High Energy Scattering
in the Lab and in the Early Universe

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30 years of strong interactions
in honor of J. Cugnon and H.-J. Pirner
Spa, April 6th, 2011

Contents today

photons

$\Omega_\gamma = 0.005\ \%$

baryons

$\Omega_B = 4\ \%$

? baryon asymmetry ?

neutrinos

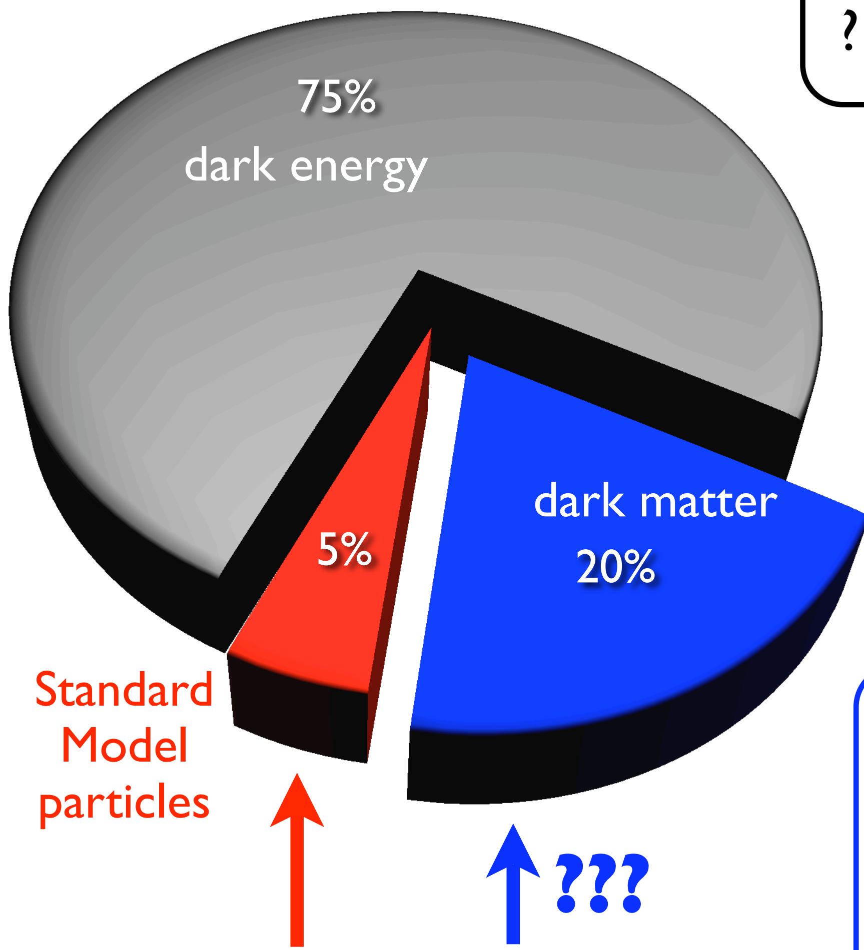
$0.1\ \% \leq \Omega_\nu \leq 1.5\ \%$

? neutrino mass ?

dark energy

$\Omega_{DE} = 75\ \%$

? vacuum energy ?



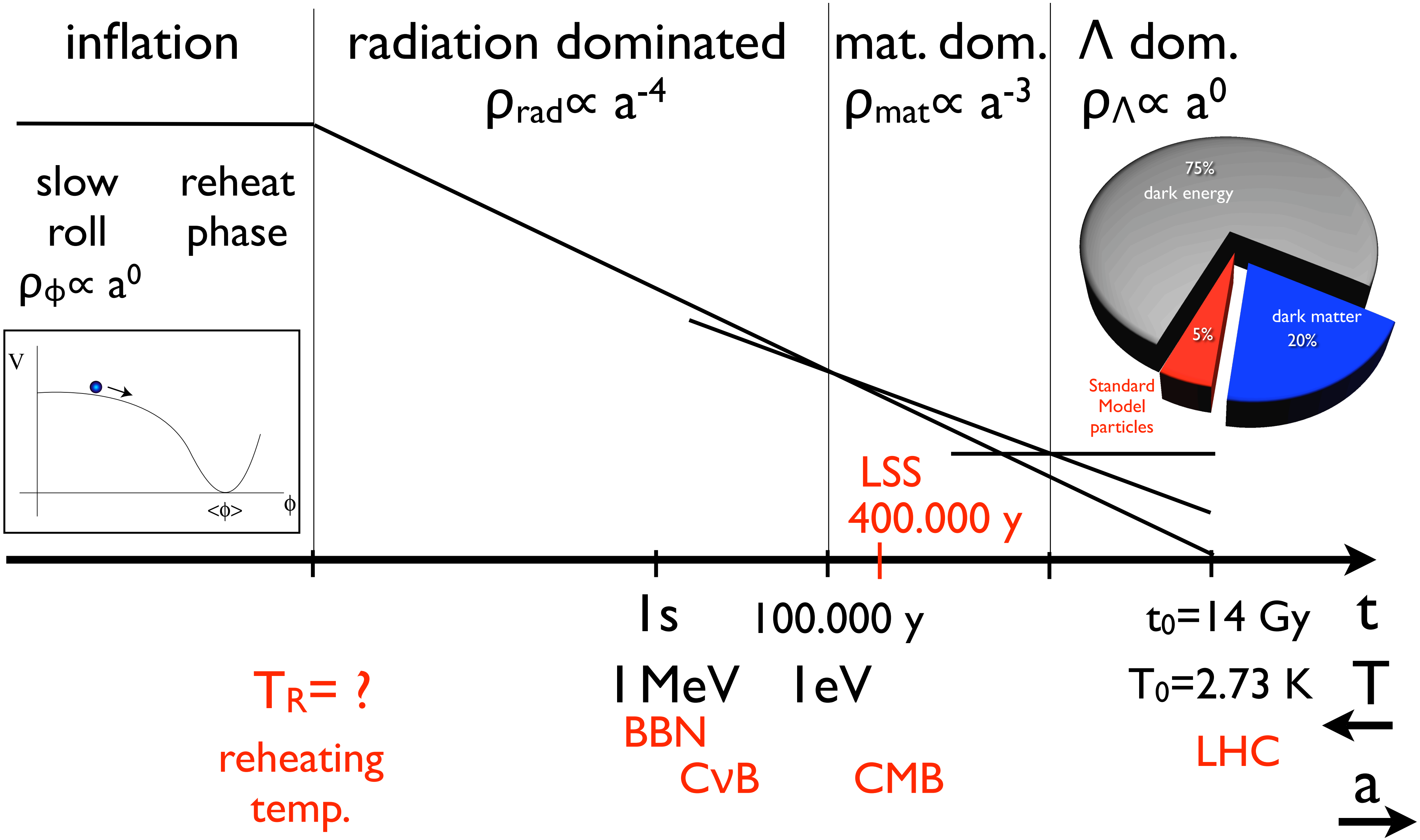
dark matter

$\Omega_{DM} = 20\ \%$

? identity ?

14 billion years of strong interactions

Standard Thermal History of the Universe

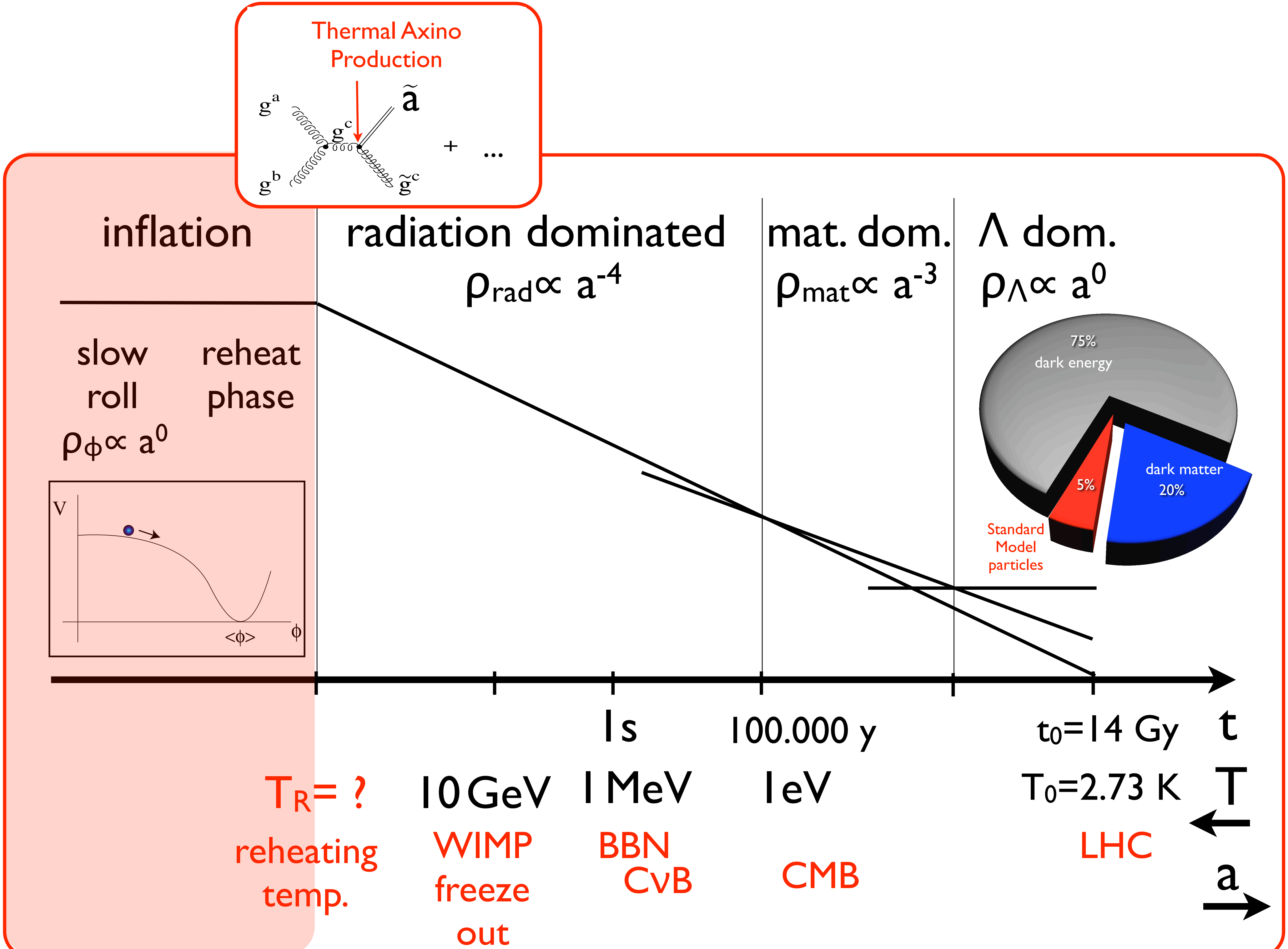


Extremely Weakly Interacting Particles (EWIPs)

