Finite-Volume effects in QCD and functional RG methods

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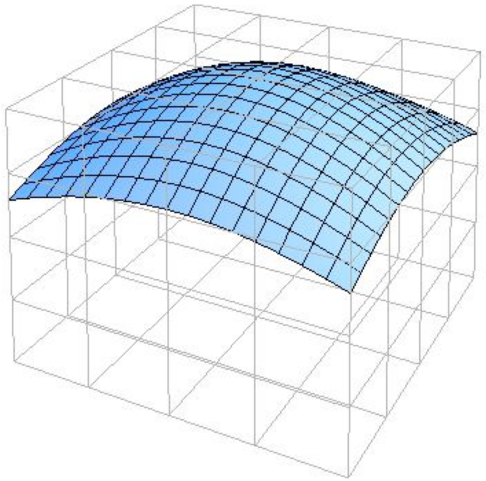
30 years of strong interactions: a three-day meeting in honor of Joseph Cugnon and Hans-Jürgen Pirner

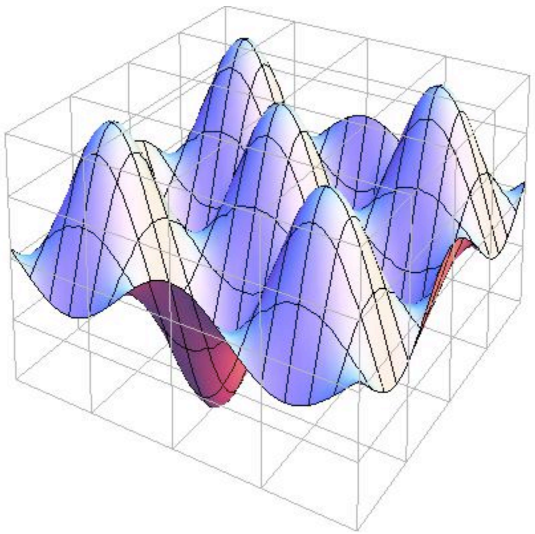
Spa, April 6, 2011

QCD in a finite volume

- confinement
  - no associated light degrees of freedom
- spontaneous chiral symmetry breaking
  - long-range fluctuations determine finite-volume behavior
- pions as Goldstone modes of spontaneous chiral symmetry breaking are essential to understand finite-volume behavior!

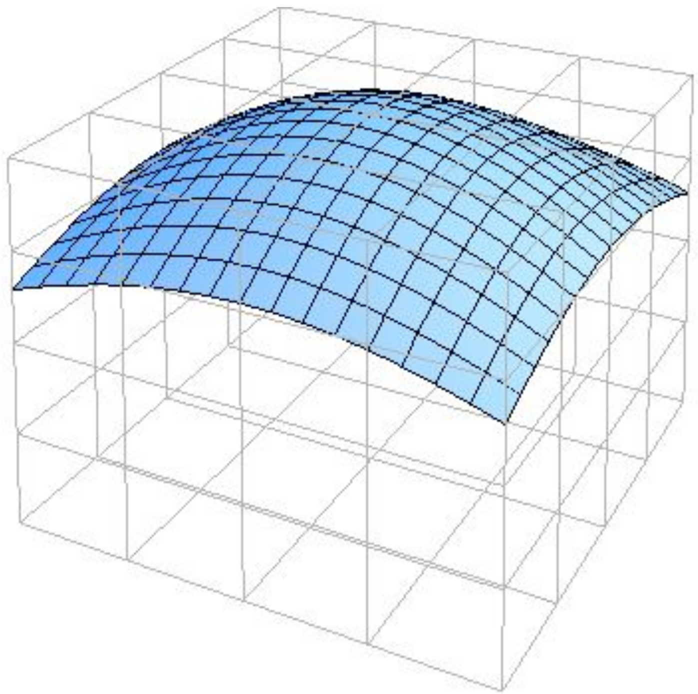
From small to large volumes





- $1/m_\pi \gg L$: pion does not fit into the box, static pion modes, Random Matrix Theory (RMT)
- $1/m_\pi \approx L$: pion wavelength of the order of the box size, transition effects and chiral symmetry restoration, **models**
- $1/m_\pi \ll L$: pion wavelength much smaller than box size, chiral perturbation theory (ChPT)

