The Yang-Mills spectrum with arbitrary simple gauge algebras

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Prologue

Yang-Mills

$$\mathcal{L}_{YM} = -\frac{1}{4} F_a^{\mu\nu} F^a_{\mu\nu}$$ + simple gauge algebra

- Nonabelian
- Asymptotic freedom
- Glueballs
  - Lattice : su(N)
  - Effective approaches ?

Why other algebras?

Better understand confinement dynamics

- Check the robustness of standard frameworks
- Maybe see what makes su(3) special

Example

- A_r : su(r+1)
  - QCD and large N limit
  - Center, Z_{r+1}
- G_2 : No center

Confinement ~center symmetry ?
~Polyakov loop

Outline

Quasigluons

- Constituent picture
- Relevance

Low-lying Yang-Mills spectrum

- Gluelumps, glueballs

Static energy

- Two and three adjoint sources

Large N limit

- Comparison to lattice data

Conclusions

Quasigluons

Constituent approaches

Relevance for quarks

- Heavy quarks: $1/m_Q$ expansion
- Light quarks
  - Dynamically generated mass thanks to chiral symmetry breaking

What about gluons?

- Dynamically generated mass
  - Finite zero momentum limit of the gluon propagator
  - Not universally accepted
- Justification of constituent picture

Gluon mass gap (I)

Coulomb gauge QCD

- Instantaneous potential

$$V_{(r)}(z) = \frac{C_2^{(r)}}{C_2^{(adj)}} \left(\sigma_0 \, z - \frac{\alpha_0}{z} \right)$$

  - One-gluon exchange
  - Casimir scaling

- Gluon mass gap equation

$$\omega(q)^2 = q^2 + \int \frac{d^3k}{4(2\pi)^3} \tilde{V}_{(adj)}(|\vec{k}+\vec{q}|) \left[1 + (\hat{\vec{k}} \cdot \hat{\vec{q}})^2\right] \frac{\omega(k)^2 - \omega(q)^2}{\omega(k)}$$

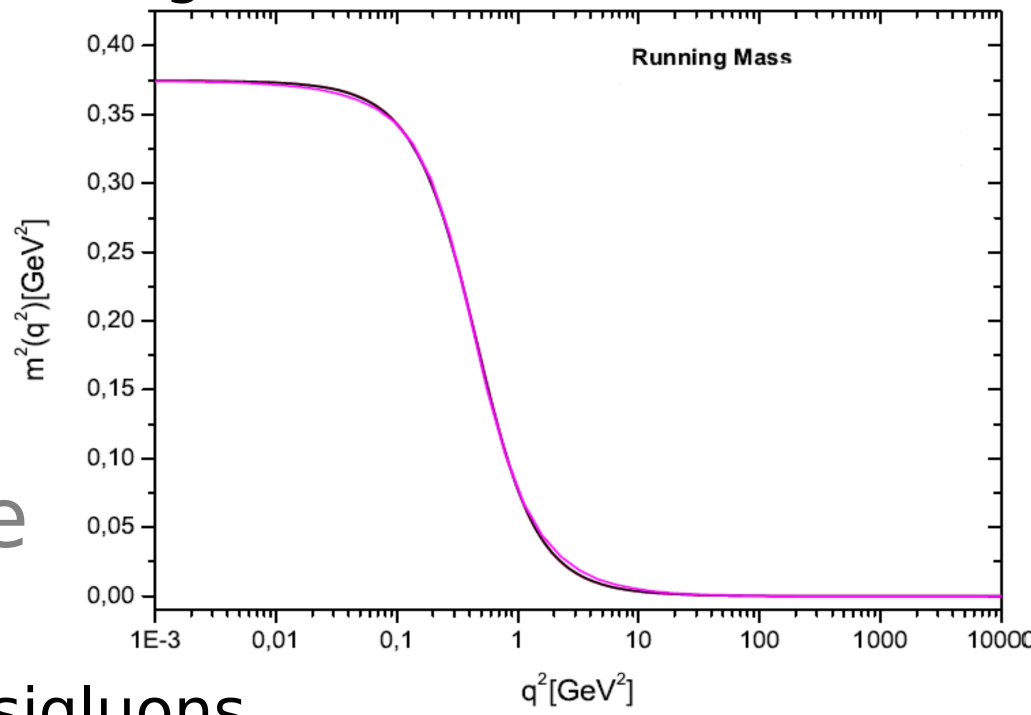
A. P. Szczepaniak and E. S. Swanson, Phys. Rev. D **65**, 025012 (2002)

