
- large production rates
- good signal to bkg ratios
- very clean environment
- possibility of tuning energy
- initial beam polarisation
- more collider options...



2. Precision SUSY measurements: the χ sector

- **Charginos:** mixtures of the charged higgsinos and gauginos

$$\tilde{W}^\pm, \tilde{h}_{2/1}^\pm \longrightarrow \chi_{1,2}^\pm$$

The general chargino mass matrix, in terms of M_2 , μ and $\tan \beta$, is

$$\mathcal{M}_C = \begin{bmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{bmatrix}, \quad s_\beta \equiv \sin \beta \text{ etc}$$

- **Neutralinos:** mixtures of the neutral higgsinos and gauginos

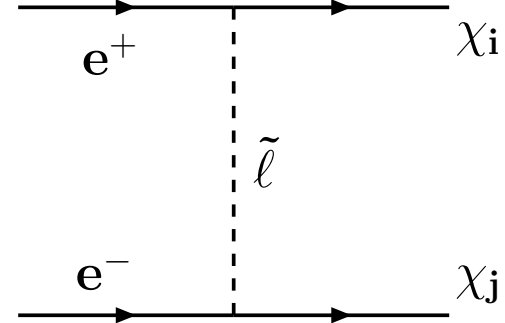
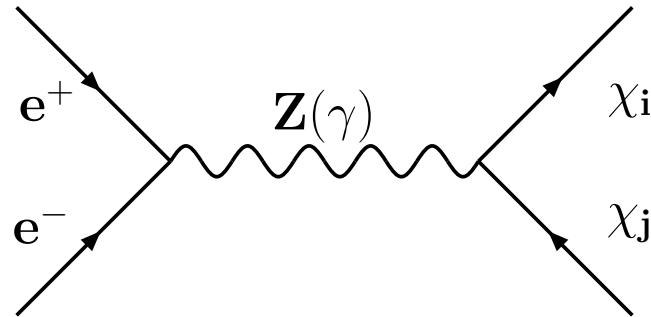
$$\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0 \longrightarrow \chi_{1,2,3,4}^0$$

The 4x4 mass matrix depends on μ , M_2 , $\tan \beta$, M_1 ; given by:

$$\mathcal{M}_N = \begin{bmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{bmatrix}$$

2. Precision SUSY measurements: the χ sector

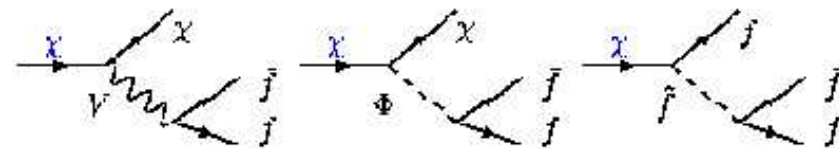
χ production:



- $e^+e^- \rightarrow \chi_i^\pm \chi_j^\pm$: s -channel γ , Z and t -channel $\tilde{\nu}_e$; large σ for $i=j$
- $e^+e^- \rightarrow \chi_i^0 \chi_j^0$: s -channel Z and t -channel \tilde{e} ; $\sigma = \mathcal{O}(10 \text{ fb})$.
- e^\pm beam polarisation selects various production channels
- cross section for χ^\pm rises steeply near threshold, $\sigma \propto \beta$
- cross sections for χ^0 rise less steeply in general, $\sigma \propto \beta^3$

χ decays:

- in general $\chi_i \rightarrow V \chi_j, \Phi \chi_j, f \tilde{f}$
- possibility of cascade decays
- signature: \cancel{E}_T from escaping χ_1^0



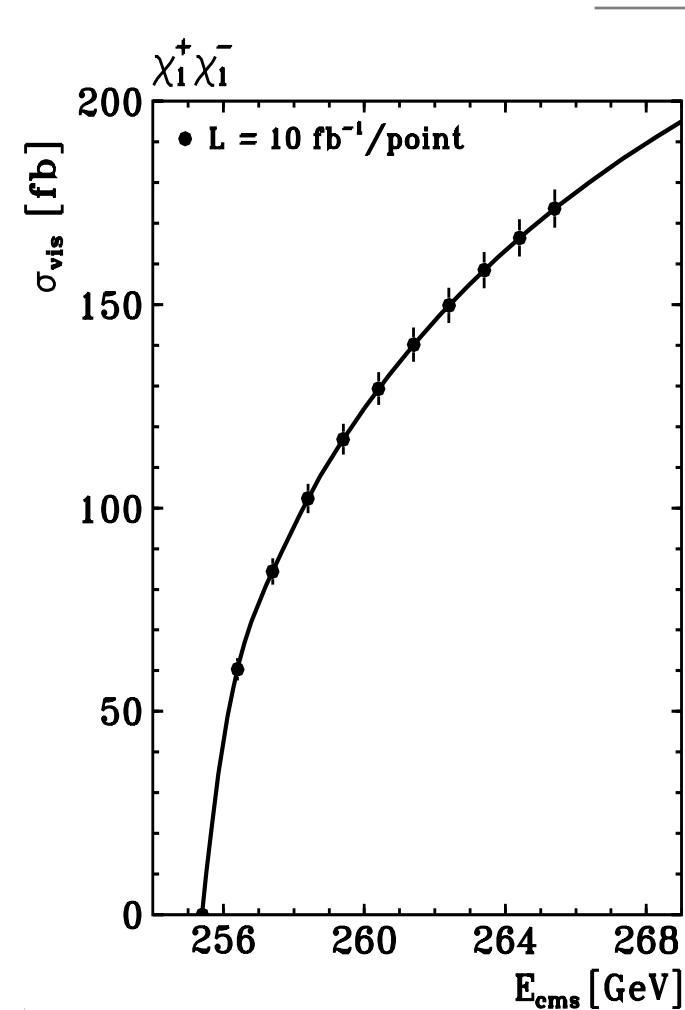
2. Precision SUSY measurements: the χ sector