Extensions of the Standard Model

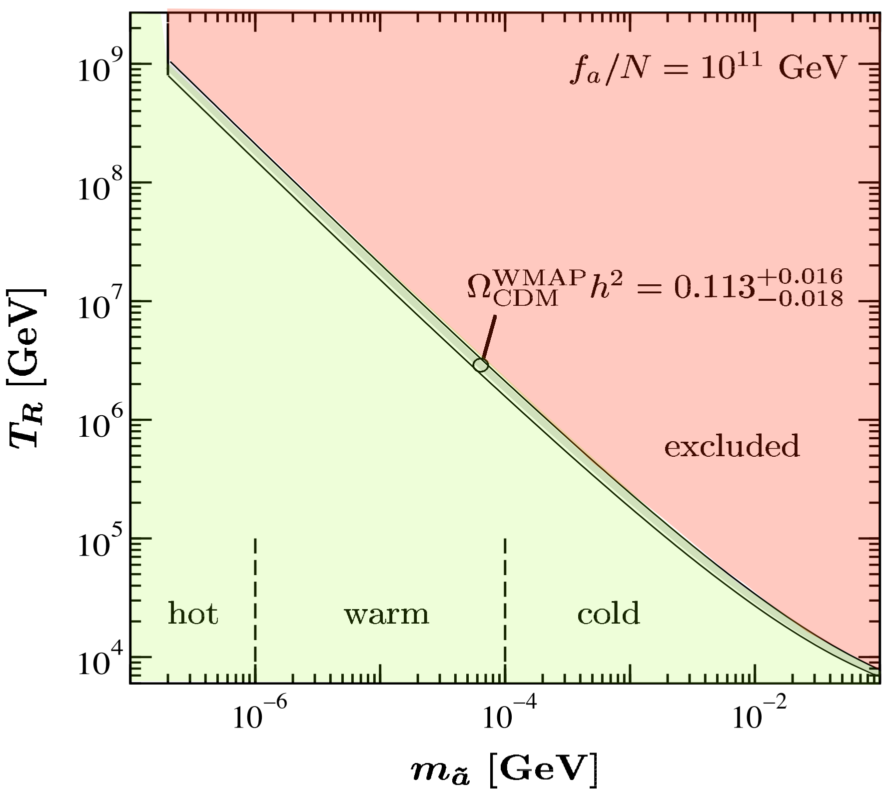
Peccei-Quinn Symmetry & Supersymmetry

	Axions $f_a > 10^9$ GeV	Axinos $f_a > 10^9$ GeV	Gravitinos $M_{Pl} = 2.4 \times 10^{18}$ GeV
spin	0	1/2	3/2
mass	< 10 meV	?	eV-TeV
int.	$\propto (p/f_a)^n$	$\propto (p/f_a)^n$	$\propto (p/M_{Pl})^n$


Thermal Axino Production
g^a
g^b
g^c
ã
g̃^c
+ ...
inflation
radiation dominated
ρ_rad ∝ a^-4
mat. dom.
ρ_mat ∝ a^-3
Λ dom.
ρ_Λ ∝ a^0
slow roll
ρ_φ ∝ a^0
reheat phase
V
φ
<φ>
75%
dark energy
dark matter
20%
5%
Standard Model particles
1 s
100.000 y
t_0=14 Gy
t
T_R= ?
10 GeV
1 MeV
1 eV
T_0=2.73 K
T
reheating temp.
WIMP freeze out
BBN
CνB
CMB
LHC
a

Axino LSP Case

Thermal $\tilde{a}$ Production



[Brandenburg, FDS, '04]

see also [Covi et al., '01]

and [Strumia, '10]

Well-motivated DM Candidates

candidate	identity	mass	interactions	production	constraints	experiments
a	axion	< 0.01 eV	$(p/f_a)^n$	misalign. mech.	$\leftarrow$ cold	direct searches with
	(spin 0)		extremely weak			microwave cavities
	N.-Goldst. boson		$f_a \gtrsim 6 \times 10^8$ GeV			$\hookrightarrow m_a,\ f_a,\ g_{a\gamma\gamma}$
	PQ symm. break.				CMB	
$\widetilde{\chi}^0_1$ LSP	lightest neutralino	$\mathcal{O}(100\,\mathrm{GeV})$	g, g', y_i	therm. relic	$\leftarrow$ cold	indirect searches
	(spin 1/2)		weak	$\widetilde{G}$ decay	$\leftarrow$ warm/hot	direct searches
	mixture of		$M_\mathrm{W} \sim 100$ GeV			collider searches
	$\widetilde{B}, \widetilde{W}, \widetilde{H}^0_u, \widetilde{H}^0_d$				BBN	$\hookrightarrow m_{\widetilde{\chi}^0_1},\ \widetilde{\chi}^0_1$ coupl.
$\widetilde{G}$ LSP	gravitino	eV–TeV	$(p/\mathrm{M_P})^n$	therm. prod.	$\leftarrow$ cold	$\widetilde{\tau}_1$ prod. at colliders
	(spin 3/2)		extremely weak	NLSP decay	$\leftarrow$ warm	$+\ \widetilde{\tau}_1$ collection
	superpartner		$\mathrm{M_P} = 2.4 \times 10^{18}$ GeV			$+\ \widetilde{\tau}_1$ decay analysis
	of the graviton				BBN	$\hookrightarrow m_{\widetilde{G}},\ \mathrm{M_P}$ (?), T_R
$\widetilde{a}$ LSP	axino	eV–GeV	$(p/f_a)^n$	therm. prod.	$\leftarrow$ cold/warm	$\widetilde{\tau}_1$ prod. at colliders
	(spin 1/2)		extremely weak	NLSP decay	$\leftarrow$ warm/hot	$+\ \widetilde{\tau}_1$ collection
	superpartner		$f_a \gtrsim 6 \times 10^8$ GeV			$+\ \widetilde{\tau}_1$ decay analysis
	of the axion				BBN	$\hookrightarrow m_{\widetilde{a}}$ (?), f_a, T_R (?)

Well-motivated DM Candidates

candidate	identity	mass	interactions	production	constraints	experiments
a						
$\widetilde{\chi}_1^0$ LSP						
$\widetilde{G}$ LSP						
$\widetilde{a}$ LSP						

For a review (including an extensive list of references),

see

[FDS, *Dark Matter Candidates*, Eur. Phys. J. C59 (2009) 557, arXiv:0811.3347]

in

Well-motivated DM Candidates

candidate	identity	mass	interactions	production	constraints	experiments
a	axion	< 0.01 eV	$(p/f_a)^n$	misalign. mech.	$\leftarrow$ cold	direct searches with
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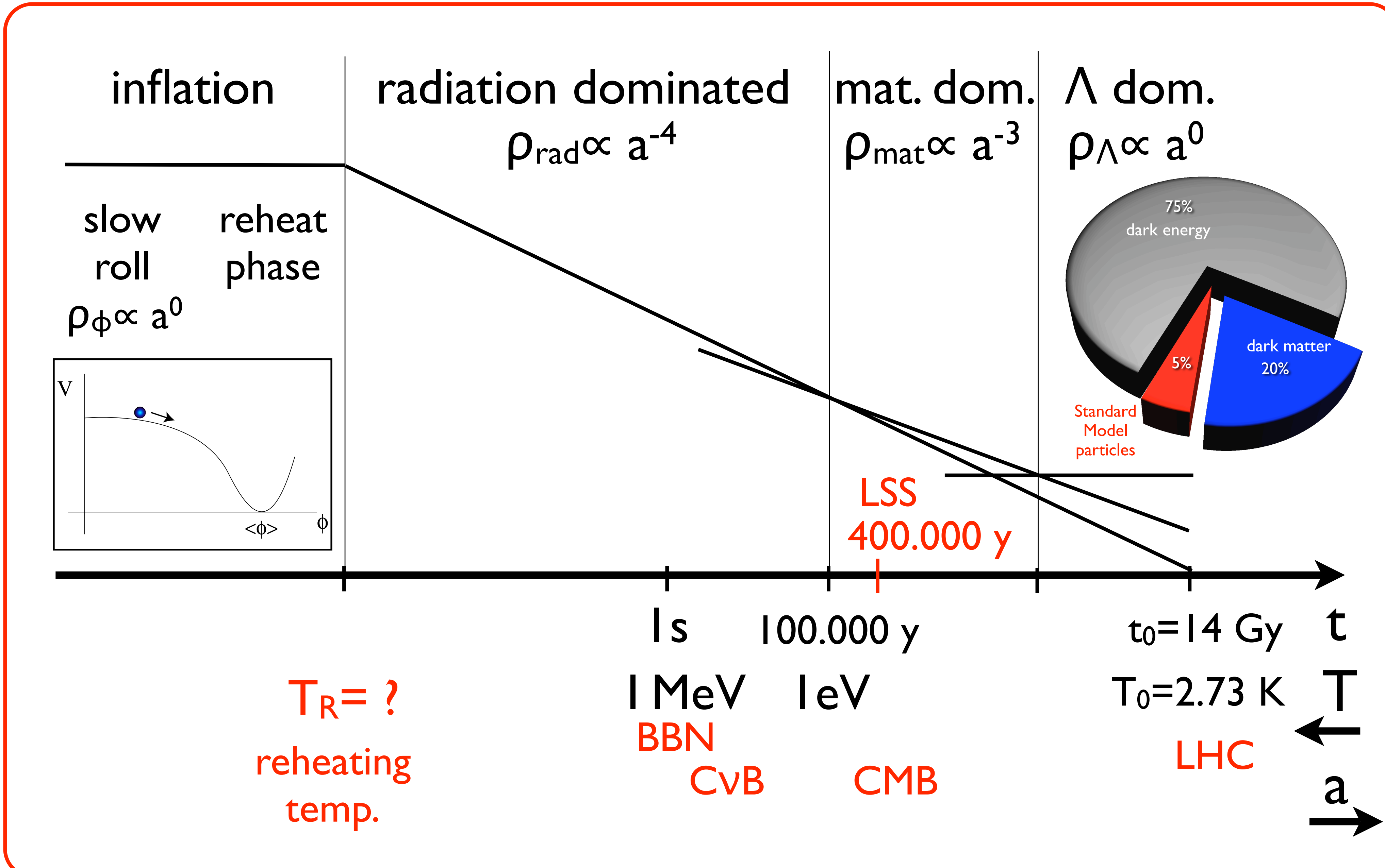
	(spin 1/2)		weak	$\widetilde{G}$ decay	$\leftarrow$ warm/hot	direct searches
	mixture of		$M_{\mathrm{W}} \sim 100$ GeV			collider searches
	$\widetilde{B},\ \widetilde{W},\ \widetilde{H}_u^0,\ \widetilde{H}_d^0$				BBN	$\hookrightarrow m_{\widetilde{\chi}_1^0},\ \widetilde{\chi}_1^0$ coupl.
$\widetilde{G}$ LSP	gravitino	eV–TeV	$(p/\mathrm{M_P})^n$	therm. prod.	$\leftarrow$ cold	$\widetilde{\tau}_1$ prod. at colliders
	(spin 3/2)		extremely weak	NLSP decay	$\leftarrow$ warm	$+\ \widetilde{\tau}_1$ collection
	superpartner		$\mathrm{M_P} = 2.4 \times 10^{18}$ GeV			$+\ \widetilde{\tau}_1$ decay analysis
	of the graviton				BBN	$\hookrightarrow m_{\widetilde{G}},\ \mathrm{M_P}$ (?), T_{R}
$\widetilde{a}$ LSP	axino	eV–GeV	$(p/f_a)^n$	therm. prod.	$\leftarrow$ cold/warm	$\widetilde{\tau}_1$ prod. at colliders
	(spin 1/2)		extremely weak	NLSP decay	$\leftarrow$ warm/hot	$+\ \widetilde{\tau}_1$ collection
	superpartner		$f_a \gtrsim 6 \times 10^8$ GeV			$+\ \widetilde{\tau}_1$ decay analysis
	of the axion				BBN	$\hookrightarrow m_{\widetilde{a}}$ (?), f_a, T_{R} (?)

