Phase Structure of Strongly Interacting Theories and Finite-Size Effects

Jens Braun

Friedrich-Schiller-University Jena

30 years of strong interactions: a three-day meeting in honor of Joseph Cugnon and Hans-Jürgen Pirner

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Research Training Group
Quantum and Gravitational Fields

“Historical” remarks - the very original motivation ...

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PHYSICAL REVIEW D VOLUME 23, NUMBER 2 15 JANUARY 1981

Universal upper bound on the entropy-to-energy ratio for bounded systems

Jacob D. Bekenstein*

Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

(Received 7 July 1980; revised manuscript received 25 August 1980)

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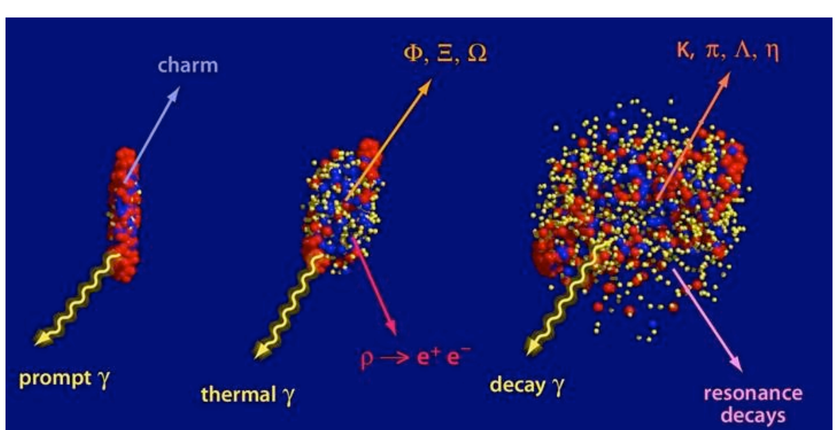
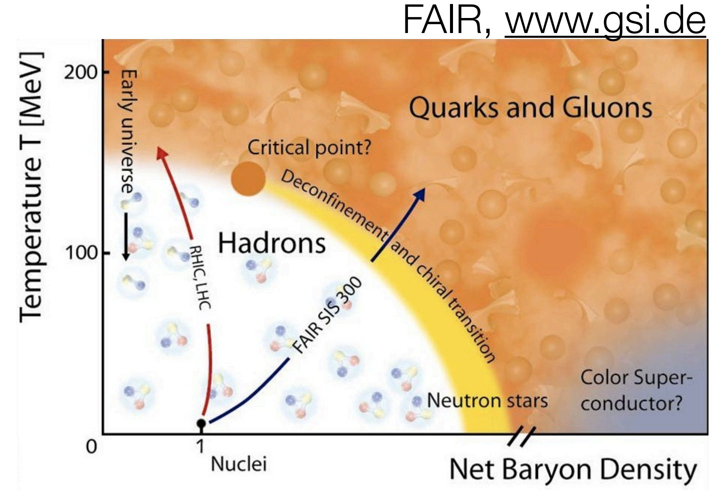
?

idea of **H. J. Pirner** in 2003



FAIR, www.gsi.de



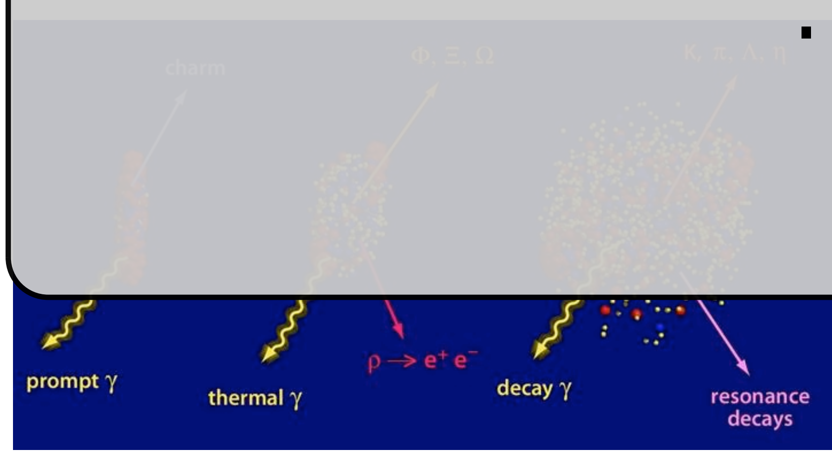
“Historical” remarks - the very original motivation ...

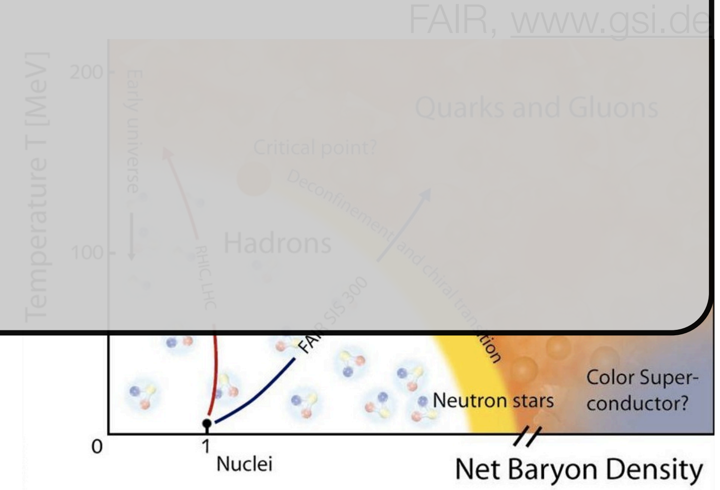
many questions:

vacuum energy?

effect of the **finite size** of a (realistic) system?

...



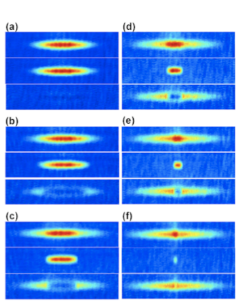


(Realistic) systems have a **finite size** ...

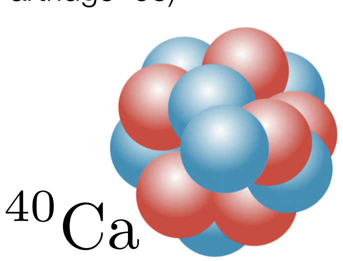
effects of a
finite system-size

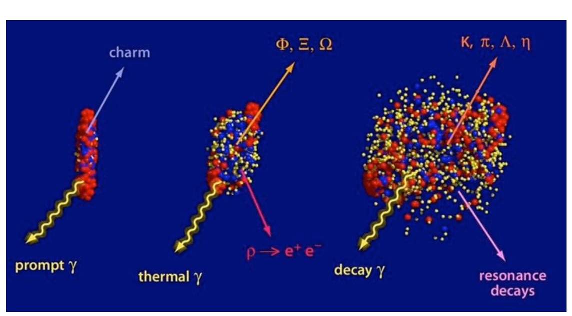
phenomenologically relevant:

- **QCD phase diagram** & HI-collision
- **experiments with ultracold atoms**
- nuclear physics
- ...



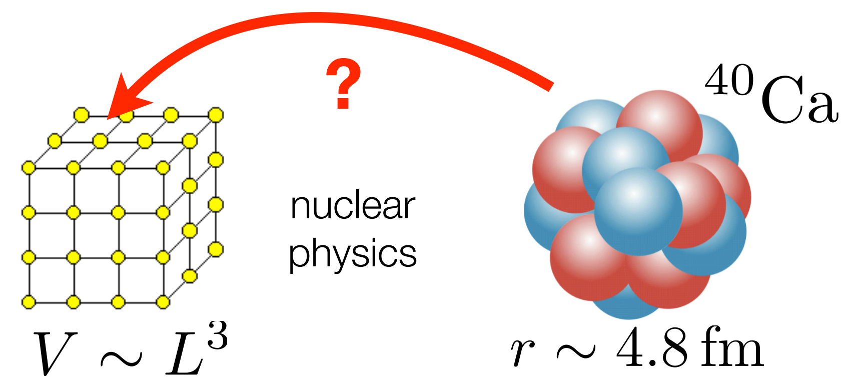
(Partridge '06)



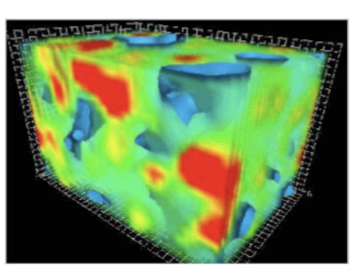


field-theoretical relevance:

Monte-Carlo simulations

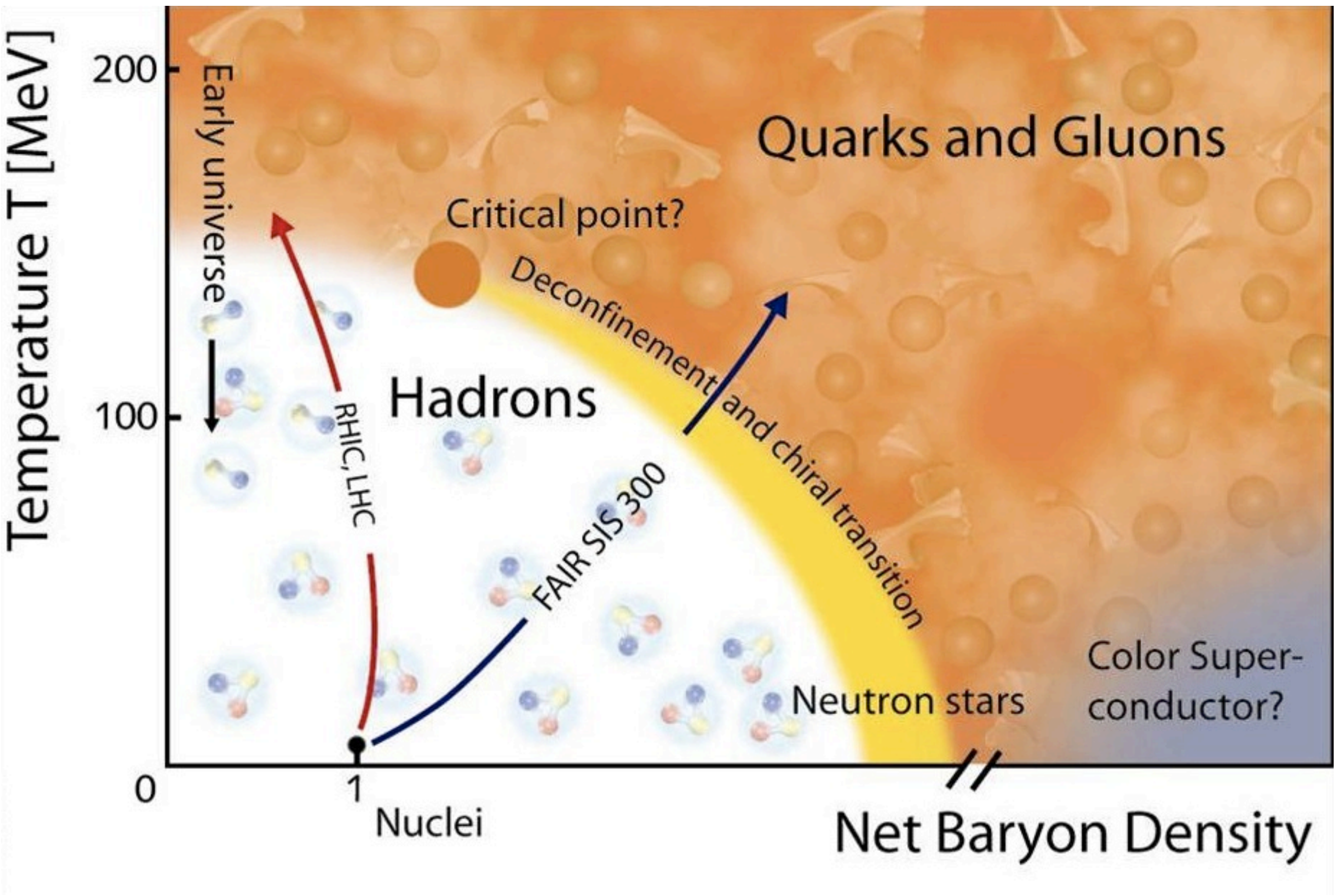


Monte-Carlo simulations, e. g. on graphic cards



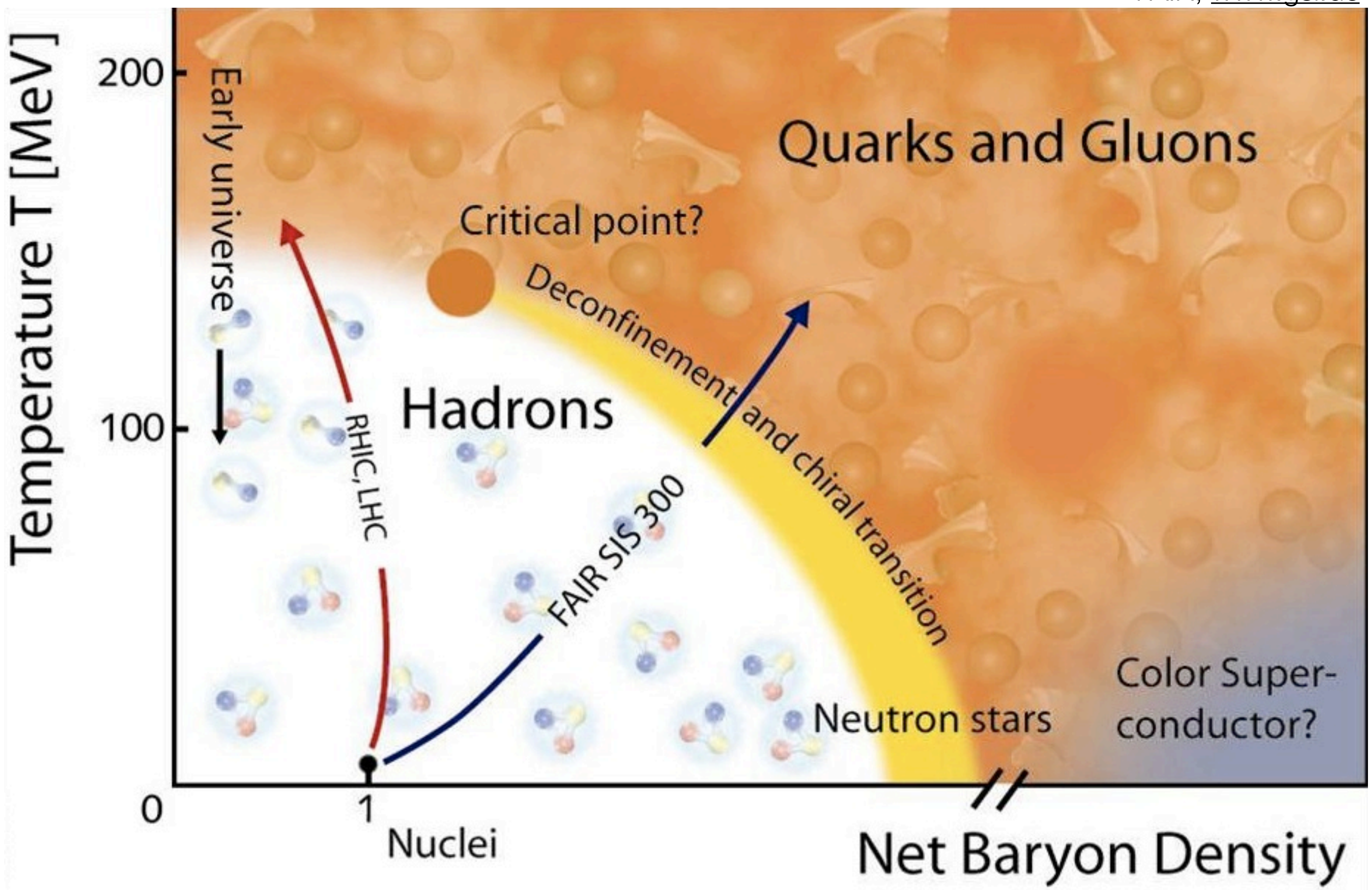
Lattice QCD: finite volume, finite quark masses

QCD Phase Diagram



QCD phase diagram

FAIR, www.gsi.de



(naive) perturbation theory fails:

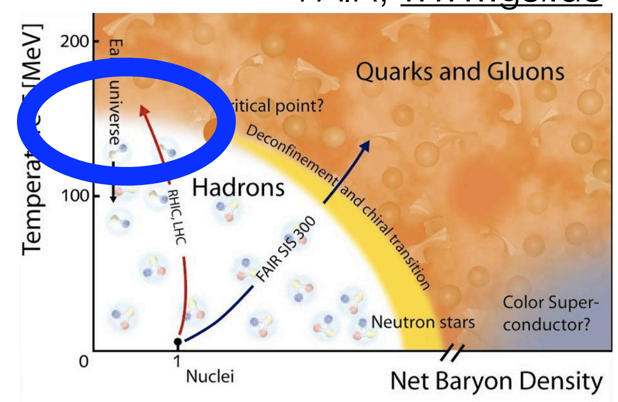
- not convergent even for very high temperatures: strongly interacting QGP
- phase transitions: long-range fluctuations are important

A simple question, many answers ...
QCD phase boundary at small chemical potentials

(JB, EPJC '09)

$$\frac{T_c(\mu_q)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_q}{\pi T_c(0)} \right)^2 + \ldots$$