Small volumes



$$\frac{1}{m_\pi} \gg L$$

Chiral symmetry and Dirac operator spectrum

- Euclidean QCD Dirac operator eigenvalues

$$\not{D}\psi_k = \lambda_k \psi_k$$

- eigenvalue density

$$\rho(\lambda) = \langle \sum_k \delta(\lambda - \lambda_k) \rangle_{\mathrm{QCD}}$$

- order parameter for chiral symmetry breaking: chiral condensate $\langle \bar{\psi}\psi \rangle$
- chiral condensate is given by eigenvalue density

$$\langle \bar{\psi}\psi \rangle = \lim_{\lambda \to 0} \lim_{V \to \infty} \frac{\pi \rho(\lambda)}{V}$$

[T. Banks and A. Casher, Nucl. Phys. B 169, 103 (1980).]

Random Matrix Theory

- **exact theory** of spectral correlations in the QCD Dirac operator spectrum [Leutwyler, Smilga, Shuryak, Verbaarschot, Zahed, Osborn, Wettig, Akemann, Klein,...]
- **schematic model** for the topology of the phase diagram [Verbaarschot, Halasz, Osborn, Klein, Splittorff, Wettig, Jackson, ...]
- spectral density determined by symmetries of QCD:
  - chiral symmetry breaking pattern $SU(N_f) \times SU(N_f) \rightarrow SU(N_f)$
  - axial symmetry: non-zero eigenvalues paired $+\lambda \rightarrow -\lambda$
  - anti-unitary symmetries (none for SU(3))
- valid for $L \ll 1/m_\pi$ (only pion zero modes relevant - static)
- valid for $L \gg 1/\Lambda$ (hadron mass scale)