Standard Thermal History of the Universe



The QCD Vacuum: Puzzles & Tools

? Chiral Symmetry Breaking

? Dynamical Mass Generation

? Color-Confinement

* Static $q\bar{q}$ Potential
* Flux-Tube Formation
* Casimir Scaling

? Hadron Structure

* van der Waals Potential

? High-Energy Scattering

* Hadronic Cross Sections
* Parton Distributions

□ Lattice QCD

* Non-Pert. + Pert. (Cooling)

□ Instantons

* Non-Pert. & Topology

□ QCD Sum Rules & Condensates

* Pert. + Non-Pert.
* Power Corrections & OPE

□ Loop-Loop Correlation Model

* Pert. + Non-Pert.(SVM)
* Gluon Field Strength Correlator
* Lattice QCD Input
* Gaussian Approximation

Unified Description: Static Potentials & High-Energy Scattering

Wegner-Wilson Loops

[Wegner '71, Wilson '74]

$$W_r[C] = \tilde{\mathrm{Tr}}_r\,\mathcal{P}\exp\left[-ig\oint_C dZ_\mu\,\mathcal{G}^a_\mu(Z)\,t^a_r\right]$$

- Ground State Energy of a Static Color-Dipole → Static $q\bar{q}$ Potential

$$E_r(R) = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W_r[C]\rangle$$

- Chromo-Field Distribution of a Color-Dipole → String Formation

$$\Delta G^2_{r\,\alpha\beta}(X) = -\lim_{R_P\to 0}\frac{1}{R_P^4}\frac{N_c}{\pi^2}\left[\frac{\langle W_r[C]P^{\alpha\beta}_{\mathrm{N}_c}(X)\rangle}{\langle W_r[C]\rangle} - \langle P^{\alpha\beta}_{\mathrm{N}_c}(X)\rangle\right]$$

- QCD van der Waals Potential between two Static Color-Dipoles

$$V_{r_1 r_2}(R,\vec{r}_1,\vec{r}_2) = -\lim_{T\to\infty}\frac{1}{T}\ln\frac{\langle W_{r_1}[C_1]W_{r_2}[C_2]\rangle}{\langle W_{r_1}[C_1]\rangle\langle W_{r_2}[C_2]\rangle}$$

- High-Energy Dipole-Dipole Scattering in the Eikonal Approximation

$$S^M_{r_1 r_2}(s,\vec{b}_\perp,z_1,\vec{r}_1,z_2,\vec{r}_2) = \lim_{T\to\infty}\frac{\langle W_{r_1}[C_1]W_{r_2}[C_2]\rangle_M}{\langle W_{r_1}[C_1]\rangle_M\langle W_{r_2}[C_2]\rangle_M}$$

$$\left\langle W_r[C] \right\rangle_E \qquad\qquad \left\langle W_{r_1}[C_1] W_{r_2}[C_2] \right\rangle_E$$

• Non-Abelian Stokes Theorem

$$W_r[C] = \tilde{\mathrm{Tr}}_r\, \mathcal{P}_S \exp\left[-i\,\frac{g}{2}\int_S d\sigma_{\mu\nu}(Z)\mathcal{G}^a_{\mu\nu}(O,Z;C_{ZO})\, t^a_r\right]$$

• Trace Trick [Berger & Nachtmann '99]

$$\tilde{\mathrm{Tr}}_{r_1}(A)\,\tilde{\mathrm{Tr}}_{r_2}(B) = \tilde{\mathrm{Tr}}_{r_1\otimes r_2}(A\otimes B)$$

$$\hat{\mathcal{G}}_{\mu\nu}(\ldots) := \begin{cases} \mathcal{G}^a_{\mu\nu}(\ldots)(t^a \otimes 1\!\!1) & \text{for} \quad X \ \epsilon \ S_1 \\ \mathcal{G}^a_{\mu\nu}(\ldots)(1\!\!1 \otimes t^b) & \text{for} \quad X \ \epsilon \ S_2 \end{cases}$$

• Matrix Cumulant Expansion

$$\left\langle \mathcal{P}\exp\left[-i\frac{g}{2}\int_S d\sigma(X)\hat{\mathcal{G}}(\ldots)\right]\right\rangle_E = \exp\left[\sum_{n=1}^{\infty}\frac{1}{n!}(-i\frac{g}{2})^n\int d\sigma(X_1)\ldots d\sigma(X_n)K_n(X_1,..,X_n)\right]$$

• Gaussian Approximation

$$K_2 = \left\langle \mathcal{P}_S\, \mathcal{G}(O,X_1;C_{X_1})\mathcal{G}(O,X_2;C_{X_2})\right\rangle_E \qquad \hat{K}_2 = \left\langle \mathcal{P}_S\, \hat{\mathcal{G}}(O,X_1;C_{X_1})\hat{\mathcal{G}}(O,X_2;C_{X_2})\right\rangle_E$$

Perturbative and Non-Perturbative QCD Components

- Bilocal Gluon Field Strength Correlator $\qquad Z = X_1 - X_2$

$$\left\langle \frac{g^2}{4\pi^2}\,\mathcal{G}^a_{\mu\nu}(O,X_1;C_{X_1O})\mathcal{G}^b_{\rho\sigma}(O,X_2;C_{X_2O})\right\rangle_E =: \frac{1}{4}\delta^{ab}\left(F^P_{\mu\nu\rho\sigma} + F^{NP_c}_{\mu\nu\rho\sigma} + F^{NP_{nc}}_{\mu\nu\rho\sigma}\right)$$

- Perturbative Gluon Exchange (P) with $m_G \neq 0$

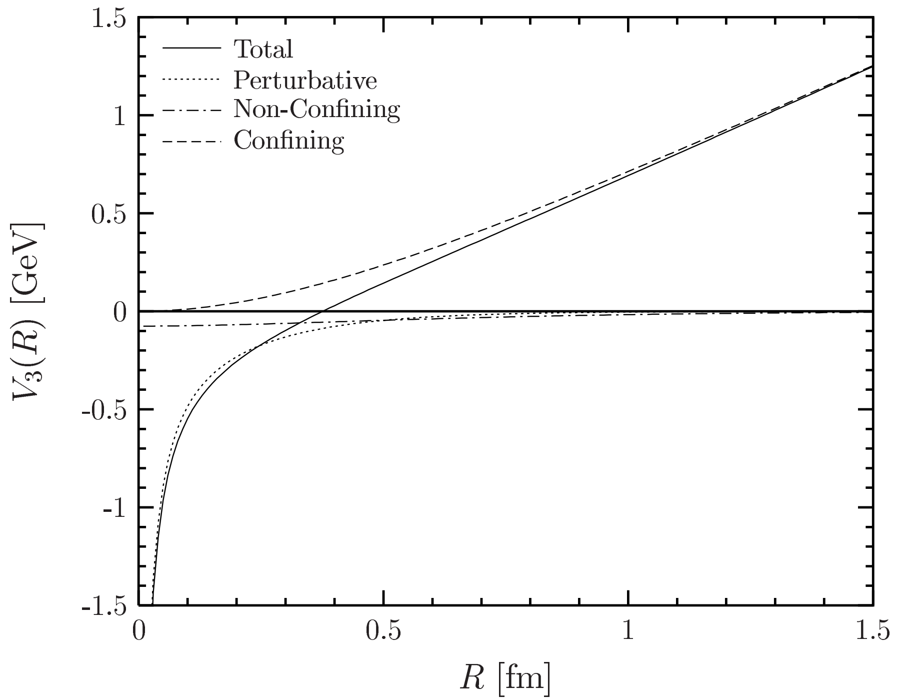
$$F^P_{\mu\nu\rho\sigma} = \frac{g^2}{\pi^2}\,\frac{1}{2}\left[\frac{\partial}{\partial Z_\nu}\left(Z_\sigma\delta_{\mu\rho} - Z_\rho\delta_{\mu\sigma}\right) + \frac{\partial}{\partial Z_\mu}\left(Z_\rho\delta_{\nu\sigma} - Z_\sigma\delta_{\nu\rho}\right)\right] D_P(Z^2, {m_G}^2)$$

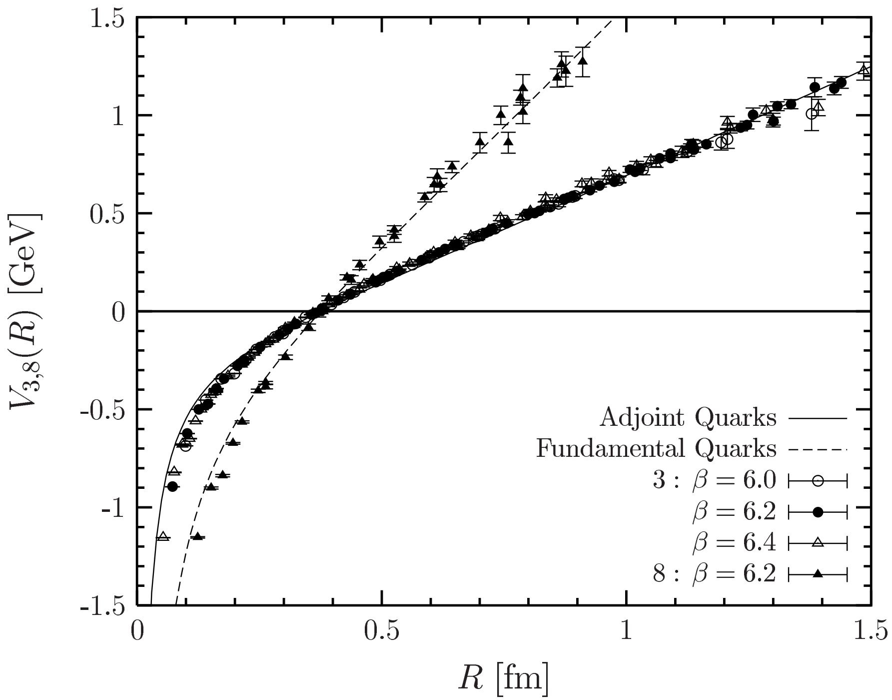
- Non-Pert. Stochastic Vacuum Model (NP) $\quad$ [Dosch '97, Dosch & Simonov '88]

$$F^{NP_c}_{\mu\nu\rho\sigma} = \frac{G_2\,\kappa}{3(N_c^2-1)}\left(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}\right) D(Z^2/a^2)$$

$$F^{NP_{nc}}_{\mu\nu\rho\sigma} = \frac{G_2\,(1-\kappa)}{3(N_c^2-1)}\,\frac{1}{2}\left[\frac{\partial}{\partial Z_\nu}\left(Z_\sigma\delta_{\mu\rho} - Z_\rho\delta_{\mu\sigma}\right) + \frac{\partial}{\partial Z_\mu}\left(Z_\rho\delta_{\nu\sigma} - Z_\sigma\delta_{\nu\rho}\right)\right] D_1(Z^2/a^2)$$