FAIR, www.gsi.de



- large N_c expansion: $\kappa \sim \frac{N_f}{N_c}$ (D. Toublan '05, JB '09)
- results from different approaches:

Method	$N_f = 1$	$N_f = 2$	$N_f = 3$	
RG: QCD flow	0.43(3)	---	---	(JB, EPJC '09)
Lattice: imag. μ	---	0.500(54)	0.667(6)	(de Forcrand et al. '02, '06)
Lattice: Taylor+Rew.	---	---	1.13(45)	(Karsch et al. '03)

latest results for 2+1 flavor QCD: $\kappa \approx 0.58(2)$ (hotQCD, Kaczmarek et al. '11)
$\kappa \approx 0.79(3)$ (Endrödi et al. '11)

- no parameters, relies solely on physical coupling: $\alpha_s(M_Z)$

Chiral Phase Diagram in a Finite Box with $V \sim L^3$

- **idea:** use **chiral low-energy QCD model** to study effects of a finite system size on the QCD phase structure
- **method:** renormalization group approach, allows to capture important long-range fluctuations

approach gives access to many **phenomenologically** important questions
in infinite volume
(see e. g. Schaefer, **Pirner** '99; Berges, Jungnickel, Wetterich '99; Berges, Tetradis Wetterich '00; Meyer, Schwenzer, **Pirner** '01; Spitzenberg, Schwenzer, **Pirner** '02; JB, Schwenzer, **Pirner** '04)

and **finite volumes**
(JB, Klein, **Pirner** '05; Klein, JB, **Pirner** '05; JB, Klein, **Pirner** '05;JB, Klein, **Pirner** '06; **Pirner**, Klein, JB '06;Klein, JB '07; JB, Klein '08; JB, Klein '09; JB, Klein, Piasecki '10; Klein, JB, Schaefer '10; **cf. talk of B. Klein**)

Chiral Phase Diagram in a Finite Box with $V \sim L^3$

$$\frac{T_c(\mu_q)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_q}{\pi T_c(0)} \right)^2 + \ldots$$

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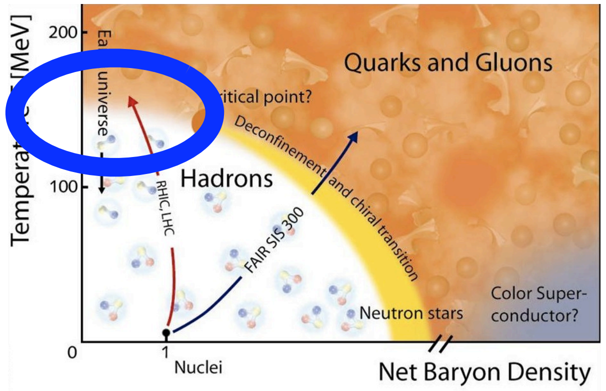
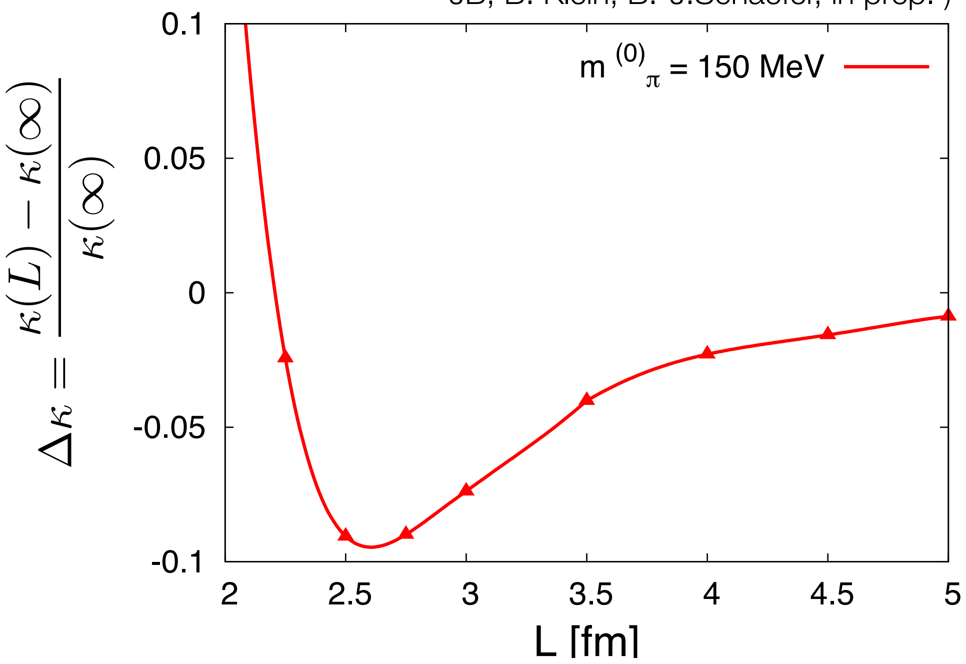
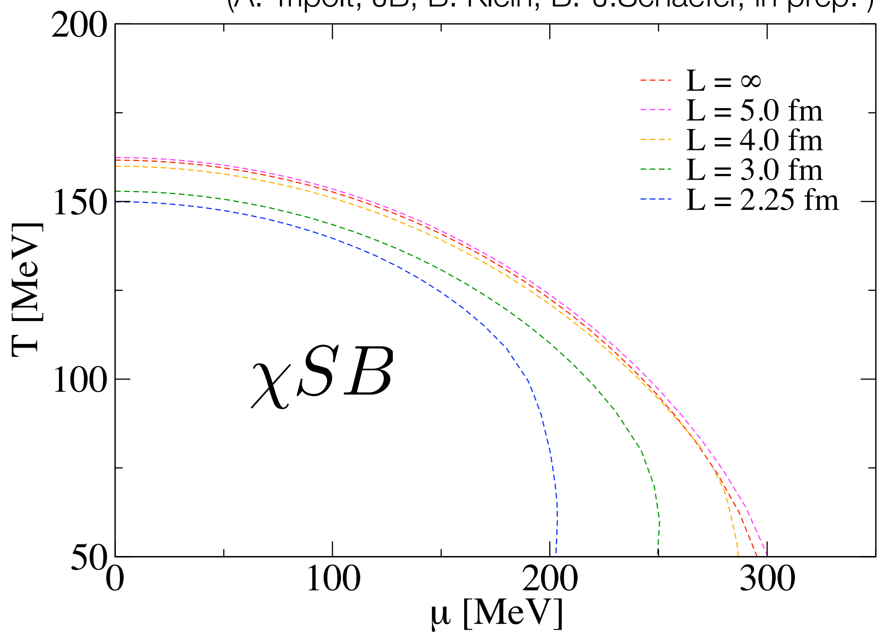
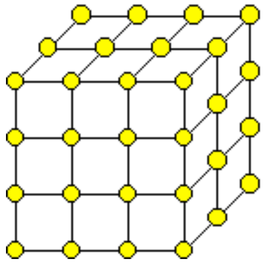


(B. Klein, JB, B.-J. Schaefer, '10; JB, B. Klein, B.-J.Schaefer, in prep.)



(A. Tripolt, JB, B. Klein, B.-J.Schaefer, in prep.)





$V \sim L^3$

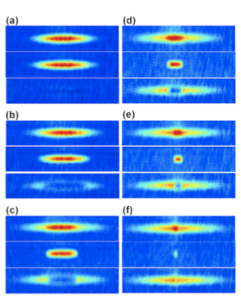
- condensate $|\langle\bar{\psi}\psi\rangle|^{1/3}$ vanishes for small volumes, cf. behavior at high temperatures
- condensate is related to the quark mass: $m_q \sim |\langle\bar{\psi}\psi\rangle|^{1/3}$
- use scaling behavior of the curvature κ to relate different lattice results?

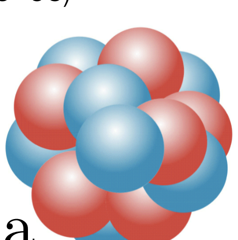
Testing finite-size effects in table-top **experiments**

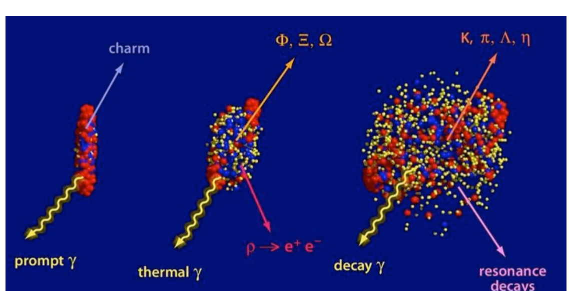
effects of a
finite system-size

phenomenologically
relevant:

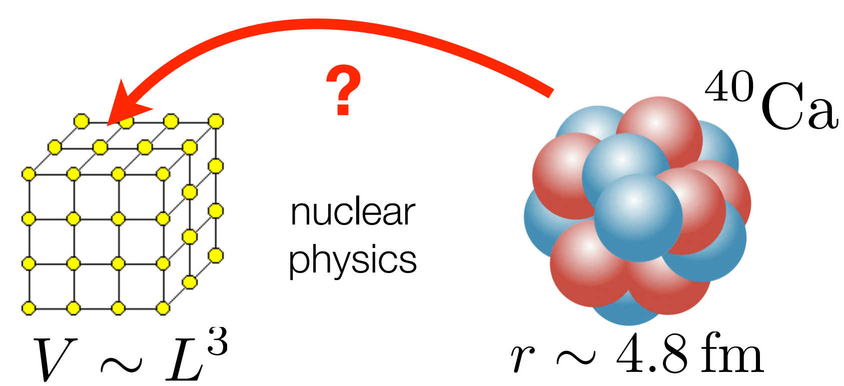
- QCD phase diagram & HI-collision
- **experiments with ultracold atoms**
- nuclear physics
- ...