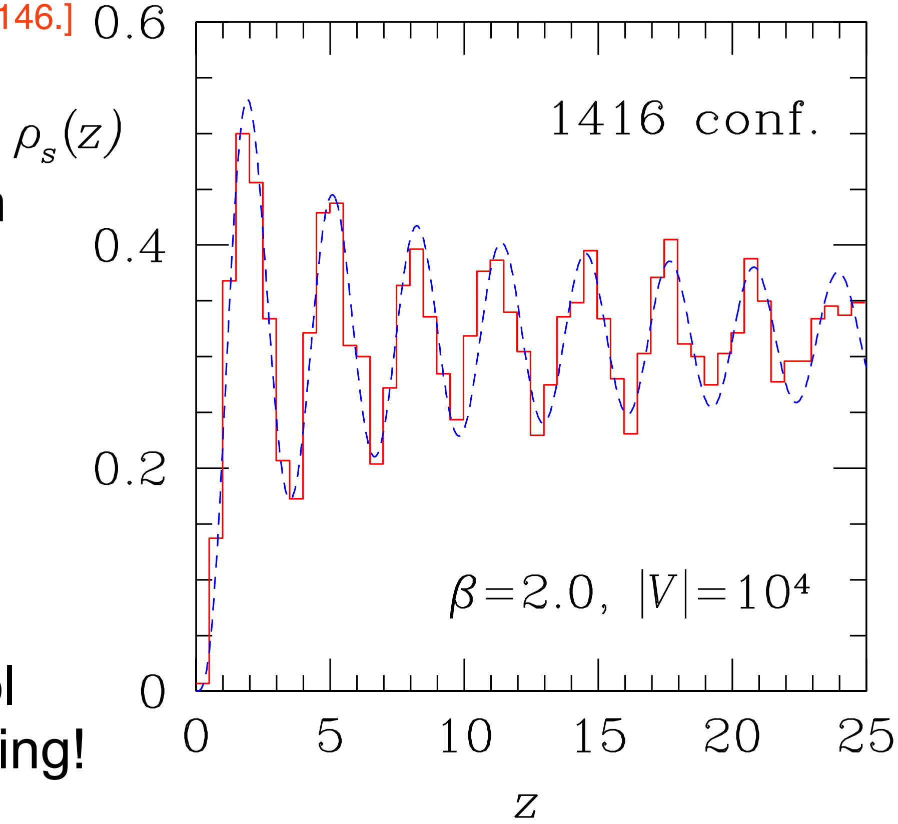
RMT and the lattice QCD spectrum

- universality: microscopic spectral density from lattice QCD agrees with RMT results
  [M.E. Berbenni-Bitsch et al. Phys. Rev. Lett. 80(1998)1146.]

microscopic spectral density:
magnification of region near the origin of the spectrum with the volume

$$\rho_s(z) = \lim_{V\to\infty} \frac{1}{V\Sigma}\rho\left(\frac{z}{V\Sigma}\right)$$

small volume used as analytical tool
to learn about chiral symmetry breaking!



Quark-meson model for 2 flavors

- Model for chiral symmetry breaking with 2 quark flavors
- no gauge degrees of freedom

$$\Gamma_\Lambda[\bar{\psi}, \psi, \sigma, \vec{\pi}] = \int \mathrm{d}^4 x \Big\{ \bar{\psi}(\mathrm{i}\,\partial\!\!\!/\,)\psi + g\bar{\psi}(\sigma + \mathrm{i}\gamma_5 \vec{\tau}\cdot\vec{\pi})\psi + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi}\,)^2 + U_\Lambda(\sigma, \sigma^2 + \vec{\pi}^2) \Big\}$$

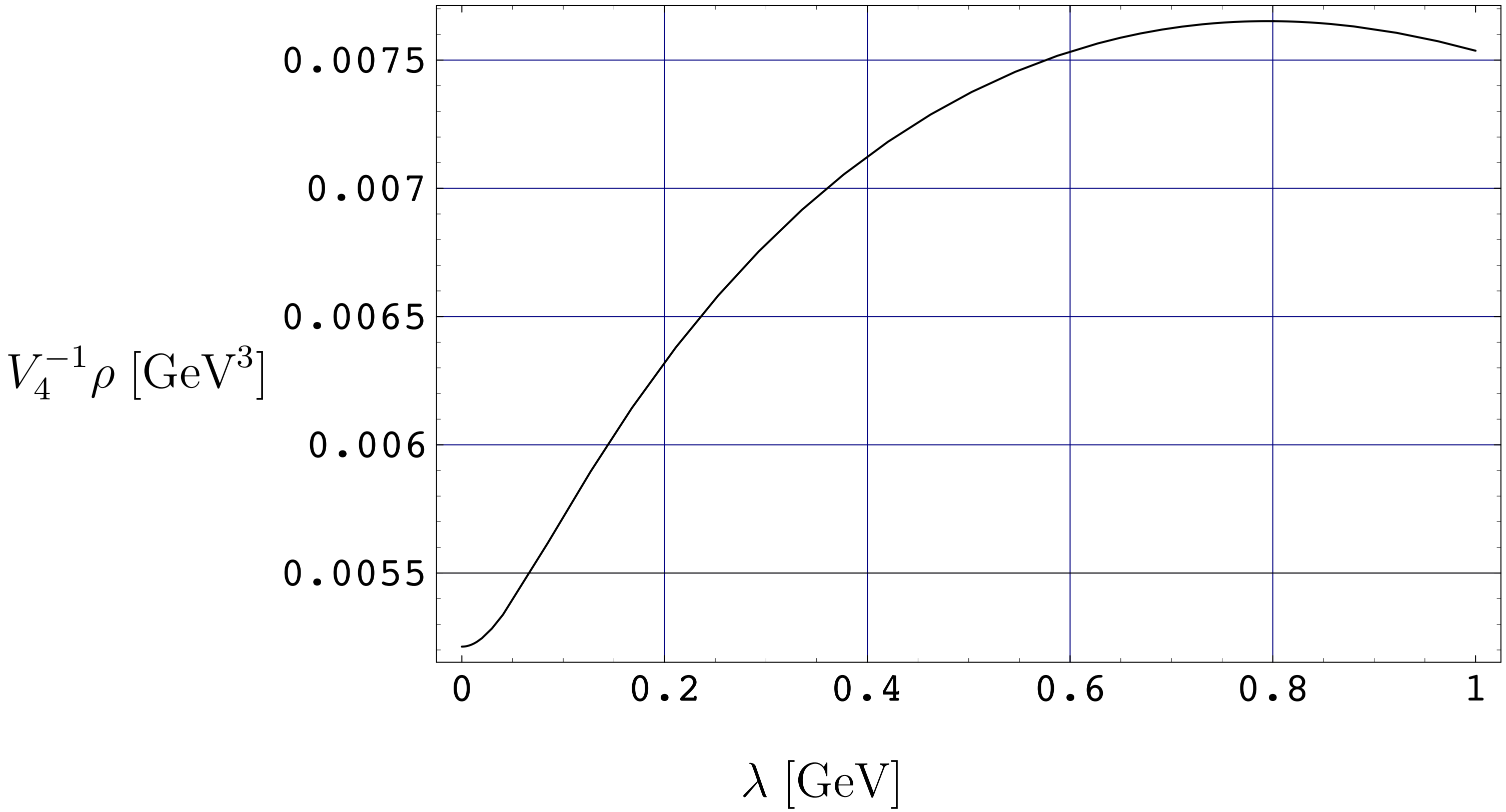
- chiral symmetry breaking: SU(2) × SU(2) → SU(2) realized as O(4) → O(3) (meson sector)