Static $q\bar{q}$ Potential $=$ Color-Coulomb $+$ Confinement





Casimir Scaling
No String Breaking

- $V_r(R) = V_r^P(R) + V_r^{NP\ nc}(R) + V_r^{NP\ c}(R)$

- $V_r(R) \;=\; -\lim_{T\to\infty} \frac{1}{T} \ln\langle W_r[C]\rangle_{\text{pot}} \;=\; \frac{C_2(r)}{2} \lim_{T\to\infty} \frac{1}{T} \chi_{SS\,\text{pot}}$

$$\chi_{SS} \;=\; -\frac{\pi^2}{4} \int_S d\sigma_{\mu\nu}(X_1) \int_S d\sigma_{\rho\sigma}(X_2) \left(F^P_{\mu\nu\rho\sigma} + F^{NP_c}_{\mu\nu\rho\sigma} + F^{NP_{nc}}_{\mu\nu\rho\sigma} \right)$$

- String Tension $\sigma = C_2(r) \frac{\pi^3 G_2 \kappa}{48} \int_0^\infty dZ^2 D(Z^2, a^2)$ [Dosch '97, Dosch & Simonov '88]

Chromo-Field Distributions of Color-Dipoles

$$\Delta G^2_{r\,\alpha\beta}(X) := \left\langle \frac{g^2}{4\pi^2} \mathcal{G}^a_{\alpha\beta}(X)\mathcal{G}^a_{\alpha\beta}(X) \right\rangle_{W_r[C]} - \left\langle \frac{g^2}{4\pi^2} \mathcal{G}^a_{\alpha\beta}(X)\mathcal{G}^a_{\alpha\beta}(X) \right\rangle_{\mathrm{vac}}$$

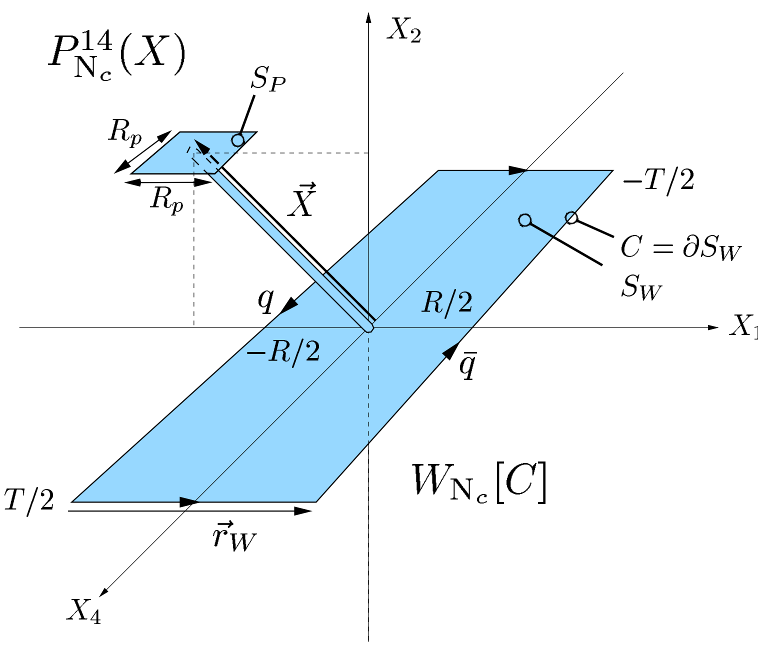
$$= -\lim_{R_P \to 0} \frac{1}{R_P^4} \frac{N_c}{\pi^2} \left[\frac{\langle W_r[C] P^{\alpha\beta}_{\mathrm{N}_c}(X)\rangle}{\langle W_r[C]\rangle} - \langle P^{\alpha\beta}_{\mathrm{N}_c}(X)\rangle \right] = \frac{g^2}{4\pi^2} \begin{pmatrix} 0 & B_z^2 & B_y^2 & E_x^2 \\ B_z^2 & 0 & B_x^2 & E_y^2 \\ B_y^2 & B_x^2 & 0 & E_z^2 \\ E_x^2 & E_y^2 & E_z^2 & 0 \end{pmatrix} (X)$$

$$\Delta G^2_{r\,\alpha\beta}(X) = -C_2(r) \lim_{R_P \to 0} \frac{1}{R_P^4} \frac{1}{4\pi^2} \chi^2_{S_P S_W}$$

- Casimir Scaling
- new derivation of SVM results [Dosch & Rüter '95]
- Chromo-Magnetic Fields $\quad \vec{B}^2(X) = 0$
- Energy & Action Densities

$$\varepsilon_r(X) = \frac{1}{2}\left(-\vec{E}^2(X) + \vec{B}^2(X)\right) = -\frac{1}{2}\vec{E}^2(X)$$

$$s_r(X) = -\frac{1}{2}\left(\vec{E}^2(X) + \vec{B}^2(X)\right) = -\frac{1}{2}\vec{E}^2(X)$$



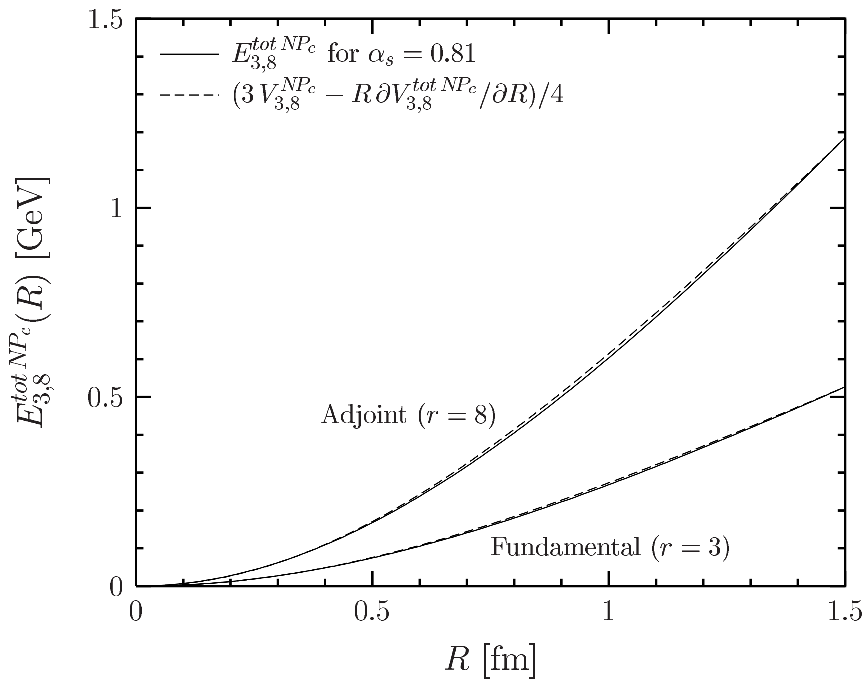
Low-Energy Theorems: Confining Non-Pert. Component

- Energy Sum Rule [Michael '86, Rothe '95]

$$E_r(R) = \int d^3X\, \varepsilon_r(X) - \frac{1}{2}\frac{\beta(g)}{g}\int d^3X\, s_r(X)$$

- Action Sum Rule [Michael '86, Dosch *et al.* '95]

$$E_r(R) + R\,\frac{\partial E_r(R)}{\partial R} = -\frac{2\beta(g)}{g}\int d^3X\, s_r(X)$$



- Total Energy Stored in the Chromo-Fields

$$E_r^{\text{tot}}(R) := \int d^3X\, \varepsilon_r(X) = \frac{3}{4}E_r(R) - \frac{R}{4}\frac{\partial E_r(R)}{\partial R}$$

- Chromo-Magnetic to Chromo-Electric

$$Q(R) := \frac{\int d^3X\, \vec{B}^2(X)}{\int d^3X\, \vec{E}^2(X)} \xrightarrow{V_r = \sigma R} \frac{2 + \beta(g)/g}{2 - \beta(g)/g}$$

- strong coupling at $\mu = \mu_{NP}$

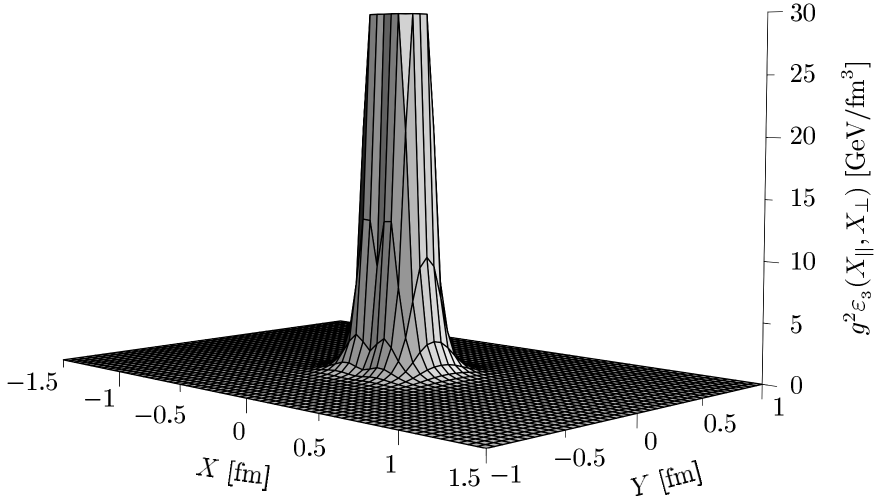
$$g^2(\mu_{NP}) = 10.2$$
$$\alpha_s(\mu_{NP}) = 0.81$$