(Partridge '06)

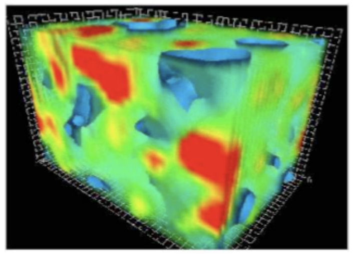

$^{40}\mathrm{Ca}$



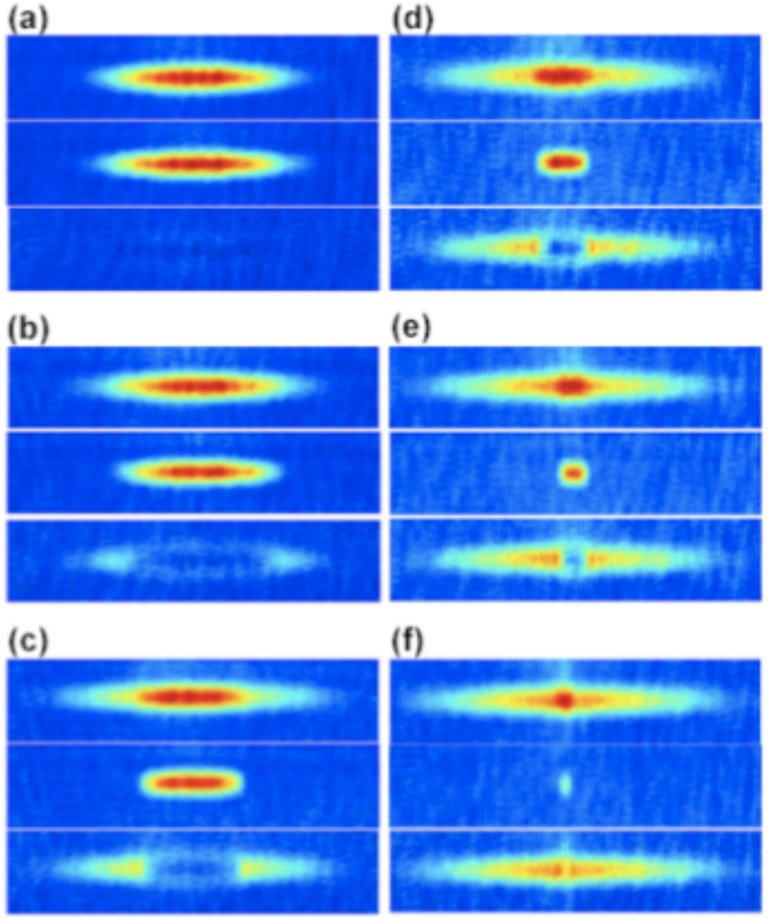
field-theoretical
relevance:
Monte-Carlo simulations




Monte-Carlo simulations, e. g. on graphic cards


Lattice QCD:
finite volume,
finite quark masses

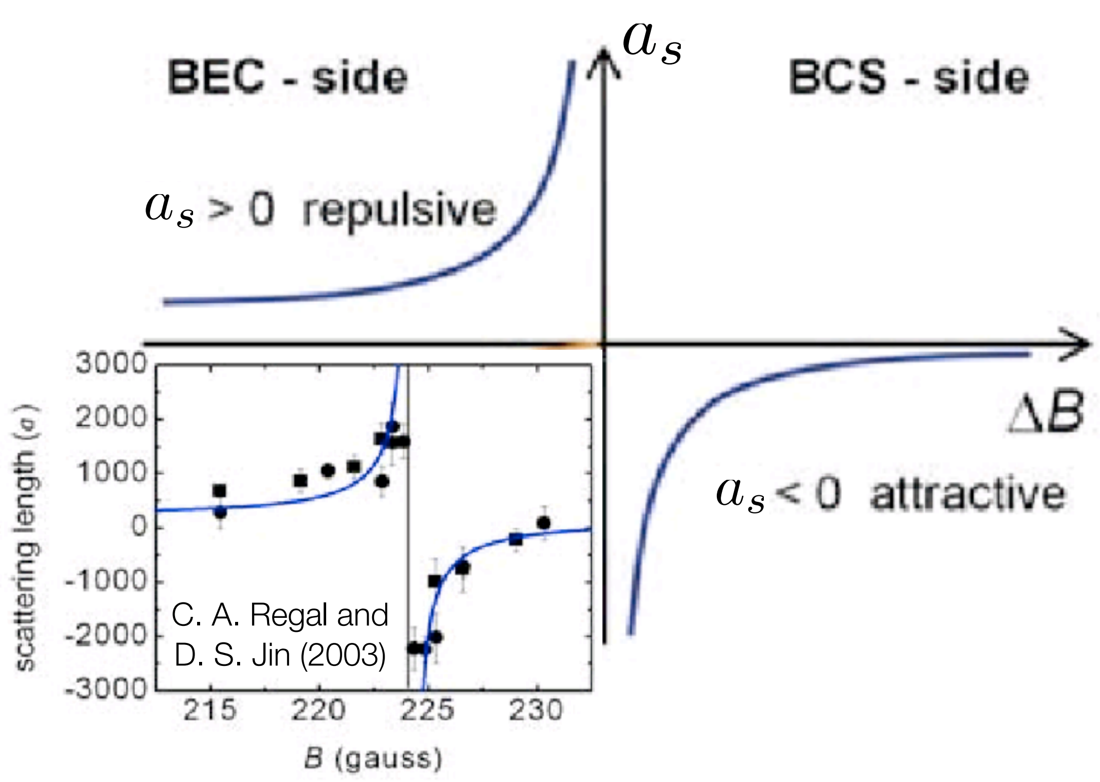
Ultracold atoms and finite-size effects



(Partridge '06)

Unitary Regime

- s-wave scattering length is tunable by an ext. magnetic field (Feshbach resonance)
- interaction strength is proportional to s-wave scattering length a_s



Unitary Regime

- s-wave scattering length is tunable by an ext. magnetic field (Feshbach resonance)
- interaction strength is proportional to s-wave scattering length a_s
- limit of infinite scattering length a_s defines a universal regime:

$$0 \approx \frac{1}{|a_s|} \ll k_\mathrm{F} \sim \frac{1}{r} \ll \frac{1}{R} \approx \infty$$

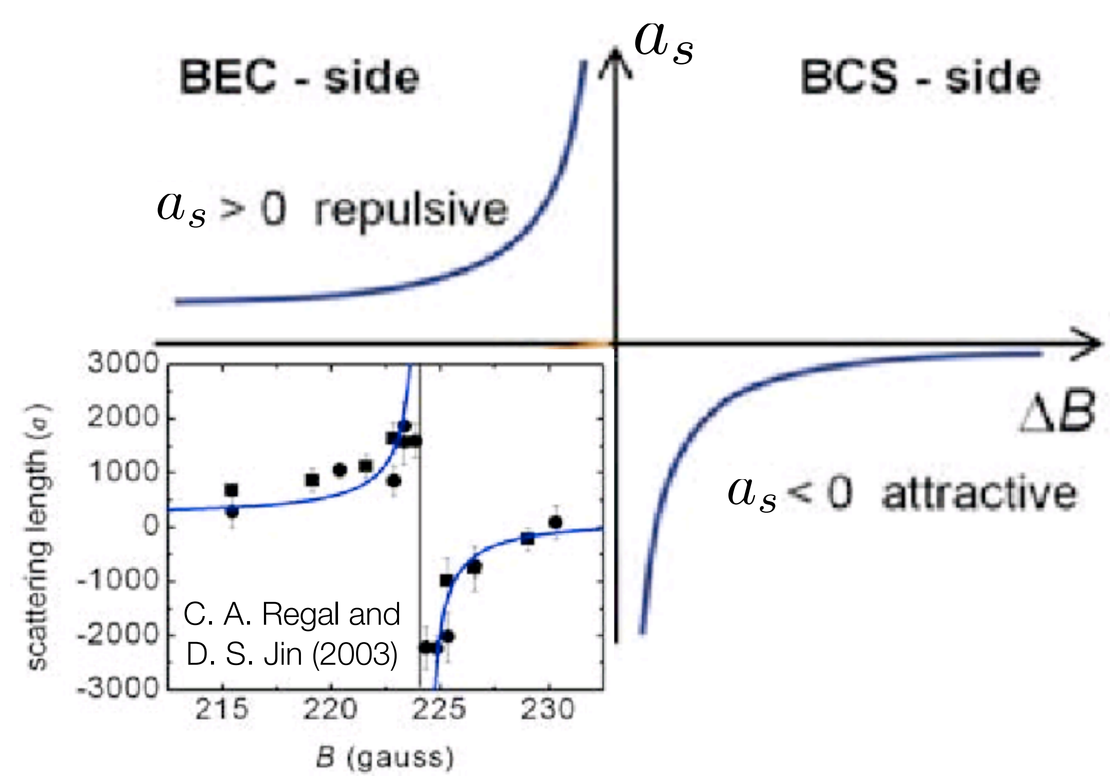
density (~Fermi momentum) is the only scale (unitarity limit)

