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THE CASIMIR EFFECT

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- *Dark energy*

- *The Casimir effect as a manifestation of quantum vacuum energy*

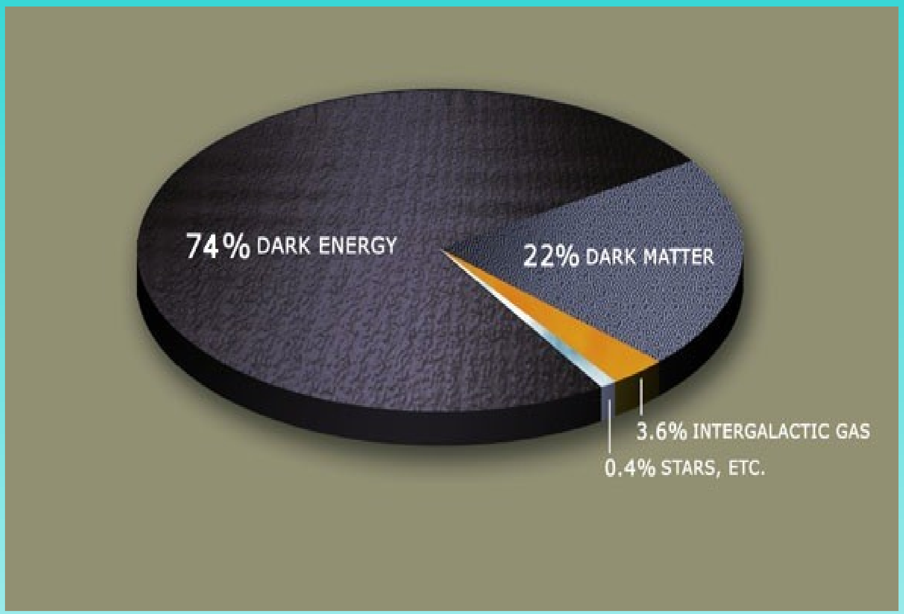
- *Dependence upon the fine structure constant*

- *The Casimir effect as a van der Waals force between giant « molecules »*

- *« Reality » of quantum fluctuations in the vacuum?*

- *Conclusion*

DARK ENERGY



from Hubble plot at large red shifts and CMB fluctuations

Dark energy has a constant energy density and a negative pressure

$$\Omega_{DE} \simeq 3/4\,\Omega_c \simeq 4\,GeV/m^3$$

Dark matter energy ⇦ zero point energy of quantum fields

- first proposed by Zeldovitch (1957)
- confirmed by Casimir effect, according...

S. Weinberg, 1987

Perhaps surprisingly, it was a long time before particle physicists began seriously to worry about [quantum zero point fluctuations contribution to Λ] despite the ***demonstration*** *in the Casimir effect of the reality of zero-point energies.*

S. M. Carroll, 2001

... and the vacuum fluctuations themselves are ***very real****, as evidenced by the Casimir effect.*

D. Perkins, « Particle Astrophysics », Oxford Univ. Press, 2008

That this concept [the vacuum energy] is ***not a figment*** *of the physicist's imagination was already* ***demonstrated*** *many years ago, when Casimir predicted that by modifying boundary conditions on the vacuum state, the change of the vacuum energy would lead to a measurable force, subsequently detected and measured by...*

For a free bosonic field:

$$\epsilon = \sum 1/2\,\hbar\omega = \hbar c \frac{\int^{k_{cut}} d^3k}{(2\pi)^3} k = \frac{1}{8\pi^2} \hbar c k_{cut}^4$$