- $\beta(g) = \mu\partial g/\partial\mu$ at $\mu = \mu_{NP}$

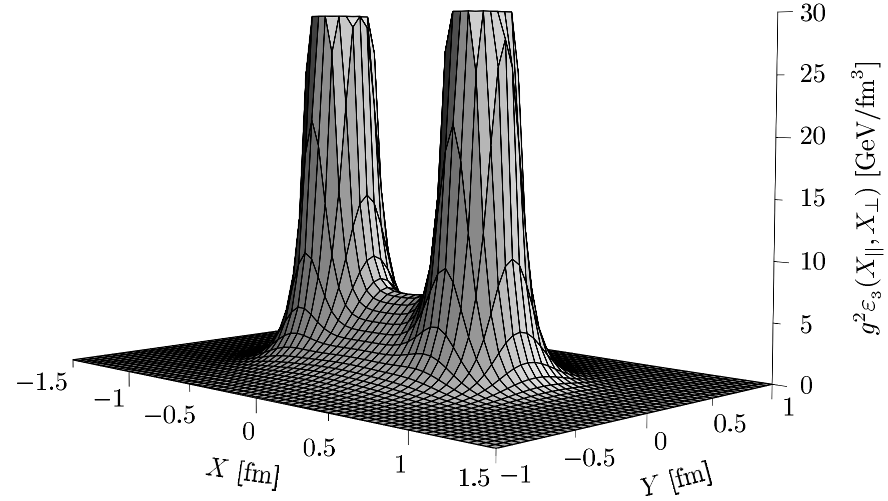
$$\left.\frac{\beta(g)}{g}\right|_{NP} = -2$$

Perturbative Dipoles $\longrightarrow$ Confining Strings

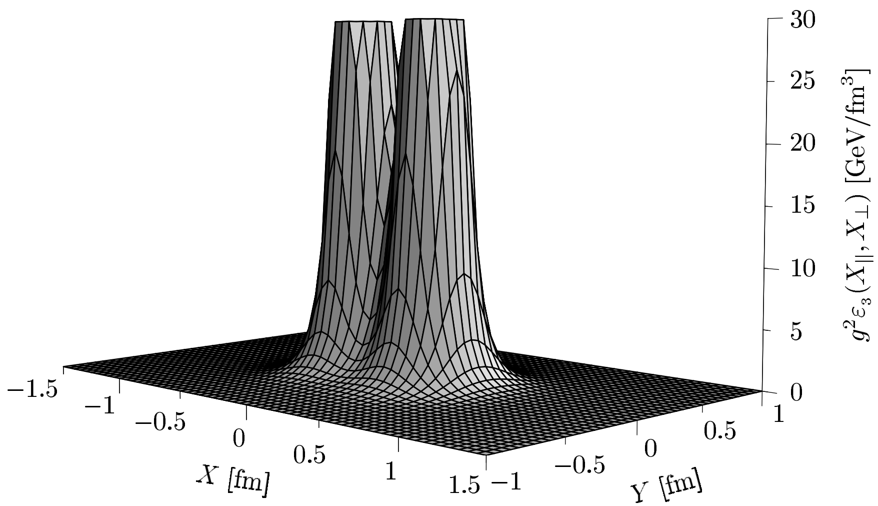
R = 0.1 fm



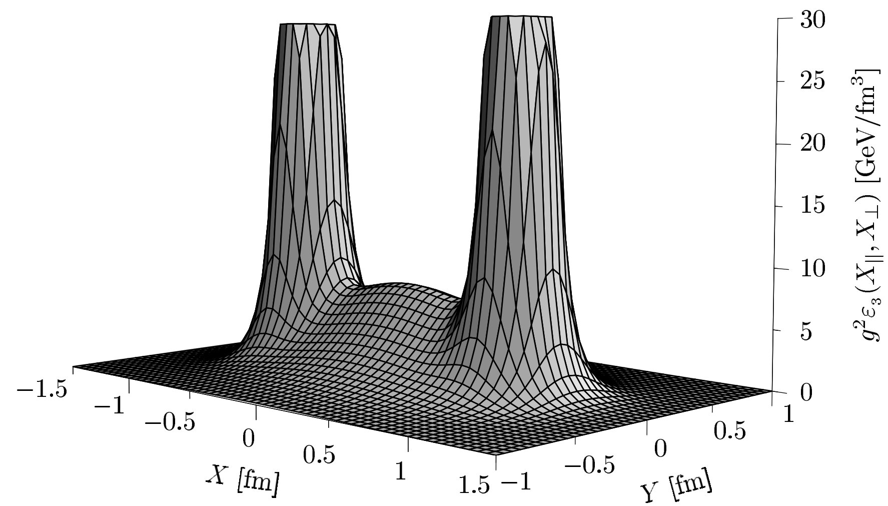
R = 1 fm

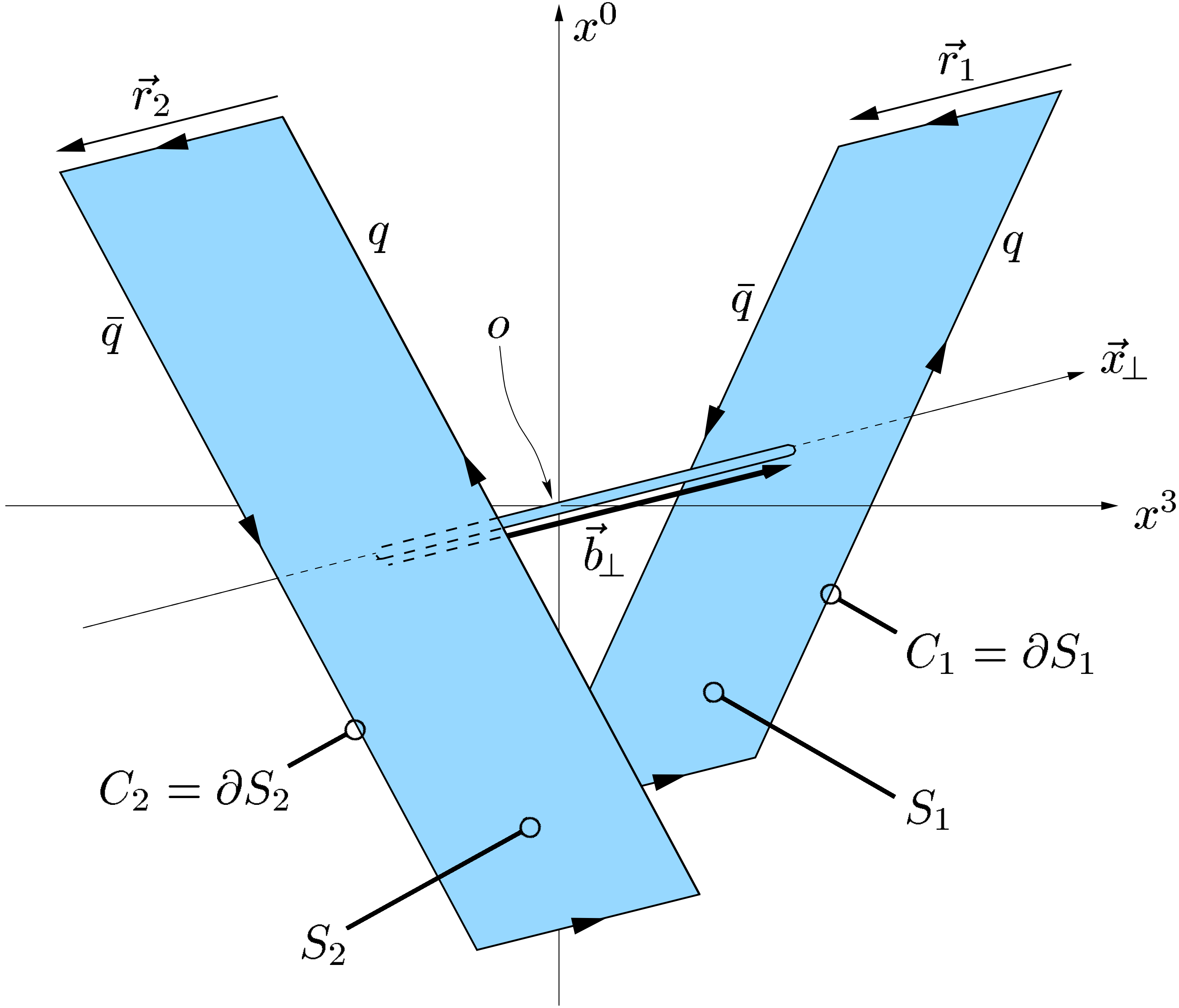


R = 0.5 fm



R = 1.5 fm




x^0
$\vec{r}_1$
$\vec{r}_2$
q
$\bar{q}$
q
$\bar{q}$
o
$\vec{x}_\perp$
x^3
$\vec{b}_\perp$
$C_1 = \partial S_1$
$C_2 = \partial S_2$
S_1
S_2

[Nachtmann 1991, Dosch *et al.* 1994]

Functional Integral Approach to High-Energy Scattering

- S-Matrix

$$S_{ab\to cd} = \delta_{fi} + i(2\pi)^4\delta^4(P_c + P_d - P_a - P_b)\,T_{ab\to cd}$$

- T-Matrix Element

$$T_{ab\to cd}(s,t) = 2is\int d^2b_\perp e^{i\vec{q}_\perp\vec{b}_\perp}\int dz_1 d^2r_1\int dz_2 d^2r_2$$

$$\times\ \psi_c^*(z_1,\vec{r}_1)\,\psi_d^*(z_2,\vec{r}_2)\left[1 - S_{DD}^M(s,\vec{b}_\perp,z_1,\vec{r}_1,z_2,\vec{r}_2)\right]\psi_a(z_1,\vec{r}_1)\,\psi_b(z_2,\vec{r}_2)$$