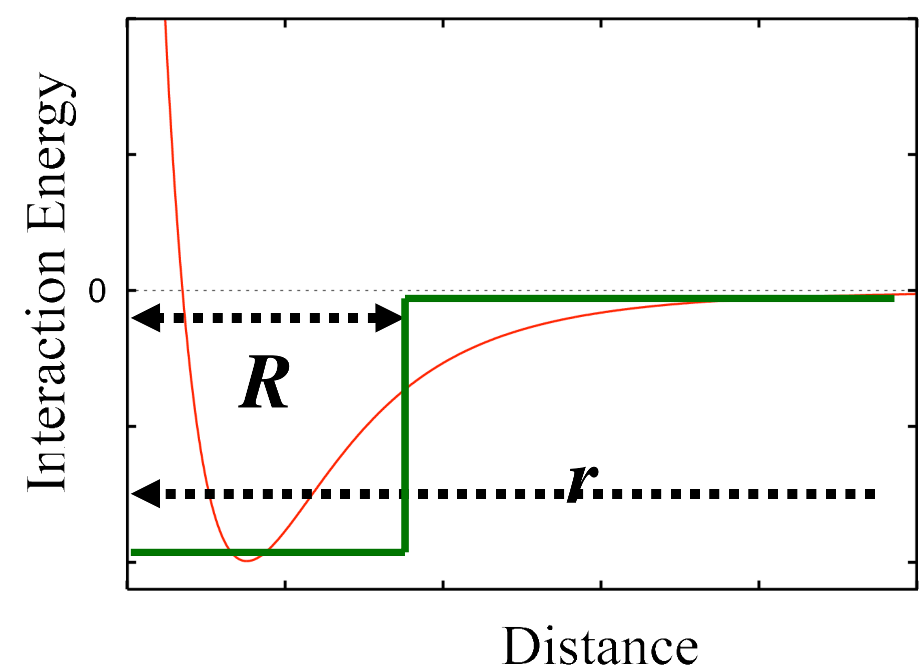
- Universal properties:

$$E/N, T_c, \cdots \propto \text{universal const(s).} \times k_\mathrm{F}$$

- Example: dilute neutron matter

$$|a_\mathrm{nn}| \sim 18.5\mathrm{fm} \gg R \sim 1.4\mathrm{fm}$$





Symmetric Fermi Gases

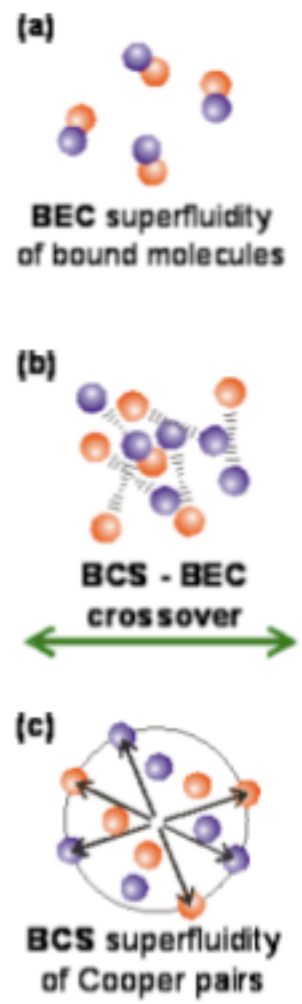
- **Experiment**: Fermions in different hyperfine states
- provides an experimentally accessible environment for a study of quantum phenomena:

(a) BEC regime: tightly bound molecule $(a_s > 0)$

(b) Unitary regime: crossover - delocalized molecule with $E_B = 0$

(c) BCS regime: delocalized Cooper pairs $(a_s < 0)$

- **symmetric regime at T=0:** smooth crossover, superfluidity persists
- **symmetric regime at finite T:** phase transition, "melting condensate"



$a_s \to \infty$

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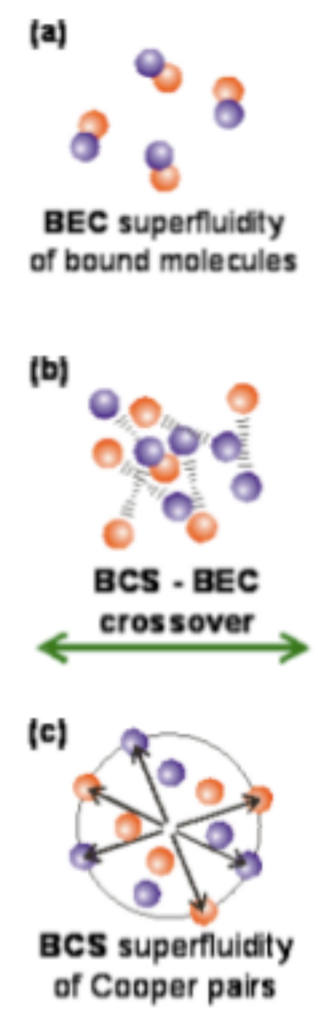


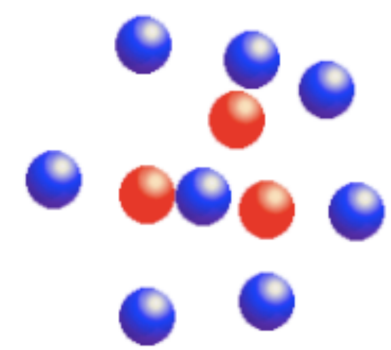
$a_s \to \infty$

- **symmetric regime at T=0:** smooth crossover, superfluidity persists
- **asymmetric systems?**

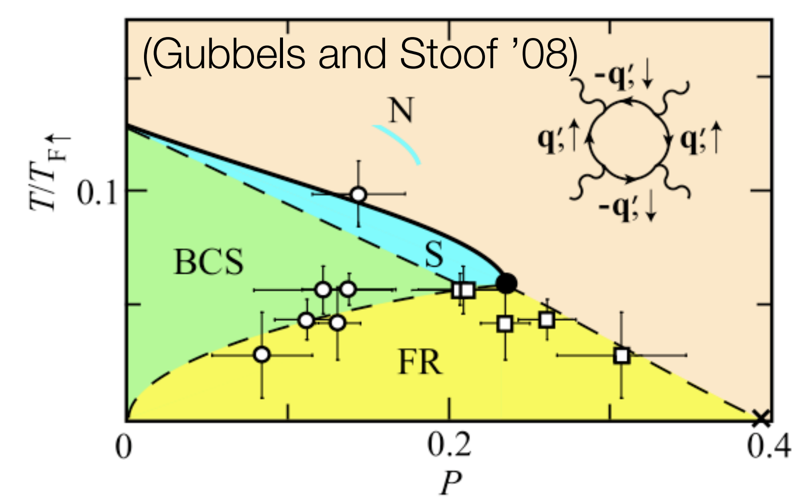
$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} > 0$$