Measurement of χ^\pm/χ^0 masses:

- from a threshold scan, $\Delta m_{\chi_1^\pm} \sim 50$ MeV for $m_{\chi_1^\pm} \sim 200$ GeV as steep rise $\sigma \propto \beta$.
- $\Delta m_{\chi_1^\pm} \sim 0.1\%$ in continuum from dijet mass in $e^+e^- \rightarrow \chi_1^+\chi_1^- \rightarrow \ell^\pm \nu q\bar{q}' \chi_1^0\chi_1^0$
- from dijet mass, $m_{\chi_1^0}$ determination with precision $\Delta(m_{\chi_1^\pm} - m_{\chi_1^0}) = \mathcal{O}(50)$ MeV.
- for small $m_{\chi_1^\pm} - m_{\chi_1^0}$, use $e^+e^- \rightarrow \chi_1^+\chi_1^-\gamma$ to measure both $m_{\chi_1^\pm}/m_{\chi_1^0}$ from spectra.
- $e^+e^- \rightarrow \chi_2^0\chi_1^0 \rightarrow \ell^+\ell^-\chi_1^0\chi_1^0$ allows an accuracy $\Delta(m_{\chi_2^0} - m_{\chi_1^0}) = \mathcal{O}(0.1\%)$

Determination of spin:

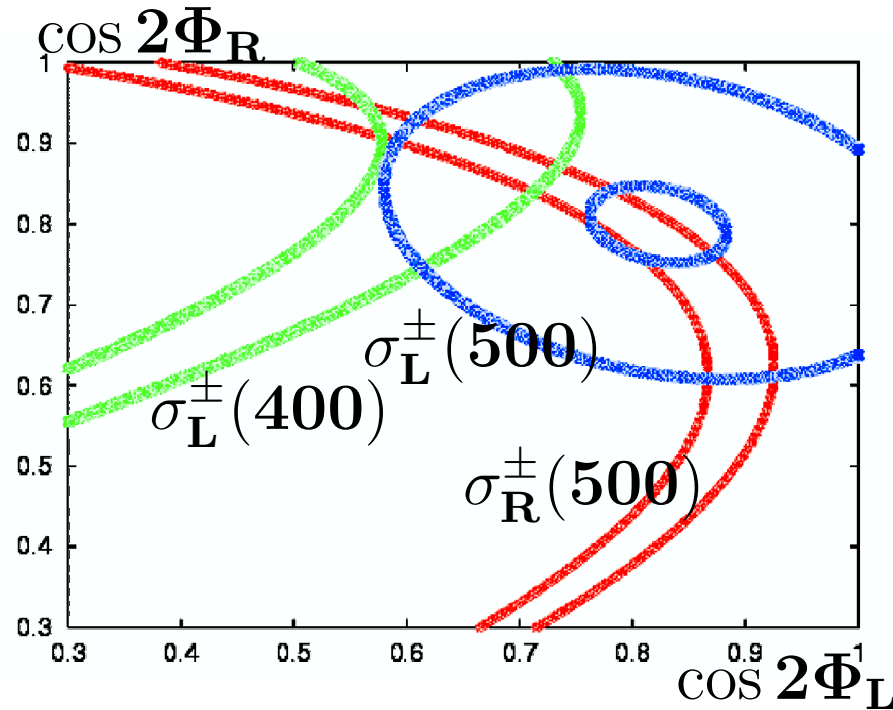
- idea from excitation curve and angular distribution from production
- sure with angular distributions of polarized χ decays with polarized e^\pm



2. Precision SUSY measurements: the χ sector

$\sigma(e^+e^- \rightarrow \chi_i^+ \chi_j^-)$ is binomial in the χ^\pm mixing angles $\cos 2\phi_{L,R}$
 \rightarrow determined in a model independent way using polarized e^\pm beams

SPS1a: $c_{2\phi_L} = [0.62, 0.72]$, $c_{2\phi_R} = [0.87, 0.91]$ at 95% CL at $\sqrt{s} = \frac{1}{2}$ TeV



- **CPC:** $e^+e^- \rightarrow \chi_i^+ \chi_j^-$ alone allows to determine basic parameters;
- sneutrinos can be probed up to masses of 10 TeV with polarisation;
- **CPV:** $e^+e^- \rightarrow \chi_i^0 \chi_j^0$ would be needed (direct probe of CPV).