- Loop-Loop Correlation Function → Dipole-Dipole Scattering

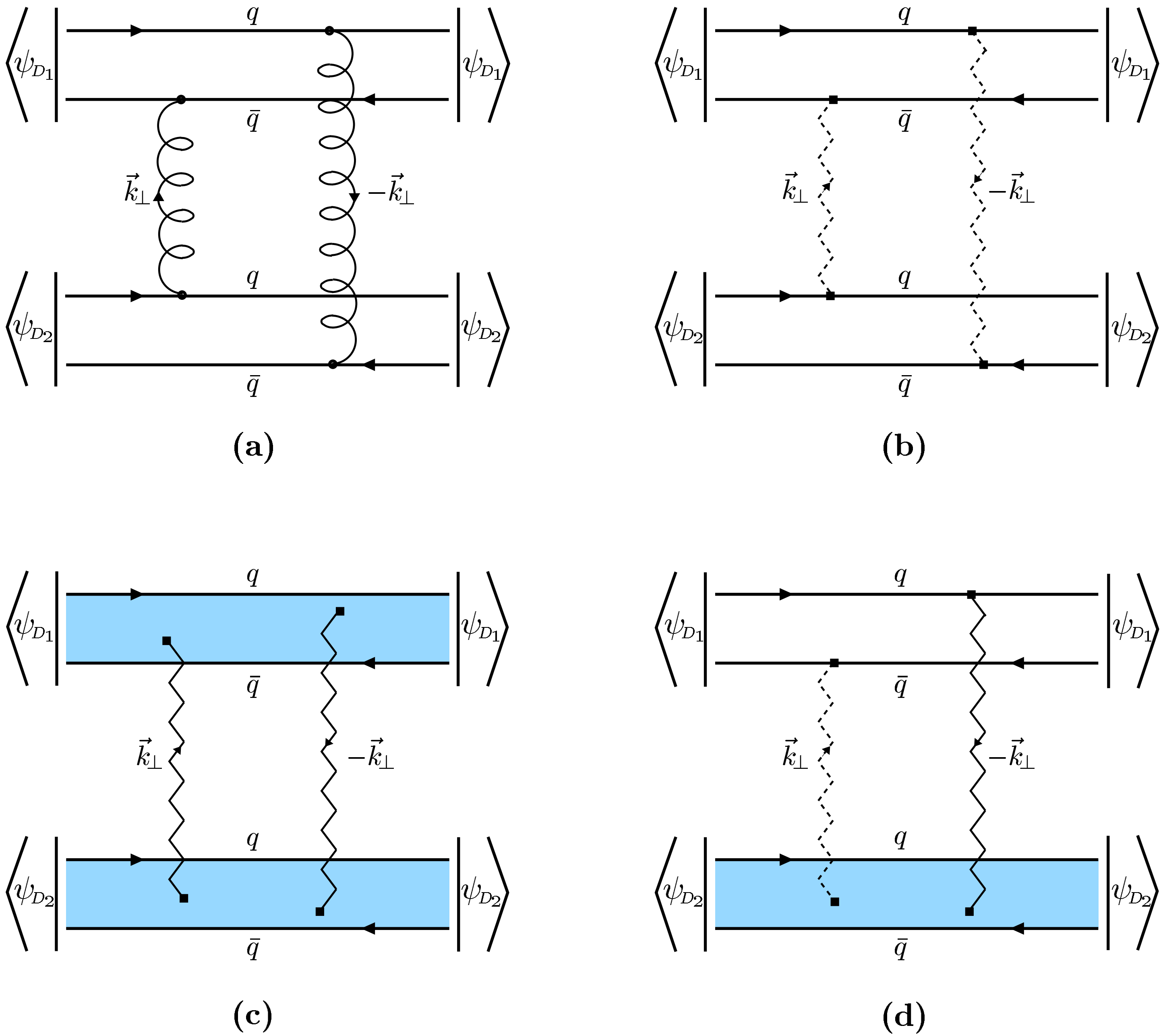
$$S_{DD}^M(s,\vec{b}_\perp,z_1,\vec{r}_1,z_2,\vec{r}_2) = \lim_{T\to\infty}\frac{\langle W[C_1]W[C_2]\rangle_M}{\langle W[C_1]\rangle_M\langle W[C_2]\rangle_M}$$

- Wegner-Wilson Loop → Color Dipole

$$W[C_{1,2}] = \frac{1}{N_c}\mathrm{Tr}\,\mathcal{P}\exp\left[-ig\oint_{C_{1,2}} dz^\mu \mathcal{G}_\mu(z)\right]$$

Unified Description of pp, $\gamma^* p$, and $\gamma\gamma$ Reactions

Perturbative and Non-Perturbative Dipole-Dipole Interactions

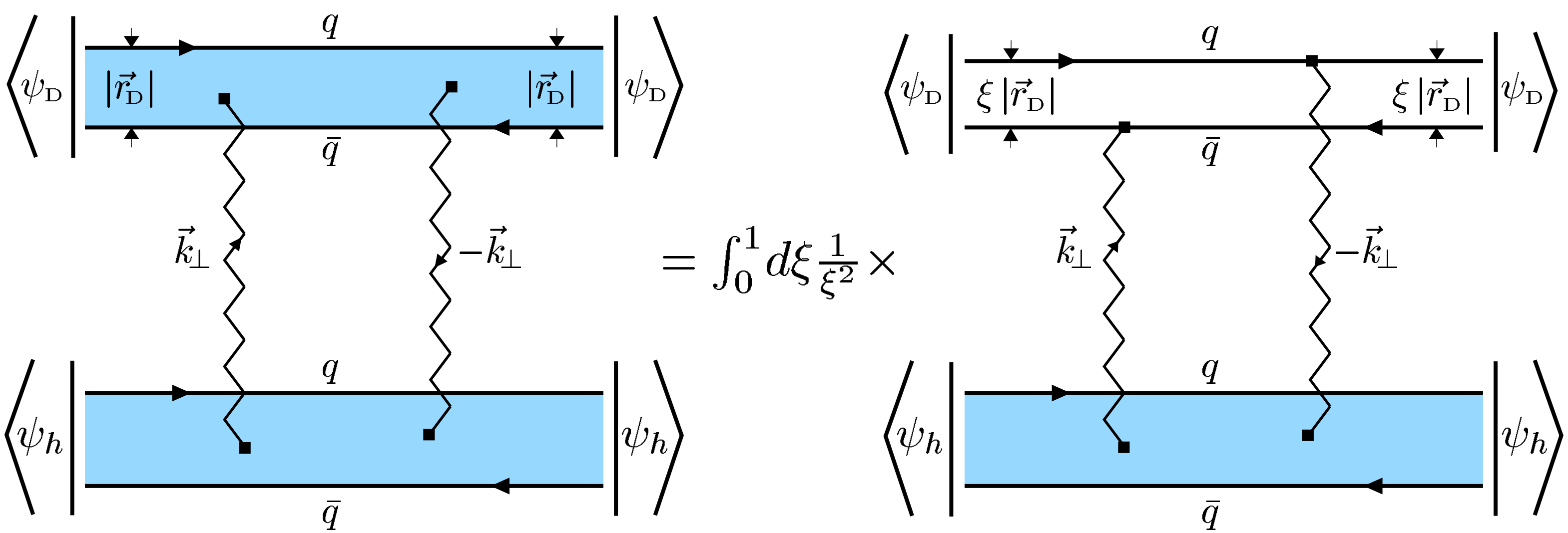


String Decomposition into Dipoles

- Mathematical Identity:

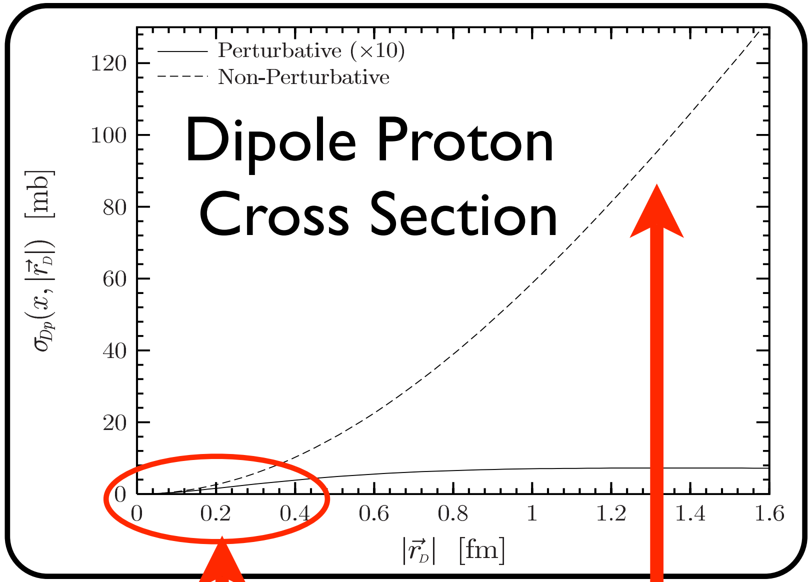
$$\left(-1 + {}_1F_2(-\frac{1}{2}; \frac{1}{2}, 1; \frac{-k_\perp^2 r_D^2}{4})\right) = \int_0^1 d\xi \, \frac{1}{\xi^2} \left(1 - J_0(|\vec{k}_\perp||\vec{r}_D|\xi)\right)$$

- String-Hadron Interaction → Stringless Dipole-Hadron Interactions



Unintegrated Gluon Distribution

[Shoshi, FDS, Dosch & Pirner, hep-ph/0207287]



- Definition via $|\vec{k}_\perp|$ - factorization:

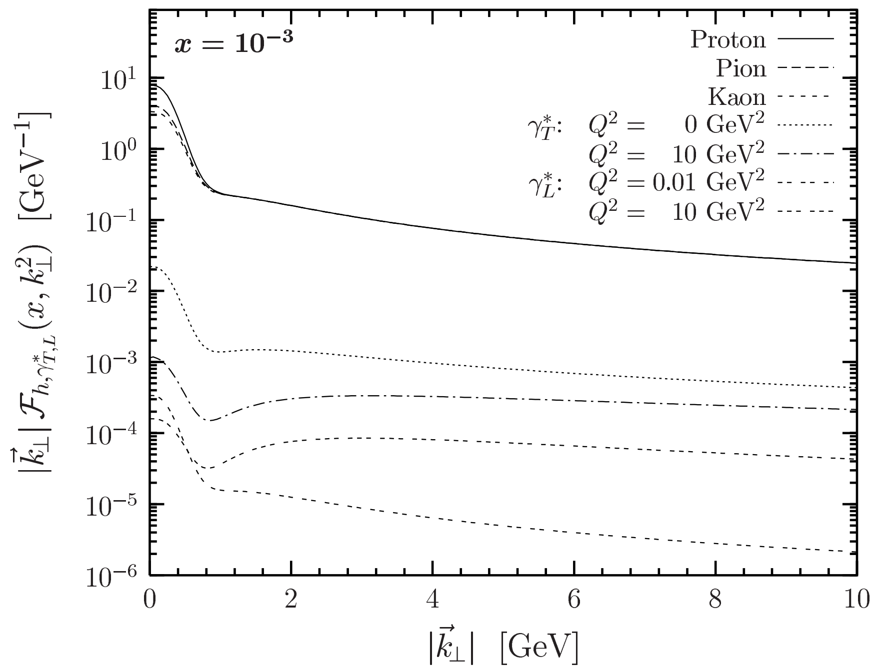
$$\sigma_{Dh}(x,|\vec{r}_D|) = \frac{4\pi^2 r_D^2}{3}\int dk_\perp^2 \, \frac{\left(1 - J_0(|\vec{k}_\perp||\vec{r}_D|)\right)}{(|\vec{k}_\perp||\vec{r}_D|)^2}\,\alpha_s(k_\perp^2)\mathcal{F}_h(x,k_\perp^2)$$