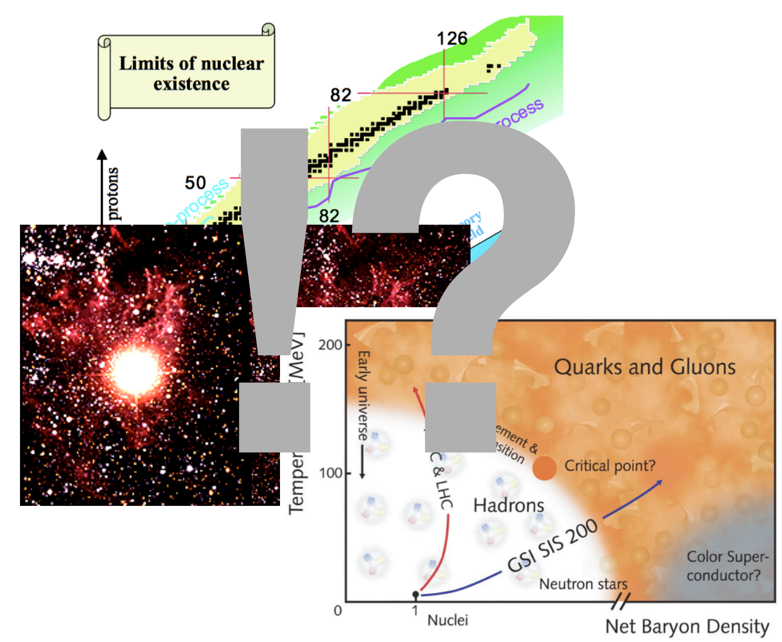
Asymmetric Fermi Gases

- Spin-polarized Fermi gases, e. g. $N_\uparrow > N_\downarrow$
  - Majority fermions $N_\uparrow$, minority fermions $N_\downarrow$
  - Polarization $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$

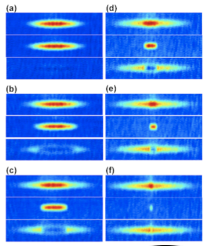


- **What happens when we have a population imbalance?**
- Relevance for various research fields, e. g.: **Clogston limit** in superconductivity, nuclear physics, astrophysics, QCD at finite T(?), ...
- Experiments with spin-polarized Fermi gases are very useful to explore asymmetric strongly-interacting Fermi systems

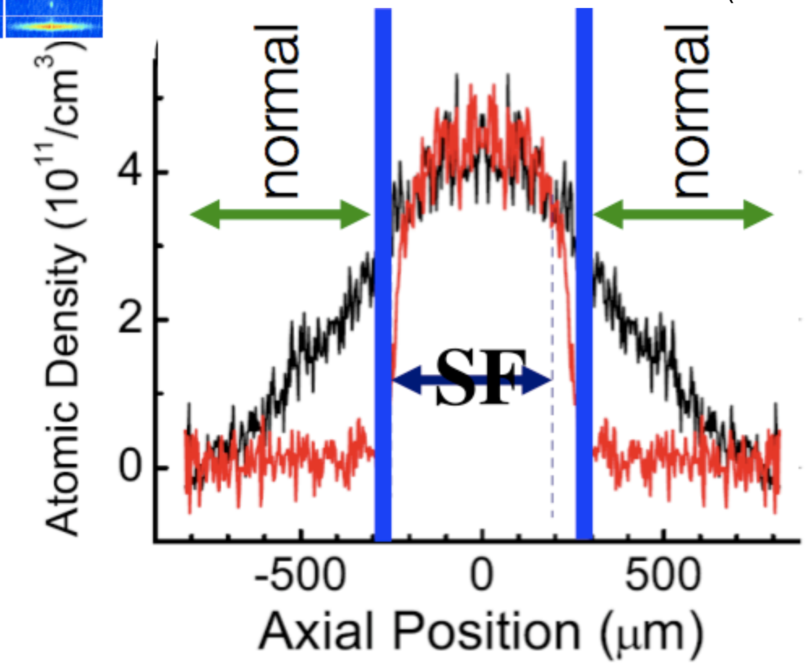




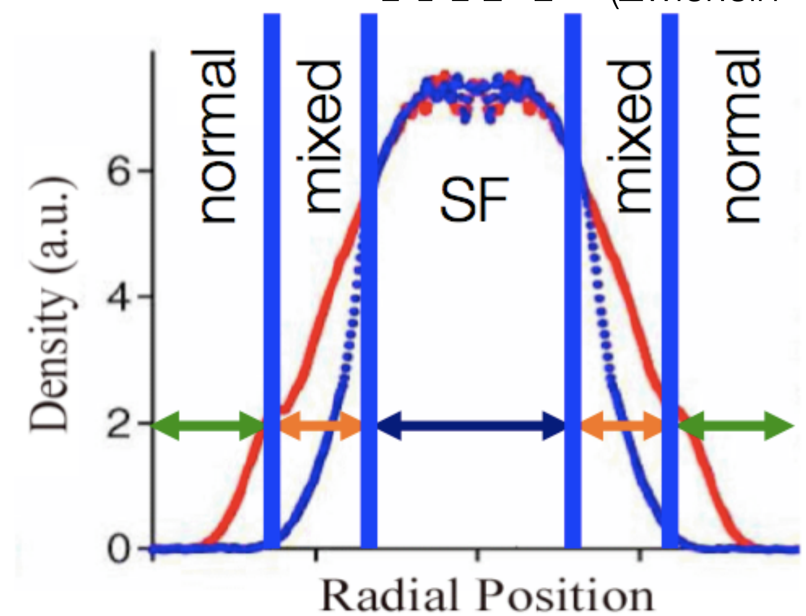
Experimental status with $^6\mathrm{Li}$:
Imbalanced spin-polarized trapped atoms



Rice U. (Partridge '06)



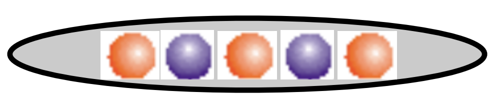
MIT (Zwierlein '06)



~ isotropic trap

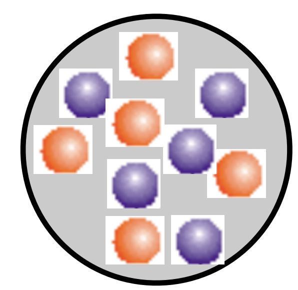
cigar-shaped trap

$N \lesssim 10^5$



Polarization

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$



$N \sim 10^6 \dots 10^7$

no superfluid core for $P_c \gtrsim 0.9$

no superfluid core above a critical polarization $P_c \sim 0.75$

(cf. **Chandrasekhar-Clogston limit** in metal superconductors '62)

Study of trap effects

(Ku, JB, Schwenk, Phys. Rev. Lett. '09)

- energy density functional ($N_\uparrow \gg N_\downarrow$):

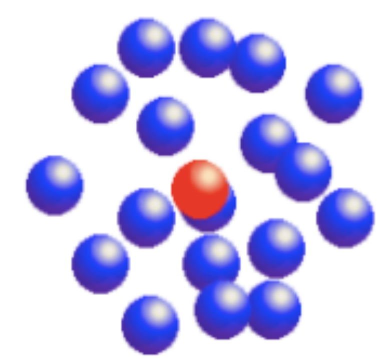
Density Functional Theory: Walter Kohn, Nobel prize '98

$$E[n_S, n_\uparrow, n_\downarrow] \;=\; 2\int_{|\mathbf{r}|<R_S} d\mathbf{r}\left\{\xi_S \frac{3}{5}\frac{(6\pi^2 n_S(\mathbf{r}))^{\frac{2}{3}}}{2m} + U(\mathbf{r}) - \mu_S\right\} n_S(\mathbf{r})$$

$$+\int_{R_S<|\mathbf{r}|<R_\uparrow} d\mathbf{r}\left\{\frac{3}{5}\frac{(6\pi^2 n_\uparrow)^{\frac{2}{3}}}{2m}\left(1-\frac{5}{3}\eta\left(\frac{n_\downarrow}{n_\uparrow}\right)+\frac{m}{m^*}\left(\frac{n_\downarrow}{n_\uparrow}\right)^{5/3}+B\left(\frac{n_\downarrow}{n_\uparrow}\right)^2\right)n_\uparrow(\mathbf{r})\right.$$

$$\left.+U(\mathbf{r})(n_\downarrow(\mathbf{r})+n_\uparrow(\mathbf{r}))-\mu_\uparrow n_\uparrow(\mathbf{r})-\mu_\downarrow n_\downarrow(\mathbf{r})\right\}$$

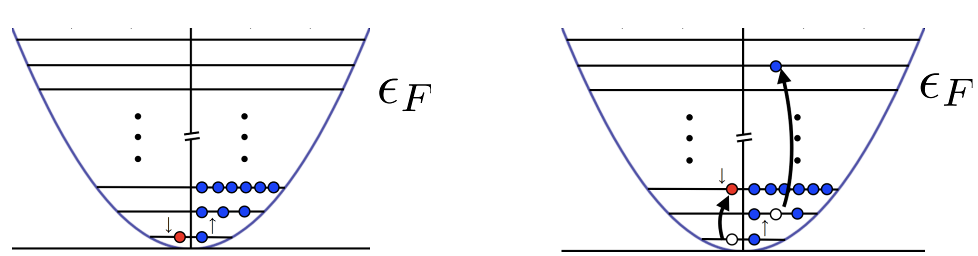
- self-consistent equation for energy gain η (N+1-body problem):



$$E_\downarrow - \varepsilon_{\mathbf{0}} = \sum_{\varepsilon_{\mathbf{h}}\leqslant\varepsilon_F}\sum_{\mathbf{S},\mathbf{L}} F(\mathbf{0},\mathbf{h},\mathbf{S})\left[M^{-1}(\varepsilon_F, E_\downarrow+\varepsilon_{\mathbf{h}})\right]_{\mathbf{S},\mathbf{L}} F(\mathbf{0},\mathbf{h},\mathbf{L}),$$