  $\langle \sigma \rangle \neq 0$

[B.-J. Schaefer, **H.-J. Pirner**, Nucl.Phys. A660 (1999) 439; J. Meyer, K. Schwenzer, **H.-J. Pirner**, Phys.Lett. B473 (2000) 25; J. Meyer, K. Schwenzer, **H.-J. Pirner**, A. Deandrea, Phys.Lett. B526 (2002) 79; J. Braun, K. Schwenzer, **H.-J. Pirner**, Phys.Rev. D70 (2004) 085016; L. Jendges, B. Klein, **H.-J. Pirner**, K. Schwenzer, hep-ph/0608056.]

Dirac Operator spectrum

- spectrum of quark-meson model in infinite volume



[T. Spitzenberg, K. Schwenzer, **H.-J. Pirner**, Phys. Rev. D70 (2004) 085016]

Universal spectrum from the model?

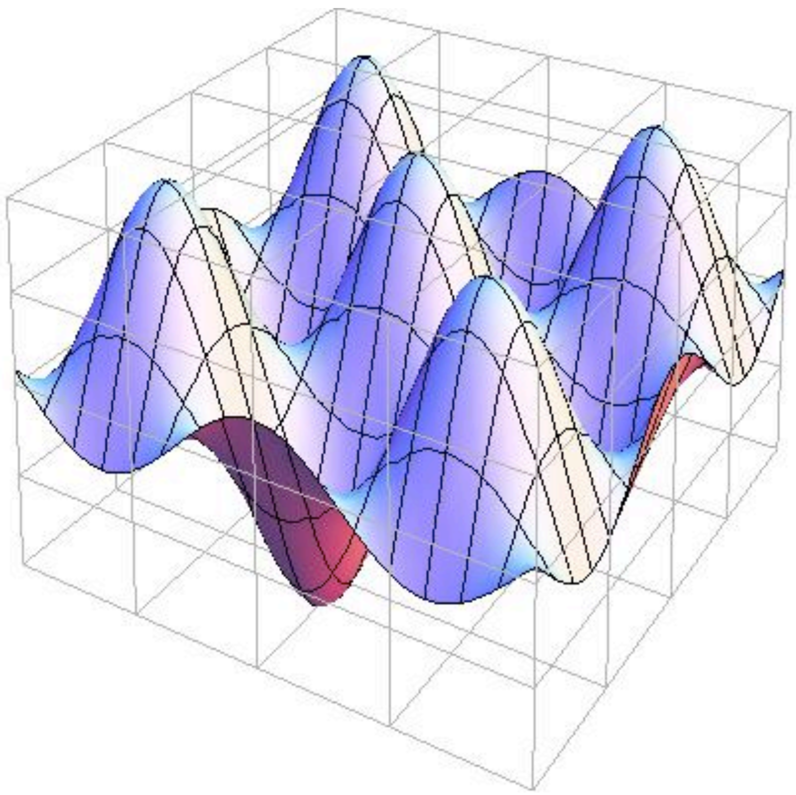
- need to combine model calculation with finite-volume approach to access universal features of spectral density