- Extraction from our Dipole-Hadron Cross Section:

$$\begin{aligned}
\mathcal{F}_h(x,k_\perp^2) \quad &= \quad \frac{k_\perp^2}{6\pi^3\alpha_s(k_\perp^2)} \\
&\times \Bigg[\left(4\pi\alpha_s(k_\perp^2)\right)^2 \left\{\left[iD_P'^{(2)}(k_\perp^2)\right]^2 \langle\psi_h|1 - e^{i\vec{k}_\perp\vec{r}_2}|\psi_h\rangle\right\}\left(\frac{x_0}{x}\right)^{\epsilon^P} \\
&+ \left(\frac{\pi^2 G_2}{24}\right)^2 \Bigg\{\left[(1-\kappa)\left[iD_1'^{(2)}(k_\perp^2)\right] + \frac{\kappa}{k_\perp^2}\left[iD^{(2)}(k_\perp^2)\right]\right]^2 \langle\psi_h|1 - e^{i\vec{k}_\perp\vec{r}_2}|\psi_h\rangle \\
&+ \frac{\kappa^2}{k_\perp^4}\int_0^1 d\xi \left[iD^{(2)}\left(\frac{k_\perp^2}{\xi^2}\right)\right]^2 \langle\psi_h|\tan^2\phi_2(1 - e^{i(\vec{k}_\perp/\xi)\vec{r}_2})|\psi_h\rangle\Bigg\}\left(\frac{x_0}{x}\right)^{\epsilon^{NP}}\Bigg]
\end{aligned}$$

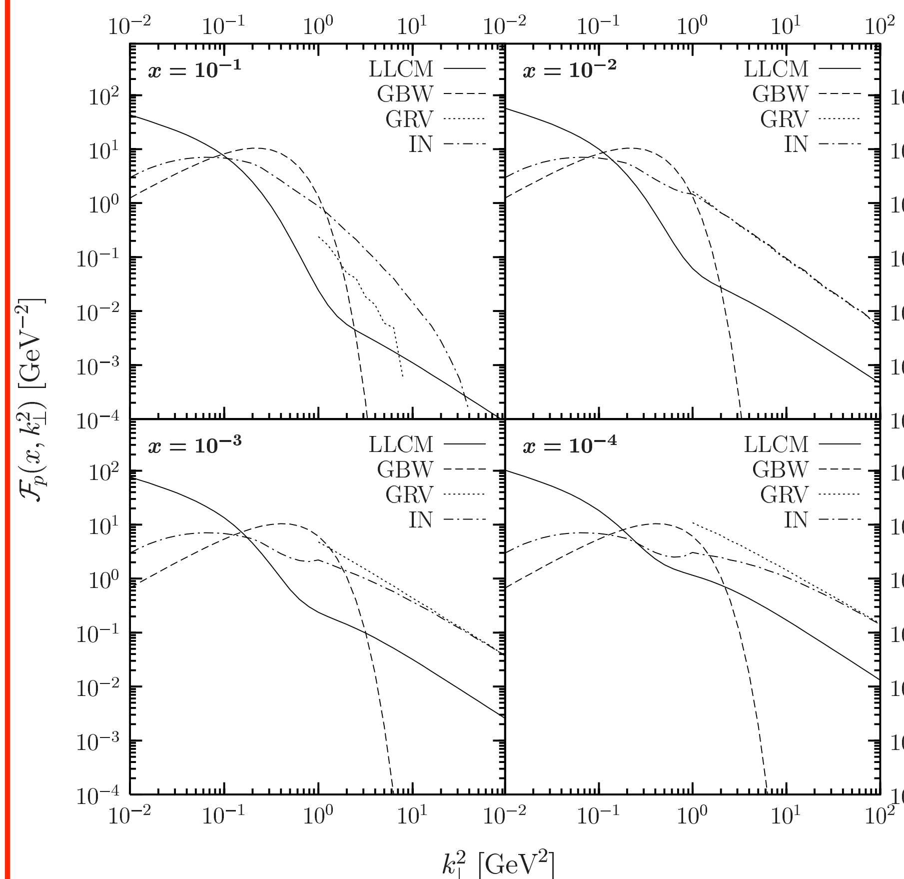
- String of the Dipole has been shifted into the Hadron!

Unintegrated Gluon Distributions of Hadrons and Photons



- small $|\vec{k}_\perp|$: $\mathcal{F}(x, k_\perp^2) \propto 1/|\vec{k}_\perp|$ $\leftarrow$ string effect!
  $\mathcal{F}(x, k_\perp^2) \propto S_h^2$ $\leftarrow$ low resolution!
- large $|\vec{k}_\perp|$: $\mathcal{F}(x, k_\perp^2) \propto 1/k_\perp^2$ $\leftarrow$ gluon propagator!
  S_h–independent $\leftarrow$ high resolution!

Comparison with Other Work



- small $|\vec{k}_\perp|$: $\mathcal{F}_{!p}^{LLCM} \propto 1/|\vec{k}_\perp|$, $\mathcal{F}_p^{GBW} \propto k_\perp^2$, $\mathcal{F}_p^{IN} \propto k_\perp^4$
- large $|\vec{k}_\perp|$: $\mathcal{F}_p^{LLCM,GRV,IN} \propto 1/k_\perp^2$,

  $\mathcal{F}_p^{GBW} \propto k_\perp^2 \exp(-R^2(x) k_\perp^2)$

[Shoshi, FDS & Pirner, hep-ph/0202012]

Energy Dependence: Two Pomeron (Soft + Hard) Picture

- Two Pomeron Picture $\quad 0 \approx \epsilon^{NP} < \epsilon^{P} < 1$

$$\left(\chi^{NP}\right)^2 \quad \rightarrow \quad \left(\chi^{NP}(s)\right)^2 := \left(\chi^{NP}\right)^2 \left(\frac{s}{s_0} \frac{\vec{r}_1^{\,2}\,\vec{r}_2^{\,2}}{R_0^4}\right)^{\epsilon^{NP}} \qquad \text{SOFT}$$

$$\left(\chi^{P}\right)^2 \quad \rightarrow \quad \left(\chi^{P}(s)\right)^2 := \left(\chi^{P}\right)^2 \left(\frac{s}{s_0} \frac{\vec{r}_1^{\,2}\,\vec{r}_2^{\,2}}{R_0^4}\right)^{\epsilon^{P}} \qquad \text{HARD}$$

- Scaling Variable $\quad s|\vec{r}_1|^2 \propto \frac{s}{Q^2} = \frac{1}{x}$

- T-Matrix Element

$$T(s,t) = 2is \int d^2b_\perp e^{i\vec{q}_\perp \vec{b}_\perp} \int dz_1 d^2r_1 \int dz_2 d^2r_2 \; |\psi_1(z_1,\vec{r}_1)|^2 \; |\psi_2(z_2,\vec{r}_2)|^2$$

$$\times \left[1 - \frac{2}{3}\cos\left(\frac{1}{3}\chi^{NP}(s)\right)\cos\left(\frac{1}{3}\chi^{P}(s)\right) - \frac{1}{3}\cos\left(\frac{2}{3}\chi^{NP}(s)\right)\cos\left(\frac{2}{3}\chi^{P}(s)\right)\right]$$

Unitarity Condition in Impact Parameter Space

Multiple Gluonic Interactions $\qquad$ Saturation Effects

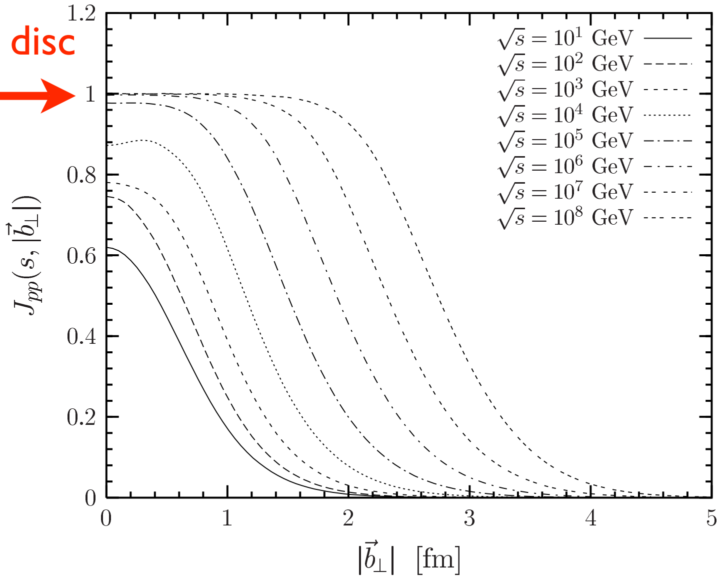
Universal Energy Dependence for pp, $\gamma^* p$, and $\gamma\gamma$ Reactions

Proton-Proton Scattering

- Profile Function

$$J_{pp}(s,|\vec{b}_\perp|) \;=\; \int dz_1 d^2r_1 \int dz_2 d^2r_2 |\psi_p(z_1,\vec{r}_1)|^2 |\psi_p(z_2,\vec{r}_2)|^2 \times \left[1 - S_{DD}(s,\vec{b}_\perp,z_1,\vec{r}_1,z_2,\vec{r}_2)\right]$$

black disc limit →



- Proton Opacity ↘ with $|\vec{b}_\perp|$ ↗
- Maximum Opacity for $|\vec{b}_\perp| = 0$ at $\sqrt{s} \gtrsim 10^6$ GeV