- choose: $E_\downarrow = \eta(\alpha, N) E_F(\alpha, N)$ with $E_F(\alpha,N) = \dfrac{(6\pi^2 n_\uparrow(0))^{\frac{2}{3}}}{2m}$

$$|\psi\rangle \;=\; \phi_0|\Omega\rangle \;+\; \sum \phi_{m,h,p}|m,h,p\rangle$$

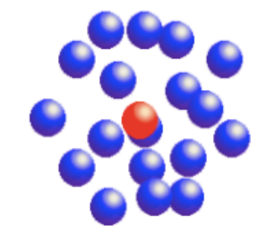


(one-particle-one-hole fluctuations)

Imbalanced spin-polarized atoms

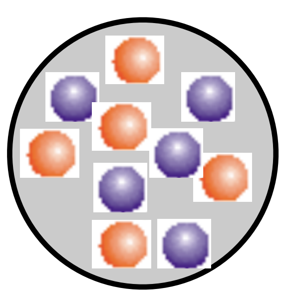
(Ku, JB, Schwenk, Phys. Rev. Lett. ’09)

- First microscopic study of trapped asymmetric systems based on a variational approach and a **controlled** expansion about a non-interacting single-species Fermi gas

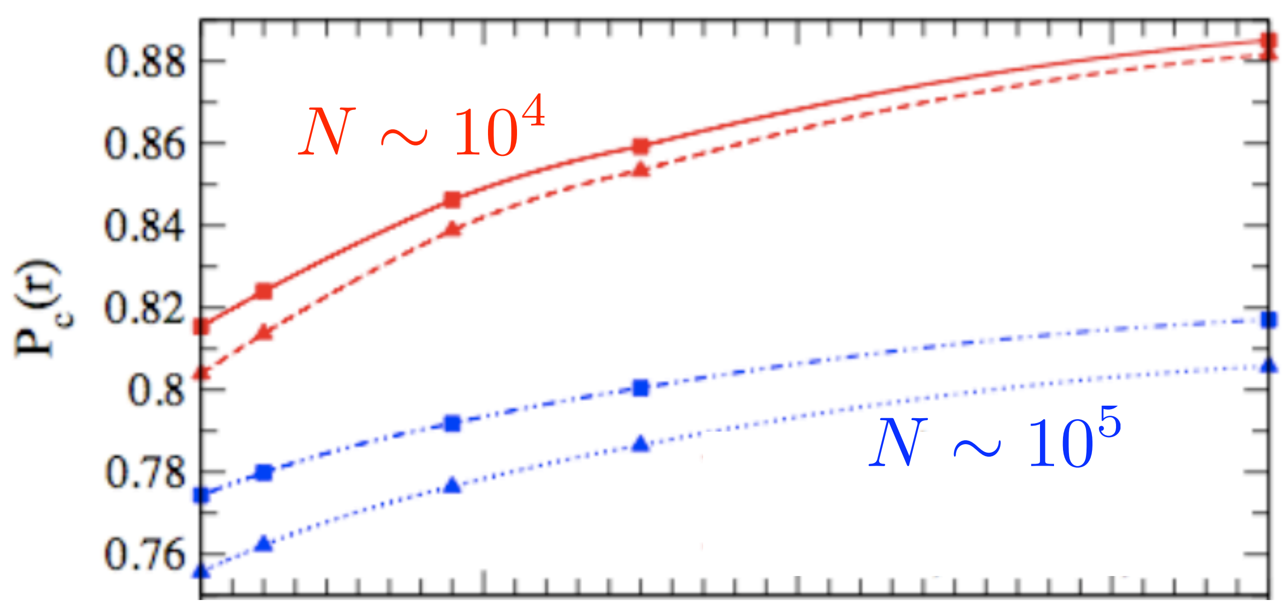


- How does the critical polarization P_c depend on the trap configuration?

isotropic trap

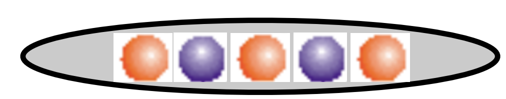


MIT



trap parameter (elongicity of the trap)

cigar-shaped trap



Rice U.

⇒ clear indications for **strong trap dependence** which helps to understand the different findings at MIT and Rice U.

- finite-temperature effects: interesting but open question ...

Another geometry: Towards a connection of continuum and Monte-Carlo studies

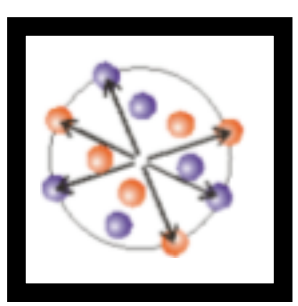
employ the **same techniques as in QCD**

(JB, Klein, **Pirner** ’05; Klein, JB, **Pirner** ’05; JB, Klein, **Pirner** ’05;
JB, Klein, **Pirner** ’06; **Pirner**, Klein, JB ’06;
Klein, JB ’07; JB, Klein ’08; JB, Klein ’09; JB, Klein, Piasecki ’10; Klein, JB, Schaefer ‘10)

to study superfluidity in **ultracold systems of fermionic atoms** at unitarity with the aid of
renormalization group techniques

$$\mathcal{L}_{\rm eff} \sim \psi^\dagger \epsilon_{\mathbf{p},t}\, \psi + \lambda \left(\psi^\dagger \mathcal{O} \psi\right)^2$$

helps to guide Monte-Carlo simulations
(see e. g. Carlson ’03, Bulgac et al. ’06, Lee et al. ’10)

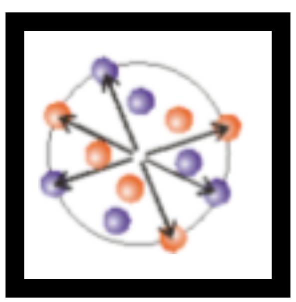


box $V \sim L^3$

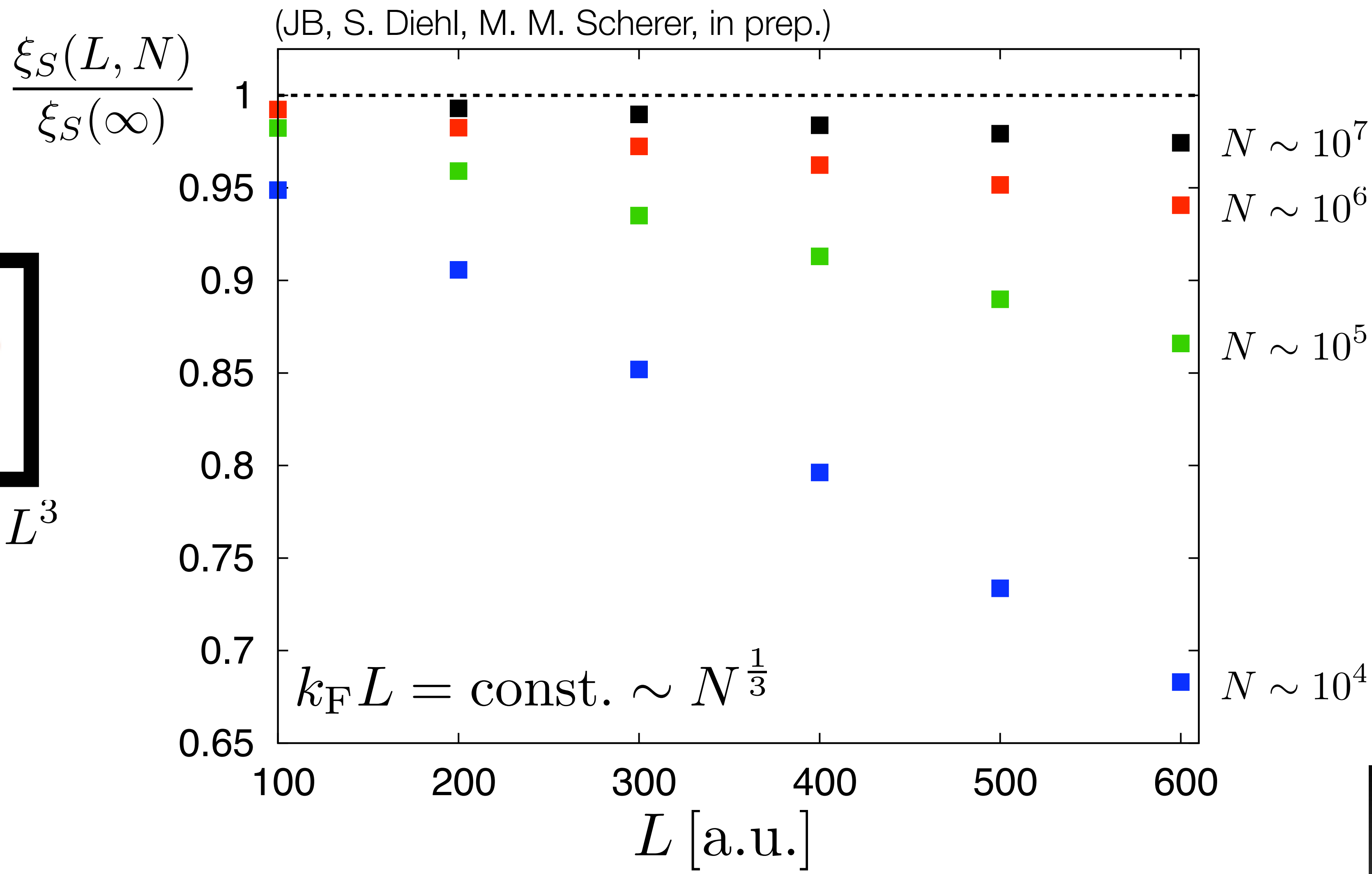
Monte-Carlo simulations,
e. g. on graphic cards