2. Precision SUSY measurements: the \tilde{f} sector

Sfermion system described by $\tan \beta$, μ and 3 param. for each species: $m_{\tilde{f}_L}$, $m_{\tilde{f}_R}$ and A_f . For 3d generation, mixing $\propto m_f$ to be included.

$$\mathcal{M}_{\tilde{f}}^2 = \begin{pmatrix} m_f^2 + m_{\tilde{f}_L}^2 + (I_f^{3L} - e_f s_W^2) M_Z^2 c_{2\beta} & m_f A_f - \mu (\tan \beta)^{-2I_f^{3L}} \\ m_f A_f - \mu (\tan \beta)^{-2I_f^{3L}} & m_f^2 + m_{\tilde{f}_R}^2 + e_f s_W^2 M_Z^2 c_{2\beta} \end{pmatrix}$$

They are diagonalized by 2×2 rotation matrices of angle θ_f , which turn the current eigenstates \tilde{f}_L, \tilde{f}_R into the mass eigenstates \tilde{f}_1, \tilde{f}_2 .

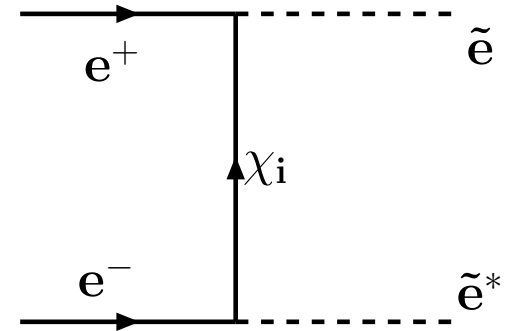
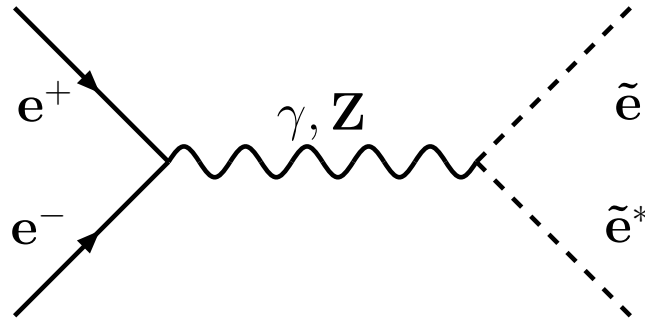
$$m_{\tilde{f}_{1,2}}^2 = m_f^2 + \frac{1}{2} \left[m_{LL}^2 + m_{RR}^2 \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4m_f^2 X_f^2} \right]$$

Note: mixing very strong in stop sector, $X_t = A_t - \mu \cot \beta$ and generates mass splitting between \tilde{t}_1, \tilde{t}_2 , leading to light \tilde{t}_1 ;

mixing in sbottom/stau sectors also for large $X_{b,\tau} = A_{b,\tau} - \mu \tan \beta$.

2. Precision SUSY measurements: the \tilde{l} sector

\tilde{l} production:



- $e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-/\tilde{\tau}^+\tilde{\tau}^-/\tilde{\nu}_{\mu,\tau}\tilde{\nu}_{\mu,\tau}$: s-channel γ, Z exchange;
- $e^+e^- \rightarrow \tilde{e}^+\tilde{e}^-$: s-channel γ, Z and t-channel χ^0 exchange;
- $e^+e^- \rightarrow \tilde{\nu}_e\tilde{\nu}_e$: s-channel Z and t-channel χ^\pm exchange;

Again, in this case:

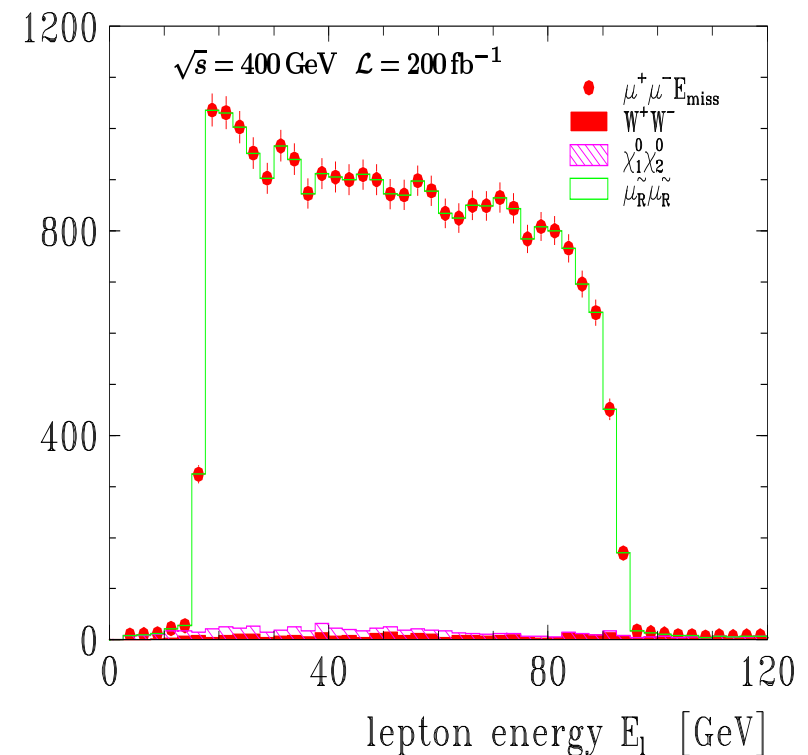
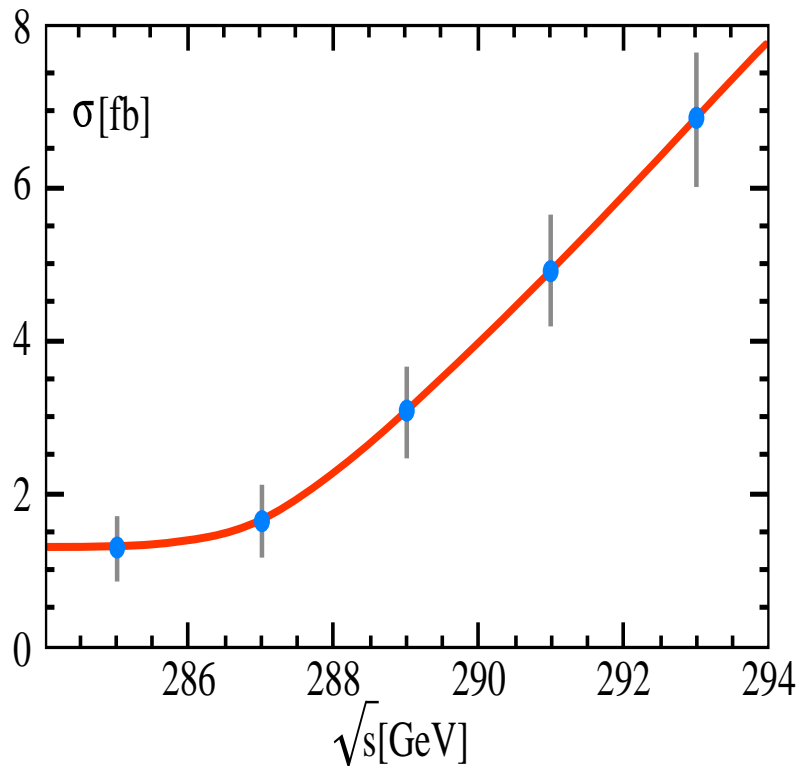
- e^\pm beam polarisation selects various channels/chiralities for $\tilde{e}, \tilde{\nu}_e$;
- $\tilde{e}_{L/R}$ production in $e_{L/R}^-e_{L/R}^-$ collisions;
- cross section for $\tilde{e}, \tilde{\nu}_e$ rises steeply near threshold, $\sigma \propto \beta$
- cross sections for 2d/3d generation rise less steeply, $\sigma \propto \beta^3$

Slepton decays: in general $\tilde{l} \rightarrow l\chi_1^0$ with possible cascades.

2. Precision SUSY measurements: the $\tilde{\ell}$ sector

Slepton mass measurement from threshold scan and in continuum:

- Polarized e^+e^- : $\Delta m_{\tilde{e}_R} = 0.2 \text{ GeV}$ and $\Delta \Gamma_{\tilde{e}_R} = 0.25 \text{ GeV}$;
- improvement by 4 using e^-e^- but 2 times worse for $\tilde{\mu}$ in e^+e^-
- from $E_{\tilde{\ell}}$ spectra in $\tilde{\ell} \rightarrow \ell \chi_1^0$ decays, 0.1% precision for $m_{\tilde{\ell}}$ and $m_{\chi_1^0}$
- $\tilde{\nu}_e$ more involved, $m_{\tilde{\nu}}$ at 1% from $e^+e^- \rightarrow \tilde{\nu}_e \tilde{\nu}_e \rightarrow \nu_e \chi_1^0 e^\pm \chi_1^\mp$



2. Precision SUSY measurements: the $\tilde{\ell}$ sector

Slepton spin determination: conceptually very simple in e^+e^-

- hint from the P-wave onset of the excitation curve (not sufficient).
- the $\sin^2 \theta$ behavior of the cross section (for \tilde{e} near threshold).