A. P. Szczepaniak and E. S. Swanson, Phys. Rev. Lett. **87**, 072001 (2001)

Gluon mass gap (II)

Solution of type $\omega(q)^2 = q^2 + m_g(q)^2$

- $m_g(q) = \sqrt{\sigma_0}\,\bar{m}(q/\sqrt{\sigma_0}\,, \alpha_0)$
- Qualitatively valid for all algebras
- Quasigluons
  - Transverse
  - Dynamical mass



Quasigluon picture

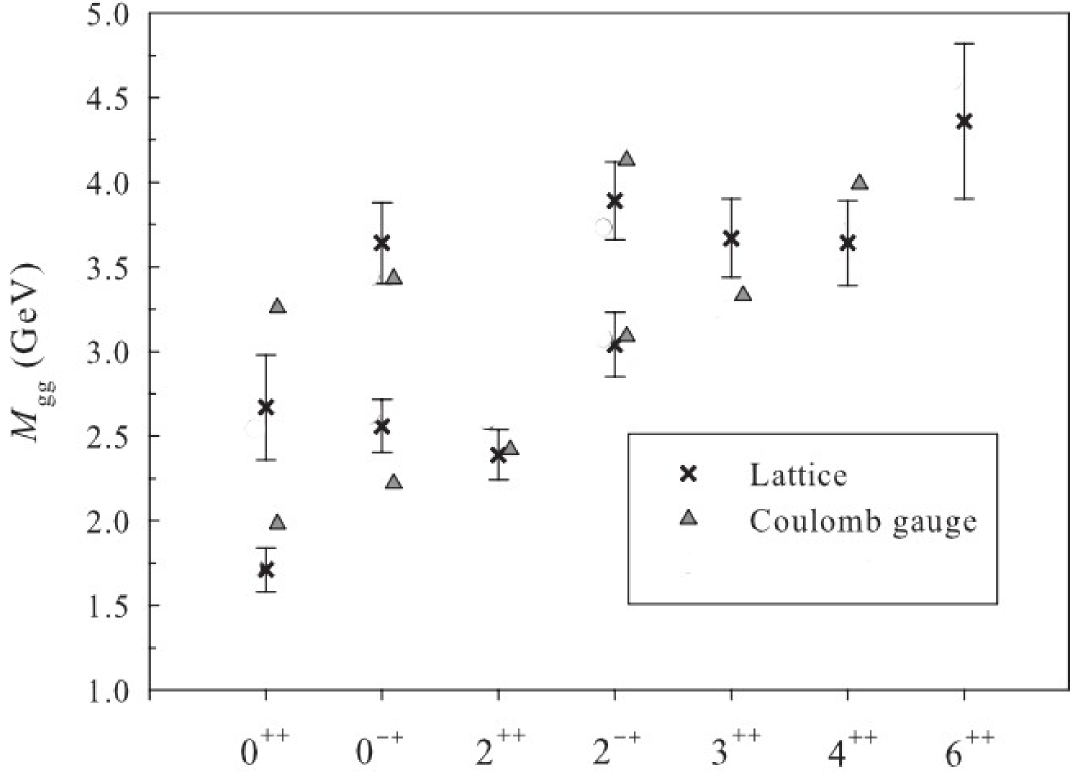
- Gluonic state

  = bound state of quasigluons

Glueball spectrum

Coulomb gauge QCD

- Two quasigluons



A. P. Szczepaniak and E. S. Swanson, Phys. Lett. B **577**, 61 (2003)

Quasigluon's features

Adjoint representation

- Charge conjugation $\hat{C}\, A_\mu\, \hat{C}^{-1} = -A_\mu^T$

Transverse particles

- Helicity 1

Bound state of n_g quasigluons

- Jacob and Wick's helicity formalism

  M. Jacob and G. C. Wick, Ann. Phys. **7**, 404 (1959).

$$\left| n_g;\, J^{PC} \right\rangle = |colour\rangle \otimes |spin - space\rangle$$