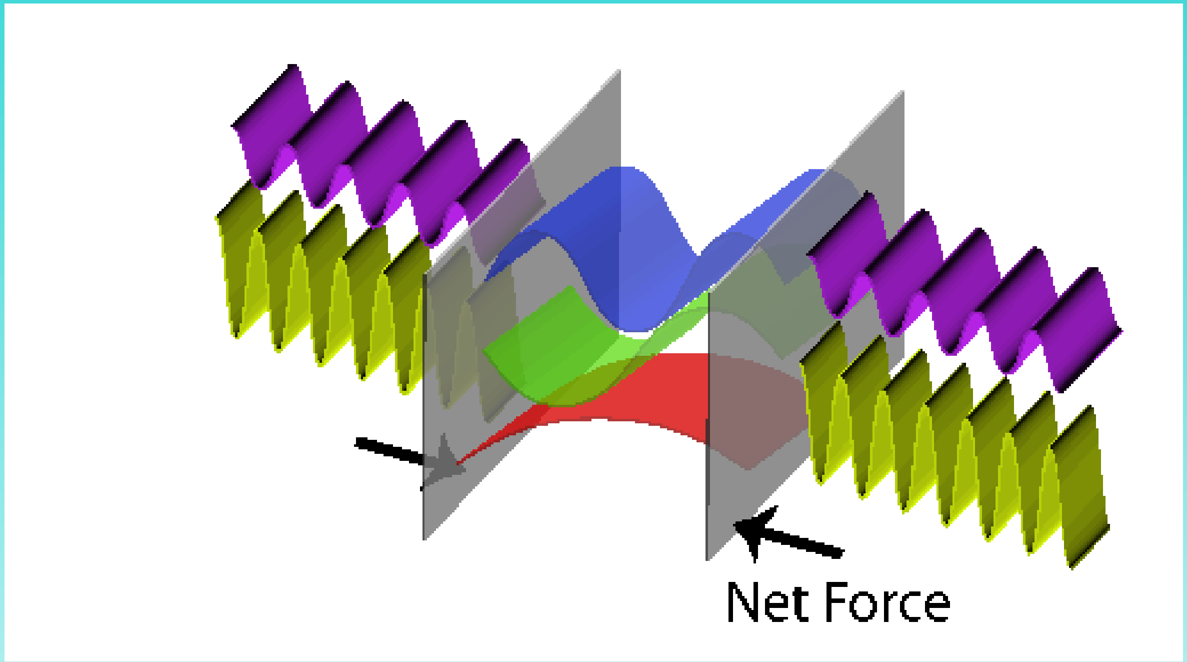
For $k_{cut} \sim 1/\ell_{PL}$, $\varepsilon \sim 10^{121}$ GeV/m^3

For a free fermionic field:

$$\epsilon = -\sum 1/2\,\hbar\omega$$

... and there are plenty of condensates in the SM !

THE CASIMIR FORCE and the ZERO POINT ENERGY of the ELECTROMAGNETIC FIELD



The modes in the cavity are not the same as in free space, especially at low frequency

Interaction energy=difference of zero-point energies

$$\vec{E}=\vec{\epsilon}\, e^{ik_x x} e^{ik_y y} e^{ik_z z} e^{-i\omega_k t} \Leftrightarrow \vec{E}=\vec{\epsilon}\, e^{ik_x x} e^{ik_y y} \sin k_z z\, e^{-i\omega_k t}$$

$$k_z = n_z \pi / d \quad for\, n_z = 1,2,3,\ldots.$$

Rm: for n_z=0, only one mode

$$\frac{\Delta E}{S}=\frac{E_{cav}}{S}-\frac{E_{free}}{S}$$

$$=\frac{\hbar c \pi^2}{4 d^3}\left\{\frac{1}{2}\int_0^\infty du\sqrt{u}+\sum_{n=1}^\infty \int_0^\infty du\sqrt{u+n^2}-\int_0^\infty dx\int_0^\infty du\sqrt{u+x^2}\right\}$$

Rm: regularized by integration factor $e^{-\alpha\sqrt{u+x^2}}, \alpha \to 0$

Euler-McLaurin theorem:

$$\frac{1}{2} f(0)+\sum_{n=1}^{\infty} f(n)-\int_0^\infty f(x)=-\frac{1}{2} f(\infty)+\frac{1}{12}\left[f'(0)-f'(\infty)\right]-\frac{1}{720}\left[f'''(0)-f'''(\infty)\right]+\ldots$$