- length scales probe different momentum scales!

- Renormalization Group methods uniquely suited to this analysis [C.Wetterich, Phys. Lett. B 301 (1993) 90.]

- description across many different length scales

Large volumes



$$\frac{1}{m_\pi} \lesssim L$$

Chiral perturbation theory (ChPT)

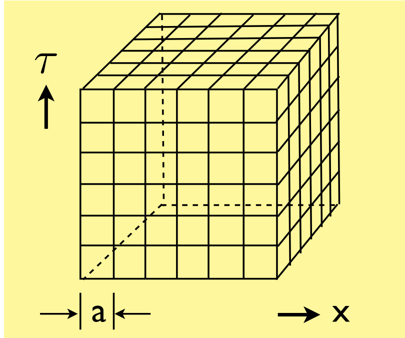
- Lagrangian constrained by the chiral symmetry breaking pattern in QCD: pion fields [Weinberg, Gasser, Leutwyler, Ecker,...]

- Effective Field Theory: systematic expansion in m_π/Λ, p/Λ where $\Lambda = 4\,\pi\, f_\pi \approx 1$ GeV (hadron mass scale)

$$\mathcal{L} = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\mu U\right) - \frac{m\Sigma}{2}\mathrm{Tr}(U^\dagger + U) + \ldots$$

- short-range physics parametrized in low-energy constants
- description of QCD below chiral transition temperature
- essential for the systematic description of nuclear forces [Yukawa, Bernard, Meissner, Epelbaum, Bedaque, Machleidt, Kaiser, Weise, ...]

Finite-volume effects in ChPT



- chiral Lagrangian in finite volume:
  no additional low-energy constants
  [for QCD with **anti-periodic** quark boundary conditions]

  [J. Gasser, H. Leutwyler, Nucl. Phys. B 307 (1988) 763.]

- pion mass shift at one loop [J. Gasser, H. Leutwyler, Phys. Lett. B 188 (1987) 477.]

- infinite-volume scattering amplitude related to mass shift in finite volume [M. Lüscher, Commun. Math. Phys. 104 (1986) 177.]

- pion mass shift from Lüscher's result
  [G. Colangelo, S. Dürr, C. Haefeli, Nucl. Phys. B 721 (2005) 136.]

- pion mass shift at NNLO [G. Colangelo, C. Haefeli, Nucl. Phys. B 744 (2006)14.]