Cold atoms in a box: Towards a connection of continuum and Monte-Carlo studies

- **RG study** of superfluid cold atoms **at unitarity**: $E/N = \xi_\mathrm{S}(L,N) E_\mathrm{F} \to \xi_\mathrm{S} \frac{3}{5} \frac{(6\pi^2 n_\mathrm{S})^{\frac{2}{3}}}{2m}$



box $V \sim L^3$



- helps to guide Monte-Carlo simulations
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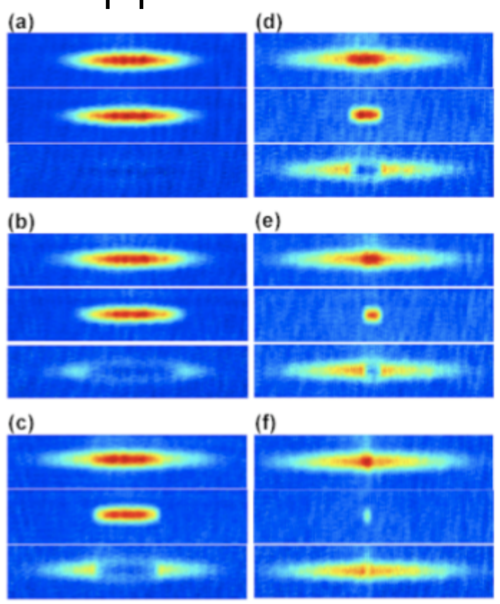
Monte-Carlo simulations, e. g. on graphic cards

Summary

ultracold
trapped fermions



(Partridge '06)

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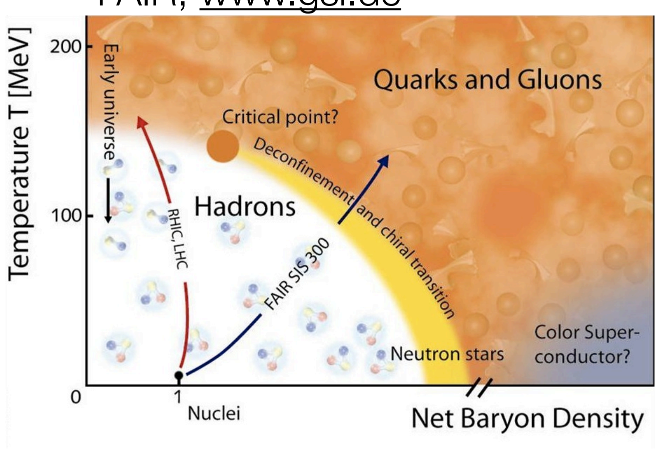
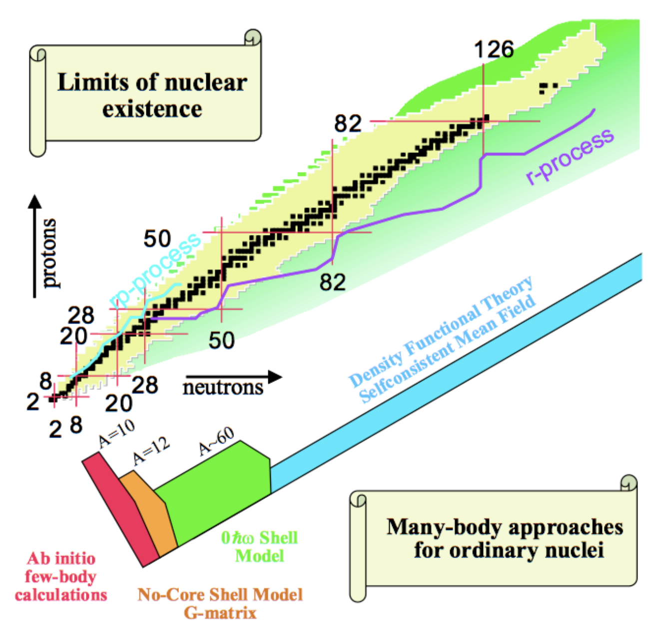


- phenomenological and technical "exchange" between different fields offers great potential
- **effects of a finite system-size** play an important role in many different physical systems: Cold atoms in a trap, heavy-ion collisions, nuclear physics, ...
- we have just started to understand the effects of a finite system-size ... **there is still more to come!**



Summary

ultracold
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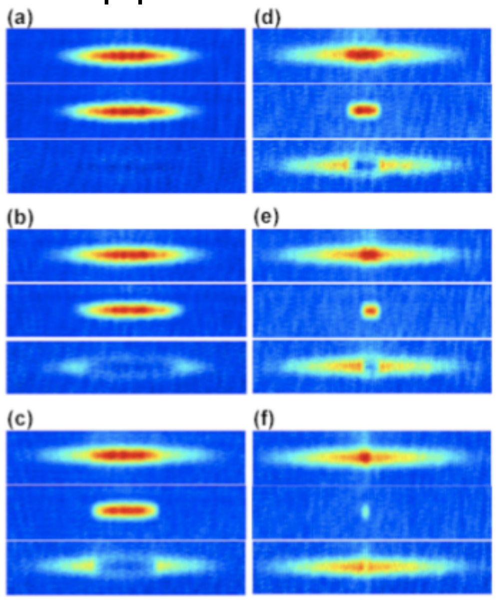

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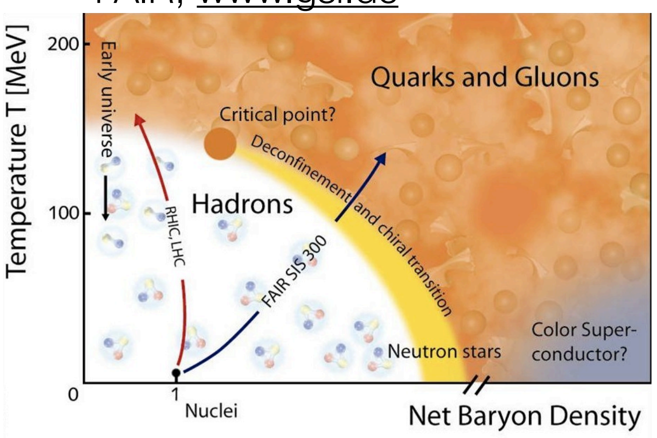

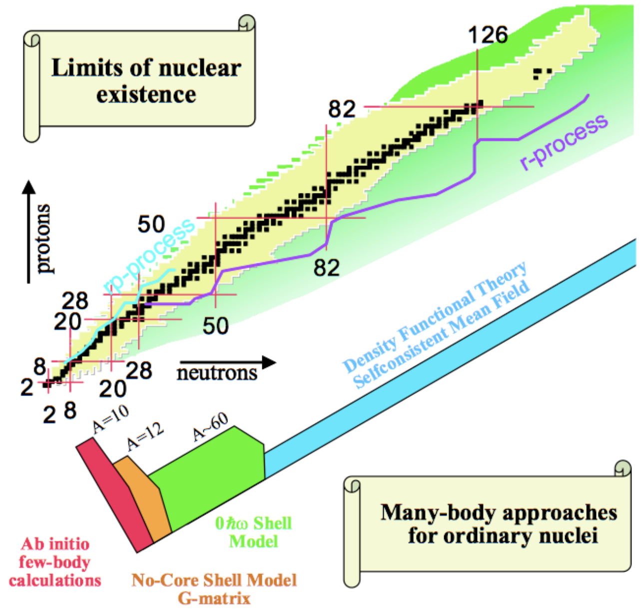

To Hans-Jürgen:

Thank you very much!