Coupling determination: check of the SUSY identity $g_{\text{gauge}} = \tilde{g}_{\text{Yukawa}}$

- from \tilde{e} and $\tilde{\nu}_e$ production cross sections (t-channel contributions)
- more efficient in χ^\pm and χ^0 production (works also for heavy $\tilde{\ell}$).

In the case of $\tilde{\tau}$: $\tilde{\tau}$ mixing and final state τ slightly complicate pattern

- mass determination as above for $\tilde{\mu}$ but accuracy ~ 3 times worse,
- complication ($\gamma\gamma$ bkg) when $\tilde{\tau}_1$ almost degenerate with the LSP χ_1^0 ,
- mixing $\theta_{\tilde{\tau}}$ measurable from $\sigma(e^+e^- \rightarrow \tilde{\tau}_1\tilde{\tau}_1)$ with \neq beam polarization,
- polarisation of τ -lepton measurable and helps for model discrimination,
- μ , A_τ and $\tan \beta$ can be determined from $\sigma(\tilde{\tau}\tilde{\tau})$ and τ polarisation
- $H, A \rightarrow \tilde{\tau}_1\tilde{\tau}_2$ decays give extra information (A_τ measurement)...

2. Precision SUSY measurements: the \tilde{Q} sector

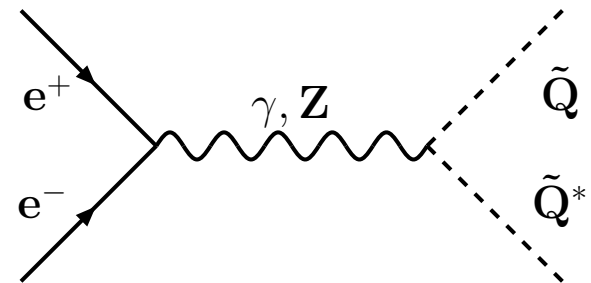
Third generation $\tilde{Q} = \tilde{t}_1, \tilde{b}_1$ possibly lightest squarks due to mixing

- In particular, \tilde{t}_1 is in general the lightest squark (RGE+mixing).
- Light stops needed in models with electroweak baryogenesis.
- Light stops are very difficult to detect at the LHC (large tt bkg)

\tilde{Q} production at ILC:

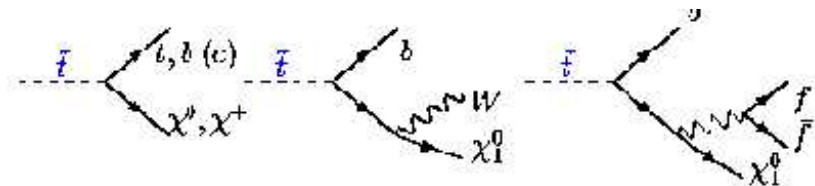
$$e^+e^- \rightarrow \tilde{t}_1\tilde{t}_1/\tilde{b}_1\tilde{b}_1:$$

via s-channel γ, Z exchange;



\tilde{t}_1 decays:

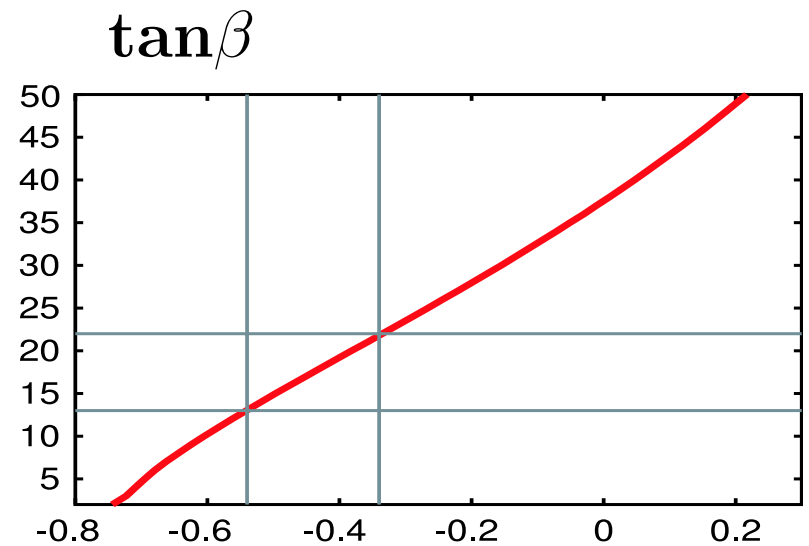
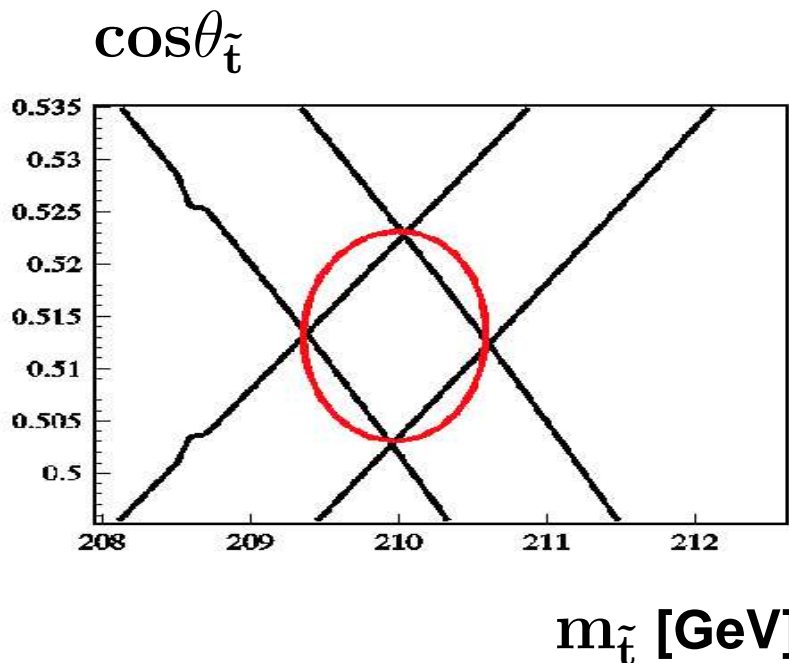
- if heavy, two-body $\tilde{t}_1 \rightarrow t\chi_1^0, b\chi_1^+$
- otherwise multi-body decays
- or loop induced $\tilde{t}_1 \rightarrow c\chi_1^0$ decays