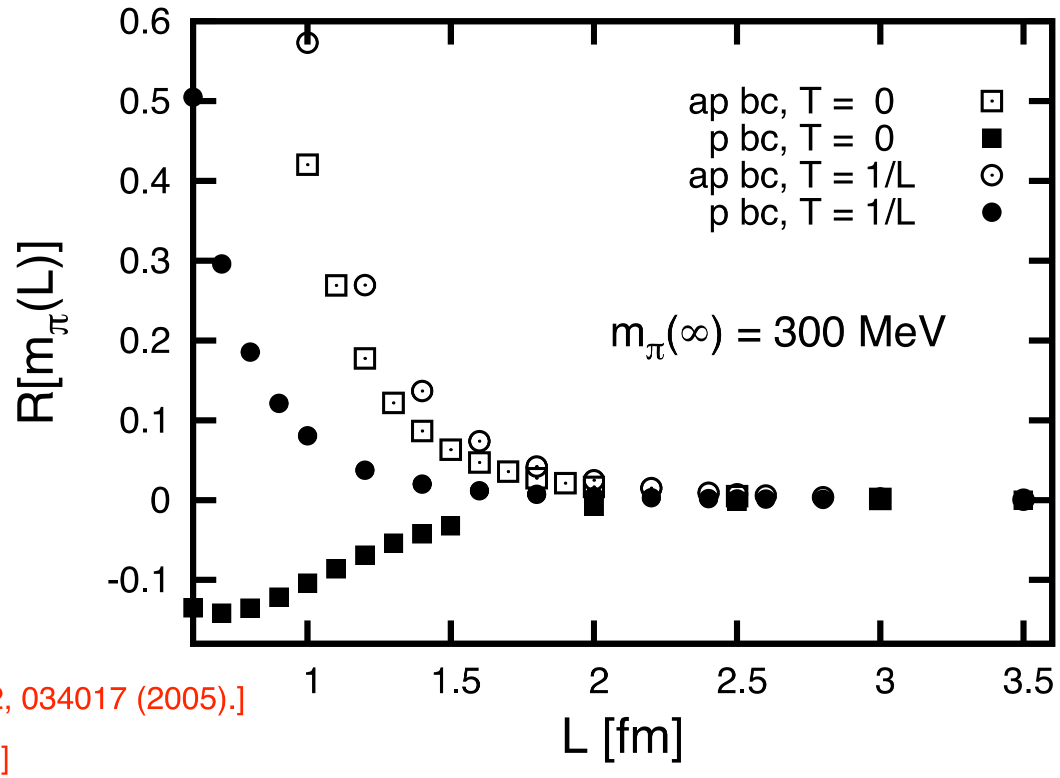
Pion mass from the quark-meson model

- Pion mass shift in V = $L^3 \times 1/T$
- periodic vs. anti-periodic quark boundary conditions (b.c.)

$$f_\pi \sim \langle \sigma \rangle$$

$$\langle \bar{\psi}\psi \rangle \sim \langle \sigma \rangle$$

$$m_\pi^2 = m \frac{\langle \bar{\psi}\psi \rangle}{f_\pi^2} \sim \frac{m}{\langle \sigma \rangle}$$

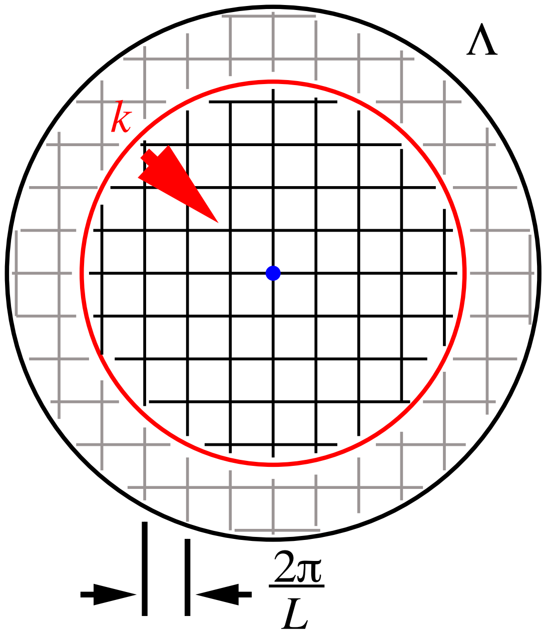


[J. Braun, B. Klein, **H.-J. Pirner**, Phys. Rev. D72, 034017 (2005).]