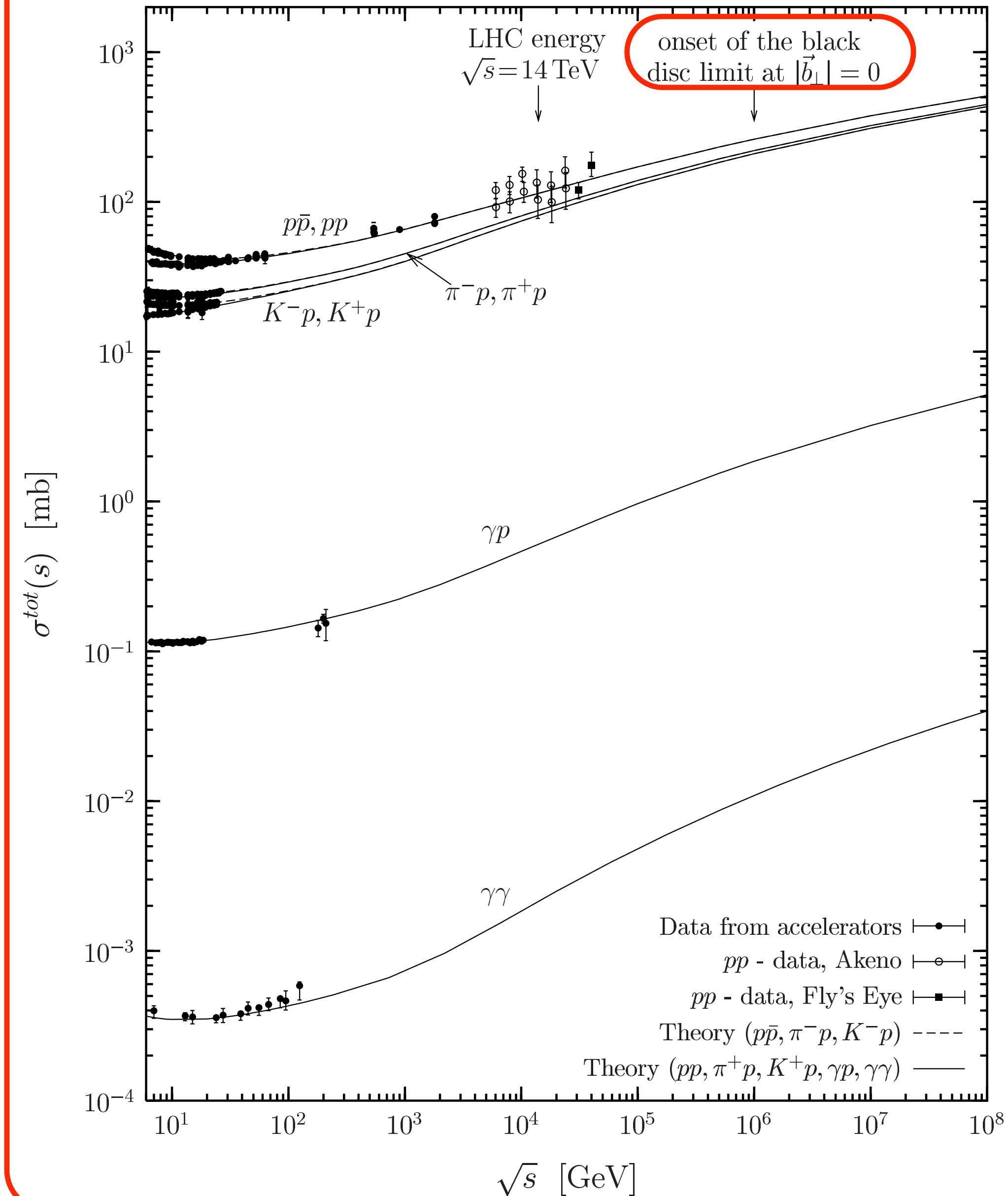
$$J_{pp}^{max} = \int dz_1 d^2r_1 \int dz_2 d^2r_2 \, |\psi_p(z_1,\vec{r}_1)|^2 \, |\psi_p(z_2,\vec{r}_2)|^2 = 1$$

since $\int dz_i d^2r_i \; |\psi_p(z_i,\vec{r}_i)|^2 = 1$

- Transverse Proton Radius ↗ with s ↗

Total Cross Sections

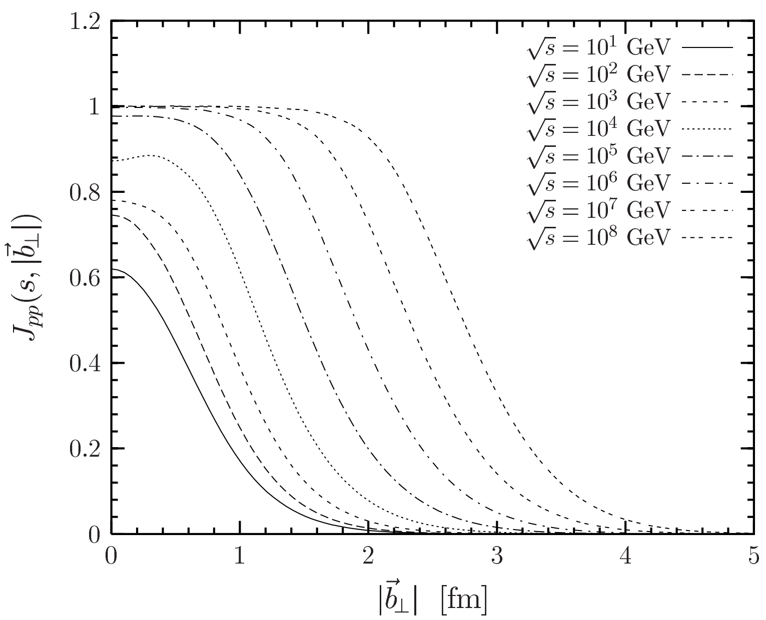
$$\sigma_{ab}^{tot}(s) \;=\; \frac{1}{s}\,\mathrm{Im}\,T(s,t=0) \;=\; 2\int d^2b_\perp \, J_{ab}(s,|\vec{b}_\perp|)$$



Proton-Proton Scattering

- Profile Function

$$J_{pp}(s,|\vec{b}_\perp|) \;=\; \int dz_1 d^2r_1 \int dz_2 d^2r_2 |\psi_p(z_1,\vec{r}_1)|^2 |\psi_p(z_2,\vec{r}_2)|^2 \times \left[1 - S_{DD}(s,\vec{b}_\perp,z_1,\vec{r}_1,z_2,\vec{r}_2)\right]$$



- Proton Opacity ↘ with $|\vec{b}_\perp|$ ↗
- Maximum Opacity for $|\vec{b}_\perp| = 0$ at $\sqrt{s} \gtrsim 10^6$ GeV

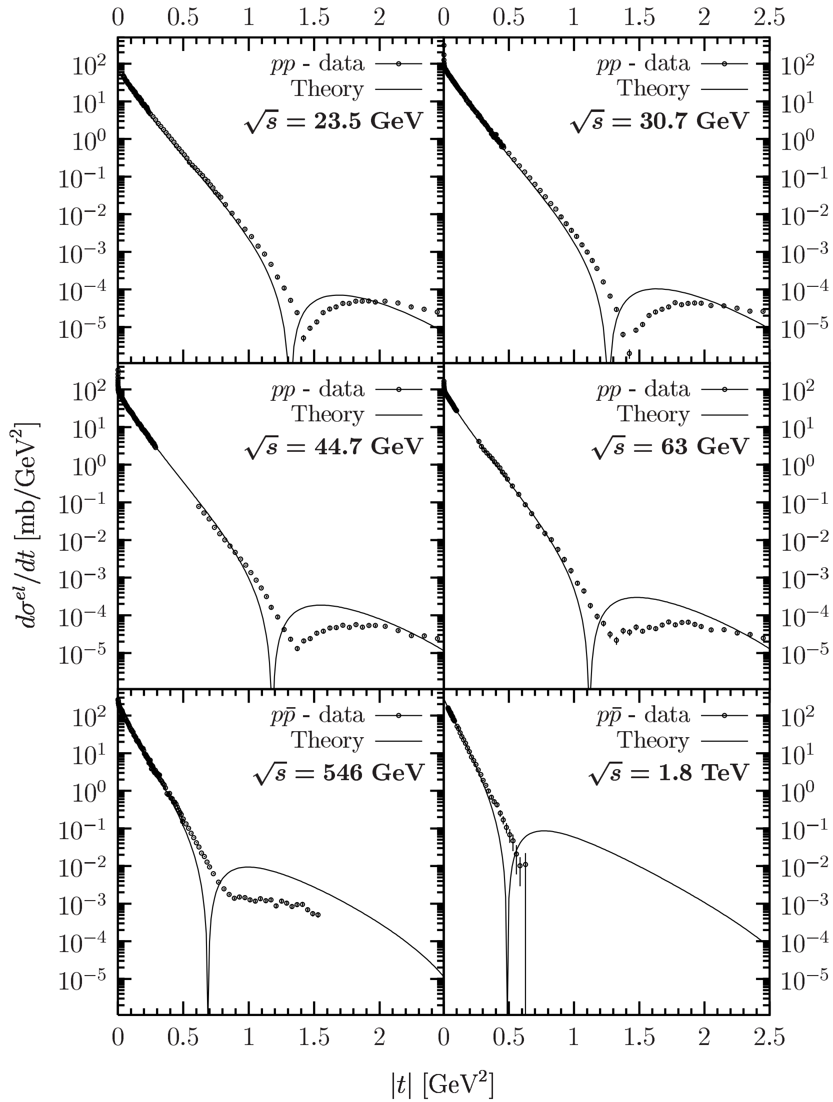
$$J_{pp}^{max} = \int dz_1 d^2r_1 \int dz_2 d^2r_2\, |\psi_p(z_1,\vec{r}_1)|^2\, |\psi_p(z_2,\vec{r}_2)|^2 = 1$$

since $\int dz_i d^2r_i\, |\psi_p(z_i,\vec{r}_i)|^2 = 1$

- Transverse Proton Radius ↗ with s ↗

The Differential Elastic Cross Section

$$\frac{d\sigma^{el}}{dt}(s,t) = \frac{1}{16\pi s^2}|T(s,t)|^2 = \frac{1}{4\pi}\left[\int d^2b_\perp\, e^{i\vec{q}_\perp\vec{b}_\perp}\, J(s,|\vec{b}_\perp|)\right]^2$$



Conclusion

- The Model
  - $\left\langle W_r[C] \right\rangle_E$ and $\left\langle W_{r_1}[C_1] W_{r_2}[C_2] \right\rangle_E \longleftarrow$ Gaussian Approximation
  - Perturbative & Non-Perturbative QCD Description $\longleftrightarrow$ Lattice QCD

    Color-Coulomb   Confinement
  - Flux-Tube Formation: Perturbative Dipole $\rightarrow$ Non-Perturbative Dipole
  - Casimir Scaling $\longleftrightarrow$ Gaussian Approximation

- Structure of Dipole-Dipole Scattering in $|\vec{k}_\perp|$-Space
  - Non-perturbative string-string interactions show a new structure different from the perturbative two-gluon exchange.

- Impact Parameter Profiles $J(s, |\vec{b}_\perp|)$
  - Unitarity Bound = Black Disc Limit or Maximum Opacity
  - Geometrical Picture of High Energy Reactions

- Comparison with Experimental Data for pp, $\gamma^* p$, and $\gamma\gamma$ Reactions
  - Total Cross Sections
  - Differential Elastic Cross Sections

Outlook

? Further Reactions

* Vector-Meson Production
* Diffractive Dissociation
* Nuclear Reactions

?? Multiple Loops

* Sea Quarks in the Nucleon
* Nuclear Interactions
* pA and AA Reactions

??? Dynamics $\longrightarrow$ String Breaking

??? Quantum Evolution $\longrightarrow$ Energy-Dependence

??? Path Dependence of the Bilocal Gluon Field Strenght Correlator

??? New Approaches to $\left\langle W_r[C] \right\rangle$ and $\left\langle W_{r_1}[C_1] W_{r_2}[C_2] \right\rangle$

Thank you, Hans-Jürgen

for the PhD research supervision and

the enjoyful/productive collaboration!!!