J^P

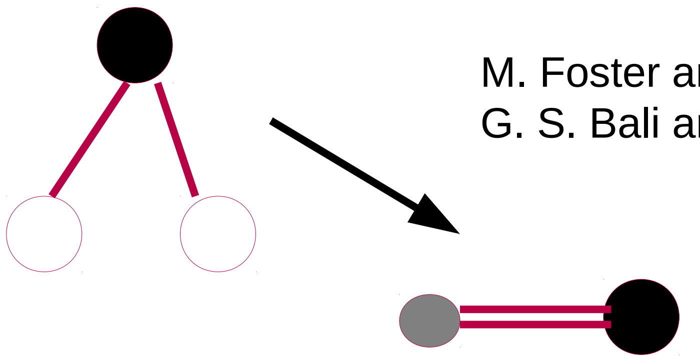
C

Low-lying Yang-Mills spectrum

Gluelumps (I)

Yang-Mills field + static adjoint source

- Static source ~ adjoint quark-antiquark pair
- Extreme simplification of heavy hybrids



M. Foster and C. Michael, Phys. Rev. D **59**, 094509 (1999)
G. S. Bali and A. Pineda, Phys. Rev. D **69**, 094001 (2004)

In general $adj \otimes adj = \bullet^S \oplus adj^A \oplus \ldots$

- One quasigluon is enough
- Lightest states

Gluelumps (II)

Colour structure

- Negative C

$$\delta_{ab}\phi^a A^b_\mu \propto \mathrm{Tr}(\phi A_\mu) = -\mathrm{Tr}(\phi^{\mathcal{C}} A^{\mathcal{C}}_\mu)$$

Helicity states: J > 0

- Lowest spin

$$|g;1^{+-}\rangle = \sqrt{\frac{2}{3}}|^3S_1\rangle + \sqrt{\frac{1}{3}}\,|^3D_1\rangle$$

$$|g;1^{--}\rangle = -|^3P_1\rangle$$

Lightest, ok with lattice

Glueballs (I)

Lightest : Two quasigluons

- $M_{gg} = O(2M_g)$
- Physical gluonic states

Colour structure $adj \otimes adj = \bullet^S \oplus adj^A \oplus \ldots$

- Positive C

$$\delta_{ab} A_\mu^a A_\nu^b \propto \mathrm{Tr}(A_\mu A_\nu) = \mathrm{Tr}(A_\mu^{\mathcal{C}} A_\nu^{\mathcal{C}})$$

Glueballs (II)

Helicity states + Pauli principle

- Color symmetric, spin-space symmetric
- No 1^{++} and 1^{-+} states
  - Yang's theorem, no $\rho \to \gamma\gamma$
  - Lattice, no light J = 1 glueball
- Examples

$$|0^{++}\rangle = \sqrt{\frac{2}{3}}\,|{}^1S_0\rangle + \sqrt{\frac{1}{3}}\,|{}^5D_0\rangle$$ Lightest

$$|0^{-+}\rangle = -\,|{}^3P_0\rangle$$

$$|2^{++}\rangle = \sqrt{\frac{2}{5}}\,|{}^5S_2\rangle + \sqrt{\frac{4}{7}}\,|{}^5D_2\rangle + \sqrt{\frac{1}{35}}\,|{}^5G_2\rangle$$

Glueballs (III)