[J. Luecker et al., Phys. Rev. D81, 094005 (2010).]

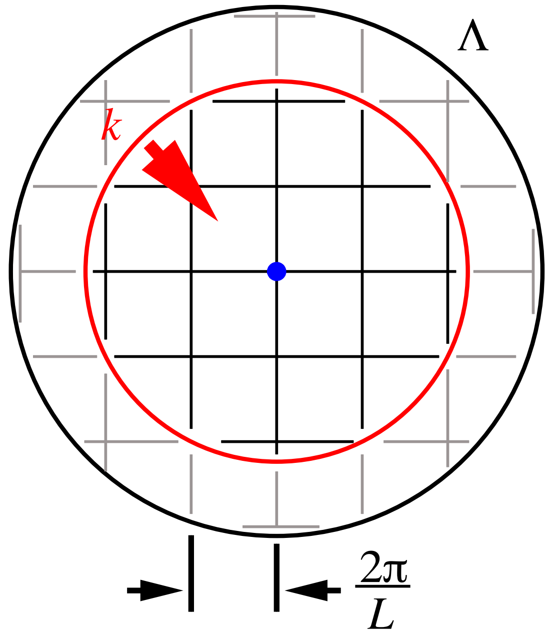
Quark contributions for a finite volume

- quark momentum modes contributing to the condensate (and the constituent quark mass) in a large finite volume
- zero-mode
  for **periodic** b.c.
- no zero mode
  for **anti-periodic** b.c.