$$\frac{\Delta E}{S}=-\frac{\hbar c \pi^2}{720 d^3}$$

See Itzykson and Zuber

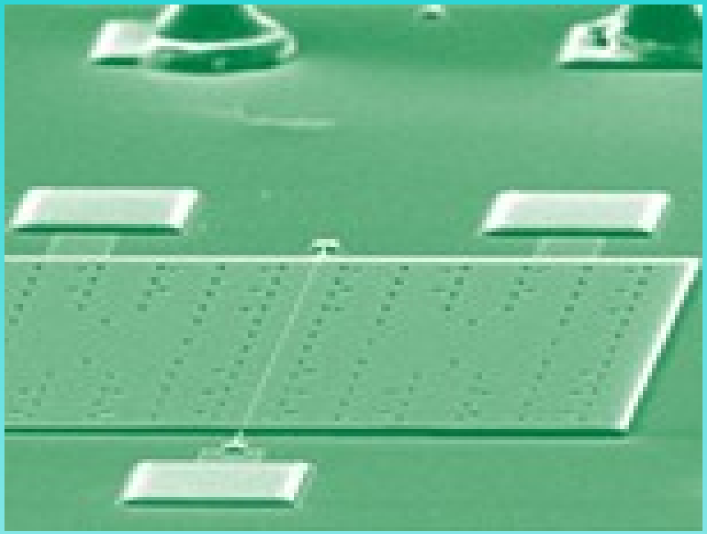
The Casimir force

$$\frac{F}{S} = -\frac{\hbar c \pi^2}{240\, d^4} = 4 \times 10^{-4}\, N/m^2 \; at \quad d = 1\, \mu m$$

- has been calculated for many « geometries »
- has been verified experimentally

  - M. J. Sparnaay, *Physica 24 (1958) 751*
    parallelism, impurities, residual charges
  - M. J. Spaarnay, P. W. J. Jochems, *1960*
  - B. V. Darjaguin et al, *« Surface forces », Plenum, 1987*
  - S. K. Lamoreaux, *PRL 78 (1997) 5*
  - *T. Ederth* (2001?) 1%

- Now, a blooming field in MEMS technology



-from attractive to repulsive
-turned to be a control tool

- A field of its own

DEPENDANCE on the FINE STRUCTURE CONSTANT

Universality? Independence of α?

Real conductors are characterized by:

- plasma frequency ω_{pl} : no propagation for $\omega < \omega_{pl}$
- skin depth δ; $\delta^{-2} = 2\pi\, \omega|\sigma| /c$: penetration depth of incident waves

Drude model: free electrons with friction force ($f=-\gamma v$)

$$\omega_{pl} = \sqrt{\frac{4\pi e^2 n}{m_e}} \qquad \delta = c\left(\frac{1}{2}\frac{\omega\,\omega_{pl}^2}{\sqrt{\gamma^2+\omega^2}}\right)^{-1/2}$$

Limit of perfect conductor: $\omega_{pl} \to \infty$, $\delta \to 0$

Perfect metal hypothesis is
justified if characteristic frequencies c/d << ω_{pl} i.e.

$$\alpha \gg \frac{m_e c}{4 \pi \hbar n d^2}$$

For typical cases (Cu, d=1μm), rhs ~ 10^{-6}

- This condition is comfortably satisfied for the physical value of α
- Casimir's result = the $\alpha \to \infty$ limit !!!!
- corrections appears as powers of $1/\alpha$

$$\frac{\Delta E}{S} \approx -\frac{\hbar c \pi^2}{720 (d + 2\delta)^3} \approx -\frac{\hbar c \pi^2}{720 d^3} \left(1 - 6 \frac{\delta}{d} + \cdots \right)$$

Small α limit

$$a_B = \frac{\hbar^2}{m_e e^2} \propto 1/\alpha, \quad n \propto \alpha^3, \quad \omega_{pl} \propto \alpha^2, \quad \delta \propto 1/\alpha^2$$

Conducting plates become transparent and Casimir effect goes away

The only distinctive feature: goes to a constant in the strong coupling limit

CASIMIR EFFECT as a VAN DER WAALS FORCE between MACROSCOPIC NEUTRAL OBJECTS

Two neutral atoms at distance r : effects of Coulomb forces

London (1937) : 2d order in α^2 and in R_{at}/r

$$\Delta E^{(2)} = -6\frac{e^4}{r^6}\sum_{k\neq 0}\sum_{l\neq 0}\frac{|a_{k0}^{(2)}|^2 |a_{l0}^{(1)}|^2}{E_k^{(2)}-E_0^{(2)}+E_l^{(1)}-E_0^{(1)}} \quad , a_{k0} = \langle k|\sum z_i|0\rangle$$