2. Precision SUSY measurements: the \tilde{Q} sector

Phenomenology of \tilde{t}_1 and \tilde{b}_1 similar to that of $\tilde{\tau}_1$:

- masses and mixing obtained from production with e_{pol}^{\pm}
 ex: study of $\sigma(e_{\text{R}}^{-}e_{\text{L}}^{+}, e_{\text{L}}^{-}e_{\text{R}}^{+} \rightarrow \tilde{t}_1\tilde{t}_1)$ for $\tilde{t}_1 \rightarrow b\chi_1^{\pm}, c\chi_1^0$ at 500GeV
- Top quark polarisation in \tilde{t}_1, \tilde{b}_1 decays provides information
 ex: top polarization in $e_{\text{L}}^{+}e_{\text{R}}^{-} \rightarrow \tilde{b}_1\tilde{b}_1 \rightarrow t\chi_1^{-} + \bar{t}\chi_1^{+}$ at 1 TeV.



2. Precision SUSY measurements: summary

	m [GeV]	Δm	Comments
$\chi_{1\pm}^\pm$	183.7	0.55	simulation threshold scan, 100 fb⁻¹
$\chi_{2\pm}^\pm$	415.4	3	estimate $\chi_{1\pm}^\pm \chi_{2\mp}^\mp$, spectra $\chi_{2\pm}^\pm \rightarrow Z\chi_{1\pm}^\pm, W\chi_{1\pm}^0$
$\chi_{1\pm}^0$	97.7	0.05	combination of all methods
$\chi_{2\pm}^0$	183.9	1.2	simulation threshold scan $\chi_{2\pm}^0 \chi_{2\pm}^0$, 100 fb⁻¹
χ_3^0	400.5	3–5	spectra $\chi_3^0 \rightarrow Z\chi_{1,2}^0, \chi_{2,4}^0 \chi_3^0$, 750 GeV, $\gtrsim 1$ ab⁻¹
χ_4^0	413.9	3–5	spectra $\chi_4^0 \rightarrow W\chi_{1\pm}^\pm, \chi_{2,3}^0 \chi_4^0$, 750 GeV, $\gtrsim 1$ ab⁻¹
\tilde{e}_R	125.3	0.05	e^-e^- threshold scan, 10 fb⁻¹
\tilde{e}_L	189.9	0.18	e^-e^- threshold scan 20 fb⁻¹
$\tilde{\nu}_e$	172.5	1.2	simulation energy spectrum, 500 GeV, 500 fb⁻¹
$\tilde{\mu}_R$	125.3	0.2	simulation energy spectrum, 400 GeV, 200 fb⁻¹
$\tilde{\mu}_L$	189.9	0.5	estimate threshold scan, 100 fb⁻¹
$\tilde{\tau}_1$	107.9	0.24	simulation energy spectra, 400 GeV, 200 fb⁻¹
$\tilde{\tau}_2$	194.9	1.1	estimate threshold scan, 60 fb⁻¹
\tilde{t}_1	366.5	1.9	estimate b-jet spectrum, $m_{\min}(\tilde{t}_1)$, 1TeV, 1 ab⁻¹

3. Determination of the SUSY parameters:

Once m_i, σ, P_i are measured, determine the low-energy SUSY parameters from inversion of the mass and cross section formulae:

• **Chargino/neutralino system:** see Jan Kalinowski

$$M_1 = \sqrt{\sum_i m_{\chi_i^0}^2 - M_2^2 - \mu^2 - 2M_Z^2}, \quad M_2 = M_W \sqrt{\Sigma - \Delta [c_{2\phi_R} + c_{2\phi_L}]}$$

$$|\mu| = M_W \sqrt{\Sigma + \Delta [c_{2\phi_R} + c_{2\phi_L}]}, \quad \tan \beta = \sqrt{(1 + \Delta') / (1 - \Delta')}$$

$$\text{with } \Delta = \frac{m_{\tilde{\chi}_2^\pm}^2 - m_{\tilde{\chi}_1^\pm}^2}{4M_W^2}, \quad \Delta' = \Delta (c_{2\phi_R} - c_{2\phi_L}), \quad \Sigma = \frac{m_{\tilde{\chi}_2^\pm}^2 + m_{\tilde{\chi}_1^\pm}^2}{2M_W^2} - 1.$$

• **Sfermion system:** see Barbara Mele

$$m_{\tilde{f}_{L,R}}^2 = M_{\tilde{f}_{L,R}}^2 + M_Z^2 \cos 2\beta (I_{L,R}^3 - Q_f \sin^2 \theta_W) + m_f^2$$

$$A_f - \mu (\tan \beta)^{-2I_f^3} = (m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2) / (2m_f) \cdot \sin 2\theta_{\tilde{f}}$$

• **Higgs system:** see e.g. Marco Battaglia

$$\text{Precise } M_h \text{ measurement: } M_h^2 = M_Z^2 |\cos 2\beta|^2 + \frac{3g^2}{2\pi^2} \frac{m_t^4}{M_W^2} \log \frac{m_{\tilde{t}}^2}{m_t^2}$$

$$\Phi \rightarrow \tilde{\tau}_1 \tilde{\tau}_2, \quad \Phi \rightarrow \chi\chi, \quad \tau\tau \rightarrow \Phi, \quad e^+e^- \rightarrow t\bar{t}\Phi, b\bar{b}\Phi, \chi\chi\Phi, \dots$$

3. Determination of SUSY parameters: summary

In reality, life is more complicated than the tree-level results above:

complete analysis with sophisticated programs: Sfittino, Sfitter, ...

	Δ LHC	Δ ILC	Δ LHC+ILC	SPS1a	Δ LHC+ILC	SPS1a'
$\tan \beta$	± 9.1	± 0.3	± 0.2	10	± 0.3	10
μ	± 7.3	± 2.3	± 1.0	344.3	± 1.1	396
M_A	fixed	± 0.9	± 0.8	399.1	± 0.8	372
A_t	± 91	± 2.7	± 3.3	-504.9	± 24.6	-565
M_1	± 5.3	± 0.1	± 0.1	102.2	± 0.1	103.3
M_2	± 7.3	± 0.7	± 0.2	191.8	± 0.1	193.2
M_3	± 15	fixed	± 11	589.4	± 7.8	571.7
$M_{\tilde{\tau}_L}$	fixed	± 1.2	± 1.1	197.8	± 1.2	179.3
$M_{\tilde{e}_L}$	± 5.1	± 0.2	± 0.2	198.7	± 0.18	181.0
$M_{\tilde{e}_R}$	± 5.0	± 0.05	± 0.05	138.2	± 0.2	115.7
$M_{\tilde{Q}_{3L}}$	± 110	± 4.4	± 39	501.3	± 4.9	471.4
$M_{\tilde{Q}_{1L}}$	± 13	fixed	± 6.5	553.7	± 5.2	525.8
$M_{\tilde{d}_R}$	± 20	fixed	± 15	529.3	± 17.3	505.7

3. Determination of SUSY parameters

Once the low-energy SUSY parameters have been obtained, try to determine the SUSY parameters at the very high scale ($M_{\text{GUT}}, M_{\text{P}}$).

- pin-down the model/SUSY-breaking (mSUGRA, AMSB, GMSB, ..),
- determine the few fundamental unified parameters of the model.

Example of mSUGRA, using all previous measurements at LHC/ILC:

	SPS1a	LHC	ILC	LHC+ILC	SPS1a'	LHC+ILC
m_0	100	± 4.0	± 0.09	± 0.08	70	0.2
$m_{1/2}$	250	± 1.8	± 0.13	± 0.11	250	0.2
$\tan \beta$	10	± 1.3	± 0.14	± 0.14	10	0.3
A_0	-100	± 31.8	± 4.43	± 4.13	-300	13