Three quasigluons

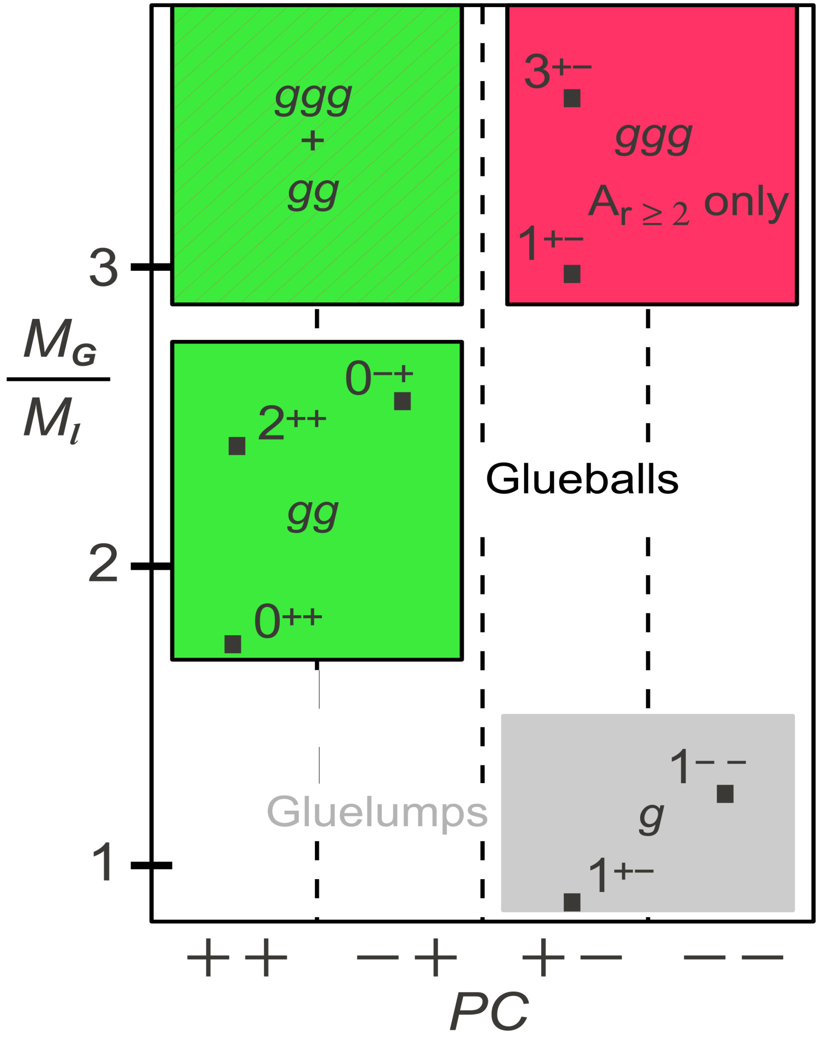
- $M_{gg} = O(3M_g)$

Colour structure

- Positive C $adj \otimes adj \otimes adj = \bullet^A \oplus \ldots$
  - Always possible
- Negative C $adj \otimes adj \otimes adj = \bullet^A \oplus \bullet^S \oplus \ldots$
  - Only for A_r gauge algebras with r>1
- Example : C = - glueballs for su(3), not for su(2)

M. J. Teper, hep-th/9812187

Partial summary



Ok with current su(3) lattice data

Higher-lying states

If only real representations

- $$T_a^{(r)} = -(T_a^{(r)})^{\mathrm{T}}$$
  $$\rightarrow A_\mu = A_\mu^C$$
- No C = – glueball
- Happens for A_1, B_2, C_r, $D_{\text{even-r>3}}$, E_7, E_8, F_4, and G_2
- Contribution to thermodynamical quantities

Static energy

Two adjoint sources

Funnel potential

- Standard form in QCD

$$V_{(adj)}(z) = \sigma_0 z - \frac{\alpha_0}{z}$$

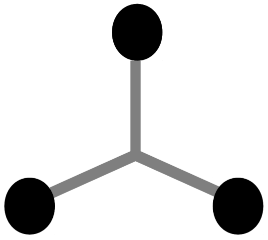
- Could be used to measure the parameter's values
- Casimir scaling assumed
  - Ok with lattice, also with G_2

G. S. Bali, Phys. Rev. D **62**, 114503 (2000)

A. Wipf, and C. Wozar, arXiv:1006.2305

Three adjoint sources (I)

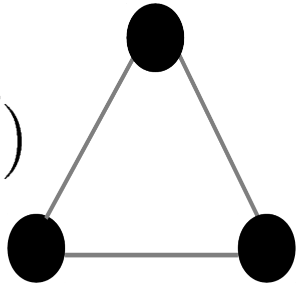
Y-junction



- Three adjoint strings

$$V_Y = \sum_i \sigma_0 \, |\vec{r}_i - \vec{Y}| - \frac{1}{2} \sum_{i<j} \frac{\alpha_0}{|\vec{r}_i - \vec{r}_j|}$$

Δ- shape



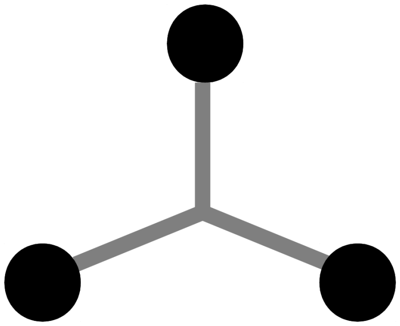
- Adjoint representation in $f \otimes f \;\; (f \otimes \overline{f})$
- Three fundamental strings

$$V_\Delta = \sum_{i<j} \left[\frac{C_2^{(f)}}{C_2^{(adj)}} \sigma_0 |\vec{r}_i - \vec{r}_j| - \frac{1}{2} \frac{\alpha_0}{|\vec{r}_i - \vec{r}_j|} \right].$$