Atom 2 1

Static atomic polarizability $\alpha_i = e^2 \sum_{k\neq 0}\frac{|a_{k0}^{(i)}|^2}{E_k^{(i)}-E_0^{(i)}}$ $\epsilon = 1+4\pi n_{at}\alpha$

HENDRIK BRUGT GERHARD CASIMIR

15 JULY 1909 · 4 MAY 2000

-Known for: Casimir operator, hyperfine interactions, cooling by adiabatic demagnetization (mK), etc

-PhD of Erhenfest, assistant of Pauli, position in Leiden

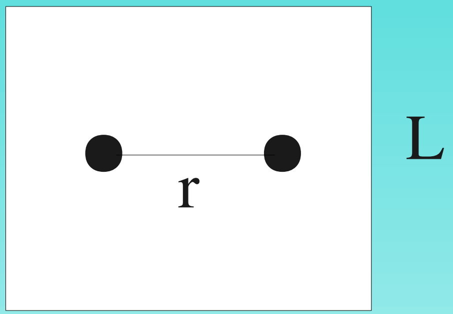
-In 1940, moved to Philips gloeilampenfabriek NV in Eindhoven

In 1947, EJW Verwey and JTG Overbeek : dilute colloidal suspensions

HBG Casimir and D. Polder, Phys. Rev 73 (1948) 360:

Interaction between atoms through Coulomb forces and the coupling to the radiation field (retardation effects)

Perturbation theory at the second order in α

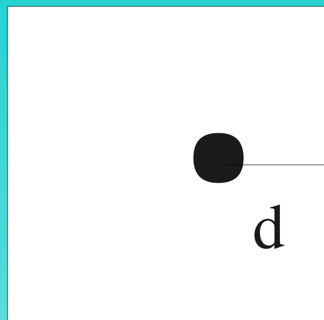


For small r, London's expression is recovered

For $r >> a_{ko}$:

$$\Delta E^{(2)} = -\frac{23\hbar c}{4\pi r^7}\alpha_1 \alpha_2 \qquad (1)$$

In the same paper, interaction of a atom with a conducting plane



For small d : $$\Delta E^{(2)}_{atom-wall} = -\frac{e^2}{4d^3}\sum_k |a_{k0}|^2$$

For d >> a_{ko} : $$\Delta E^{(2)}_{atom-wall} = -\frac{3\hbar c}{8\pi d^4}\alpha_1 \qquad (2)$$

Perturbation theory at the first order in α (first order in Coulomb forces, second order in atom-radiation coupling)

Niels Bohr: « *Why don't you calculate the effect by evaluating the differences of zero point energies of the electromagnetic field?* »

Casimir rederived results (1) and (2) by this method (in some approximation) : *Colloque sur la théorie de la liaison chimique, Paris, 12-17 avril 1948*

He calculated by the same method the interaction between parallel conducting planes

Mathematics. — *On the attraction between two perfectly conducting plates.* By H. B. G. CASIMIR.

(Communicated at the meeting of May 29, 1948.)

In a recent paper by POLDER and CASIMIR [1]) it is shown that the interaction between a perfectly conducting plate and an atom or molecule with a static polarizibility α is in the limit of large distances R given by

$$\delta E = -\frac{3}{8\pi} \hbar c \frac{\alpha}{R^4}$$

and that the interaction between two particles with static polarizibilities α_1 and α_2 is given in that limit by

$$\delta E = -\frac{23}{4\pi} \hbar c \frac{\alpha_1 \alpha_2}{R^7}.$$

These formulae are obtained by taking the usual VAN DER WAALS–LONDON forces as a starting point and correcting for retardation effects.

In a communication to the "Colloque sur la théorie de la liaison chimique" (Paris, 12—17 April, 1948) the present author was able to show that these expressions may also be derived through studying by means of classical electrodynamics the change of electromagnetic zero point energy. In this note we shall apply the same method to the interaction between two perfectly conducting plates.