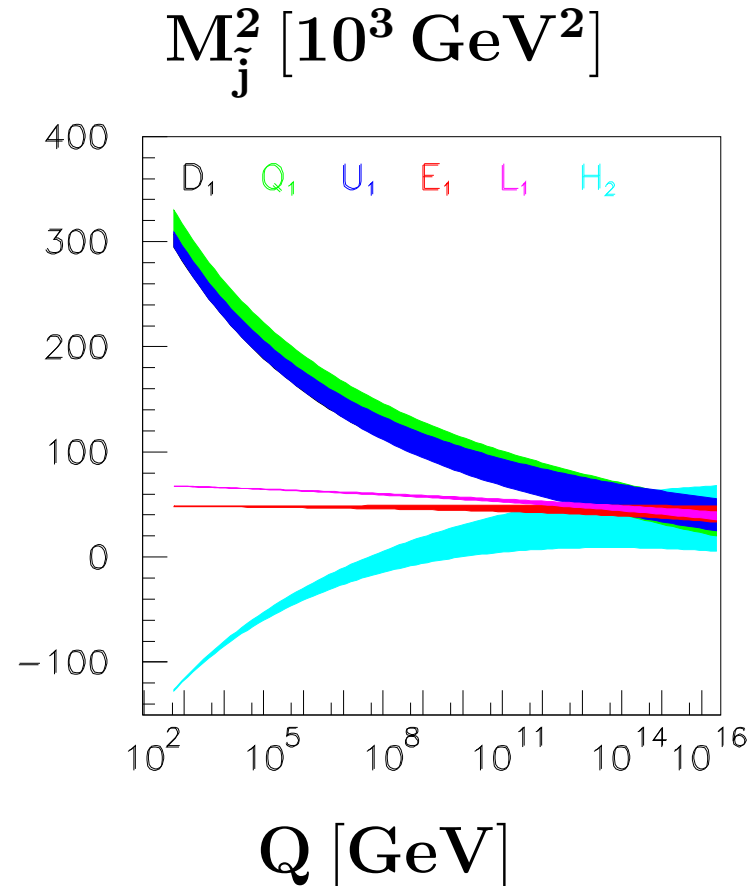
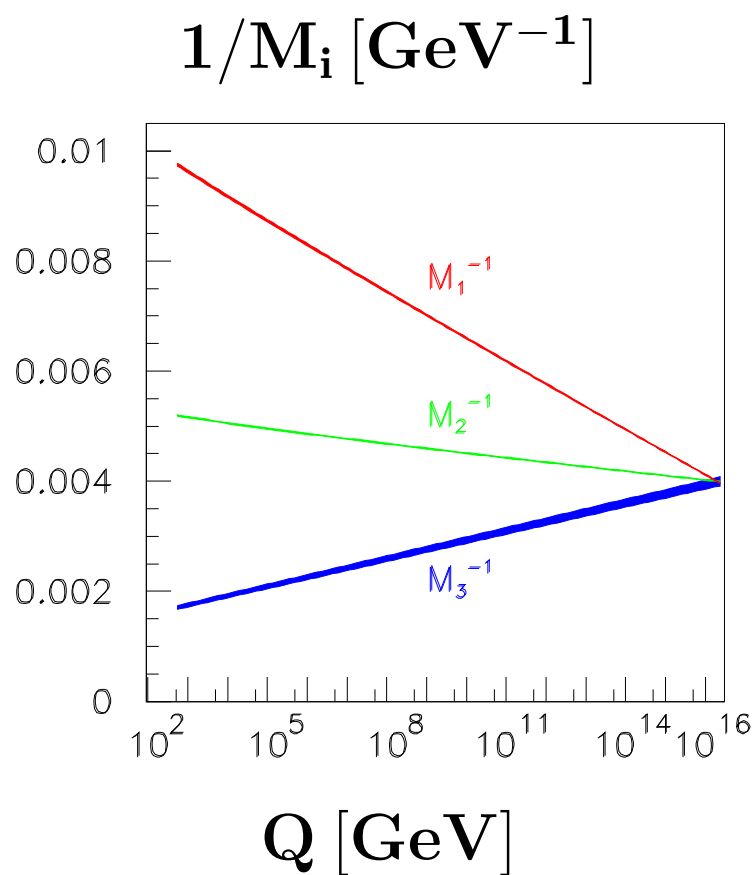
The same type of analysis in other breaking schemes/models.

To be absolutely sure: only with model dependent analyses at ILC!

3. Determination of SUSY parameters:

One can check the fundamental assumptions at high (GUT) scale.

For example: gaugino and scalar mass unification in mSUGRA....



Also: check that one is in accord with cosmology (see G. Bélanger)...

4. Summary

If SUSY is the solution to the SM pbs: SUSY particles should be light.

Colored and non-colored sparticles observable very soon at LHC.

The ILC will be needed as it will provide very crucial information:

- **very clean environment, large production rates with low backgrounds,**
- **tunable energy to perform threshold scans and increase rates,**
- **beam polarisation which allow to select various channels,**
- **additional options (e^-e^- , $\gamma\gamma$, $e\gamma$) for complementgary studies,**
⇒ **high-precision analyses and a true probe of SUSY phenomena.**

Only coherent/combined analyses of LHC+ILC data will allow for:

- **better/model independent reconstruction of low energy SUSY paramete**
- **connect weak-scale SUSY with more fundamental physics at GUT scale,**
- **provide input to predict the LSP density and connection with cosmology**

Here: gave illustration of ILC performance in mSUGRA-type MSSM,

Many interesting analyses/physics can also be done in other scenarios!