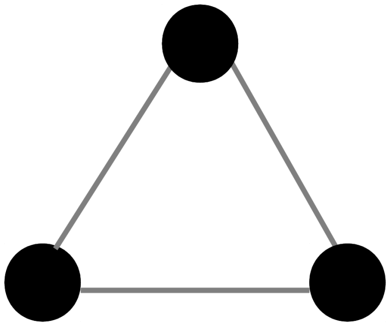
Three adjoint sources (II)

Equilateral triangle: $\frac{V_Y}{V_\Delta} = \frac{1}{\sqrt{3}} \frac{C_2^{(adj)}}{C_2^{(f)}}$

- Algebra-dependent



B_3, D_4, E_6, E_7, E_8, F_4



A_r, $B_{r\geq 4}$, $C_{r\geq 2}$, $D_{r\geq 5}$, G_2

Checked for su(3)

M. Cardoso and P. Bicudo, Phys. Rev. D **78**, 074508 (2008)

Large N limit

Two quasigluons

Gauge algebra $A_{N-1} \cong su(N)$

- Fixed 't Hooft coupling $\lambda = N g^2$
- Potential and mass gap N-independent $\alpha_0, \ \sigma_0 \propto \lambda$

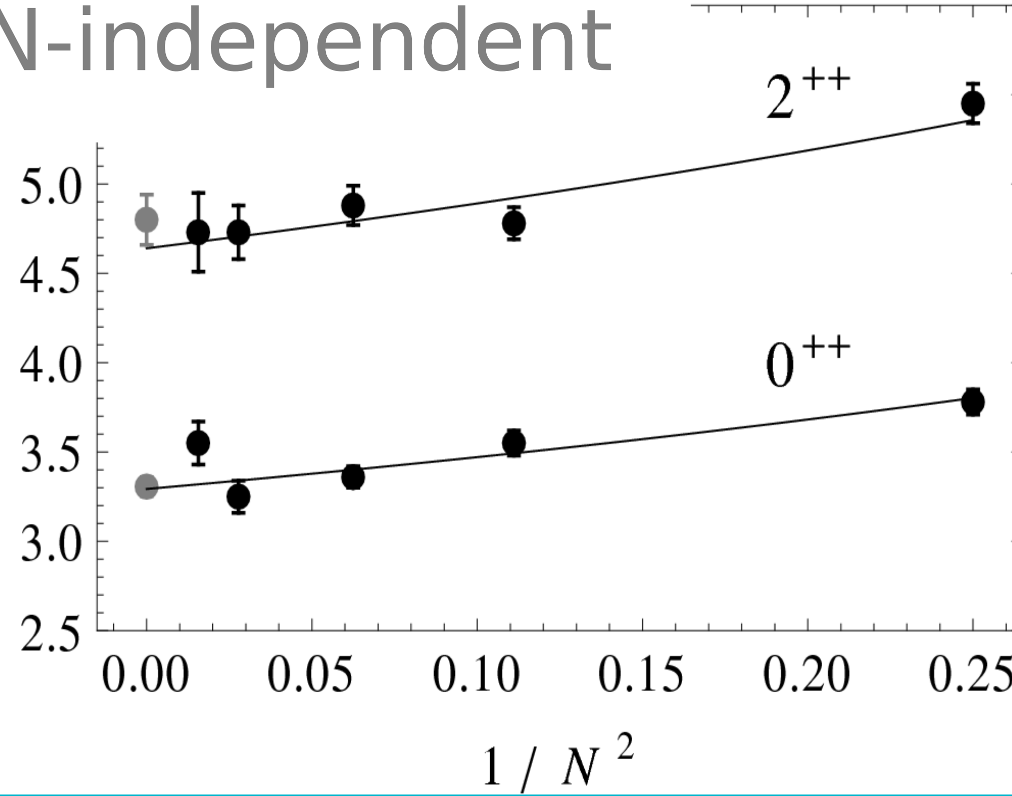
Glueball mass : N-independent

- OK with lattice

B. Lucini, A. Rago, and E. Rinaldi, JHEP **1008**, 119 (2010)

$$\frac{M_{gg}}{\sqrt{\sigma^{(f)}}} = \sqrt{\frac{2N^2}{N^2-1}}\,\theta_{gg}$$

B. Lucini, M. Teper, and U. Wenger, JHEP **0406**, 012 (20004)



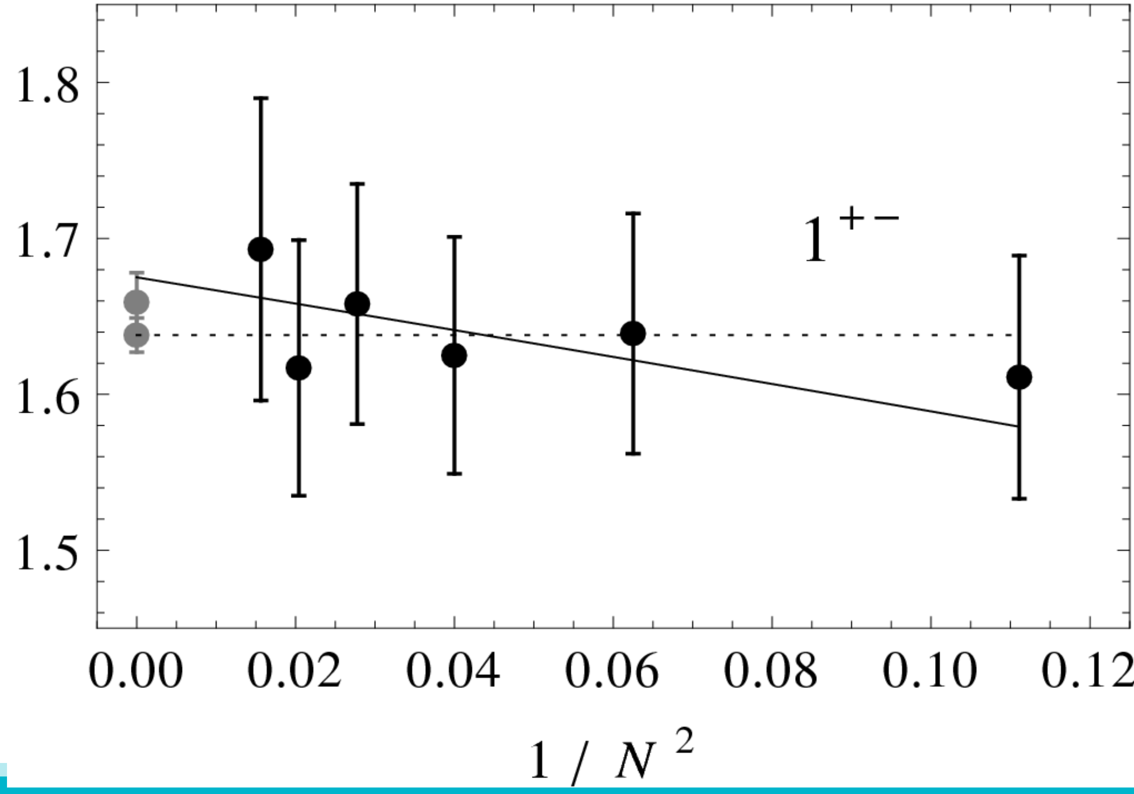
Three quasigluons

Potential

- Y- junction N-independent
- Δ shape N-dependent through the string tension

$$M^{Y}_{ggg} = \theta_{ggg}$$

$$M^{\Delta}_{ggg} = \sqrt{\frac{N^2 - 1}{2N^2}}\,\theta_{ggg}$$



B. Lucini, A. Rago, and E. Rinaldi, JHEP **1008**, 119 (2010)

Conclusions

Summary

Yang-Mills theory

- Confining for any simple gauge algebra
- Glueballs: general feature

Quasigluon picture

- Bound states of transverse adjoint particles
- Global structure of the spectrum

Static energy

- Algebra-dependent for 3 adjoint sources

Large N limit

- Glueball masses versus N agree with lattice results

Confirmation of the present ideas

- Other theoretical frameworks
- Lattice computations ?

Very last slide

Full Yang-Mills

Constituent models

- Less fundamental
- Capture essential features
- Intuitive / analytical

Reliable approximation