H.B.G. Casimir, *Proceedings of the Koninglijke Nederlandse Akademie van Wetenschappen B51, 793 (1948)*

Limit of VdW-Casimir-Polder : $\lim \alpha_i / r^3 = 1$

London-VdW	Casimir-Polder	Order
$\Delta E^{(2)} = -6\frac{e^4}{r^6}\sum_{k\neq 0}\sum_{l\neq 0}\frac{\lvert a_{k0}^{(2)}\rvert^2 \lvert a_{l0}^{(1)}\rvert^2}{E_k^{(2)}-E_0^{(2)}+E_l^{(1)}-E_0^{(1)}}$	$\Delta E^{(2)} = -\frac{23\hbar c}{4\pi r^7}\alpha_1\alpha_2$	α^2
$\Delta E^{(2)}_{atom-wall} = -\frac{e^2}{4d^3}\sum_k \lvert a_{k0}\rvert^2$	$\Delta E^{(2)}_{atom-wall} = -\frac{3\hbar c}{8\pi d^4}\alpha_1$	α
Replacing an atom by a plate : $-$infinite volume with constant n; $-n\alpha_i \rightarrow (\epsilon-1)/\epsilon$; $-\epsilon\rightarrow\infty$	$\Delta E = -\frac{\hbar c \pi^2}{720 d^4} S d$	α^0

α_i disappears, *a power of α is absorbed !!* *replaced by a volume !*

Same results by Casimir's method

Conclusion :

Casimir effect = limit of van der Waals force

for « macroscopic molecules »

in the ideal conductor (infinitely polarisable) limit

The « REALITY » of QUANTUM FLUCTUATIONS in the VACUUM

1. Casimir's result is « heuristic »: that the interaction energy is given by the difference of zero point energies is accidental and does not reveal the « reality » of quantum fluctuation energy

for a diffuse charge: $W = \frac{1}{2}\int dr \int dr' \frac{\rho(r)\rho(r')}{|r-r'|} = \frac{1}{8\pi}\int |E(r)|^2 dr$

Rm: « reality » of the field coming from light, pair production, etc

2. Zero-point energy coming from an « obscure » choice of ordering fields in the classical lagrangian (giving negative energy for fermion fields)

3. Interaction between neutral objects gives no more (or less) support to the « reality » of the vacuum energy of fluctuating quantum fields than the other one-loop effects in quantum electrodynamics, like vacuum polarisation contribution in the Lamb shift (R. Jaffe, PR D72 (2005) 021301) : the effect vanishes as $\alpha \to 0$

4. Casimir effect can be derived without reference to zero-point motion

-general theory of fluctuations (Lifshitz, *Zhur. eksp. i teor. fiz. 29 (1955) 9*, Landau and Lifshitz, *Electrodynamics of continuous media*)

$$\langle \vec{E}i(r)\vec{E}j(r)\rangle = f(\varepsilon), \ldots W = \frac{1}{8\pi}\int (|E|^2 + |H|^2)$$

no field quantization

calculation of the interaction between semi-infinite pieces of dielectrics; Casimir force obtained as the $\varepsilon \to \infty$ limit

- field theory without reference to zero-point fluctuations:
  Schwinger, 1975, scalar field
  Schwinger, DeRaad, Milton, 1978, for QED

$$\Delta E = \frac{\hbar}{2\pi} \Im \int d\omega\, \omega\, Tr \int d^3x \left[G(x, x, \omega + i\epsilon) - G_0(x, x, \omega + i\epsilon) \right]$$

G is the full Green's function in the background (plates)

$$\Delta E \propto \int d\omega\, \omega \frac{d\Delta N}{d\omega} \quad \text{i.e.} \quad \frac{1}{2} \sum (\hbar\omega - \hbar\omega_0)$$

G can be expanded in series of G_0 and α

Rm: all features of QED can be reformulated from the point of view of zero point fluctuations (Milonni, *« The Quantum Vacuum », Acad. Press, 1994*)

CONCLUSIONS

- Casimir effect often advocated as a manifestation of the quantum fluctuations of the vacuum and the support of the latter as a candidate for DM. But...

- Casimir force can (should) be viewed as a vdW force between gigantic molecules in the strong coupling limit

- Casimir forces are real in the micro- to nano-world

- Not a demonstration of the « reality » of the zero-point energy (no more than vacuum polarisation, ...): vanishes in weak coupling, formulation without reference to zero-point energy

- Reality of vacuum energy?