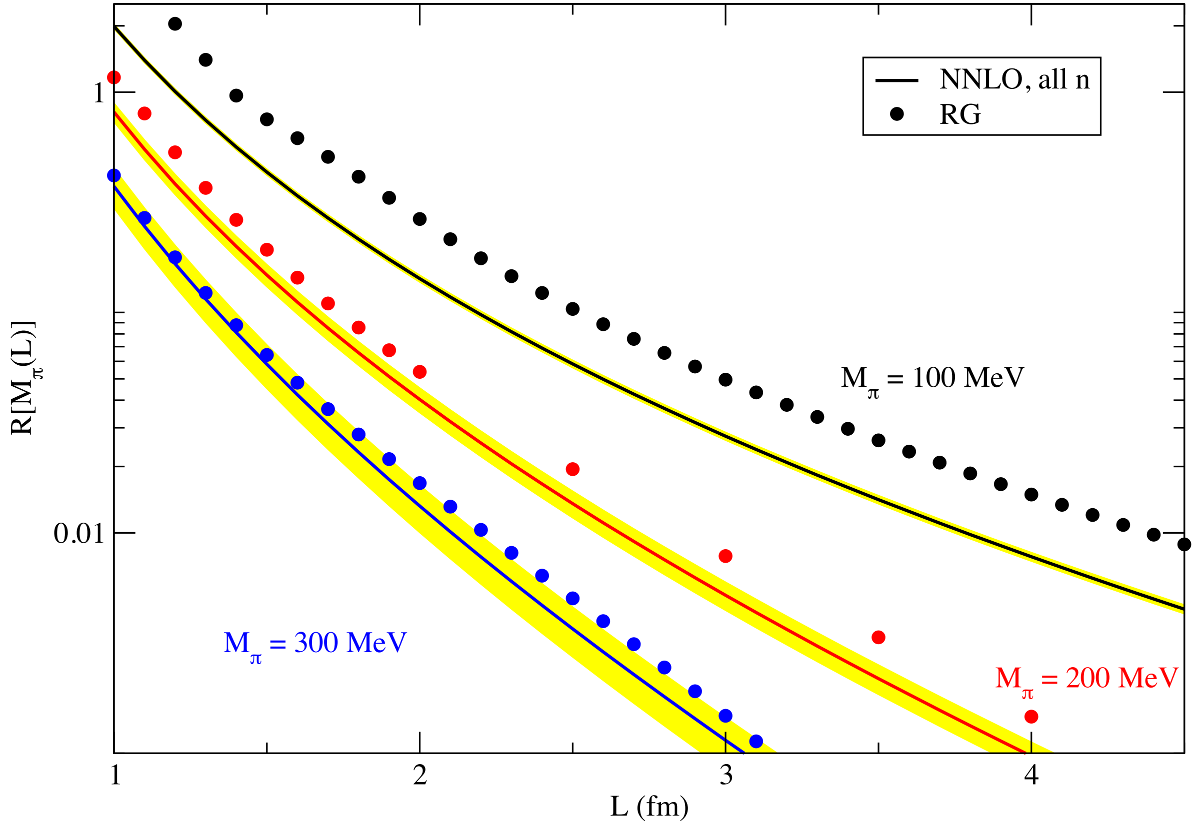
Quark contributions for a finite volume

- quark momentum modes contributing to the condensate (and the constituent quark mass) in a small finite volume
- enhancement of the zero-mode contribution ~1/V for **periodic** b. c.



Comparison to ChPT

- comparison of model results with **anti-periodic** boundary conditions from RG to ChPT in NNLO ChPT data thanks to G. Colangelo
  [G. Colangelo, S. Dürr, C. Haefeli, Nucl. Phys. B 271 (2005) 136.]

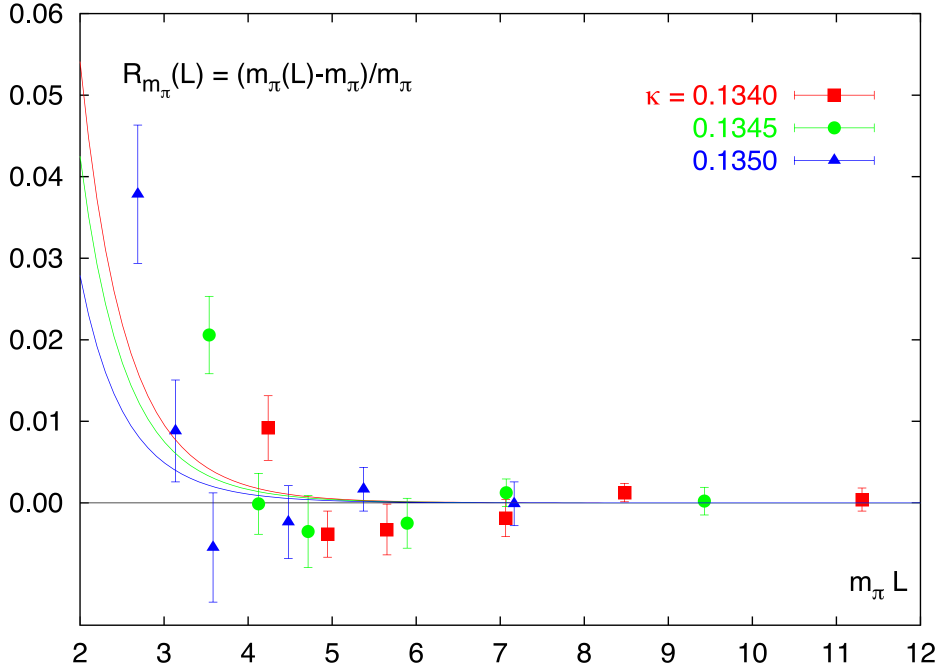


agreement only for **this** choice of boundary conditions!

keep boundary conditions in mind for the finite-volume analysis of lattice QCD results

Comparison to lattice QCD results

- pion mass shift from lattice QCD with Wilson fermions (quenched) with **periodic** boundary conditions
[M. Guagnelli, et al. [ZeRo Collaboration], Phys. Lett. B 597 (2004) 216.]98)1146.]



κ	$m_\pi(\infty)$ [MeV]
0.1340	881
0.1345	735
0.1350	559

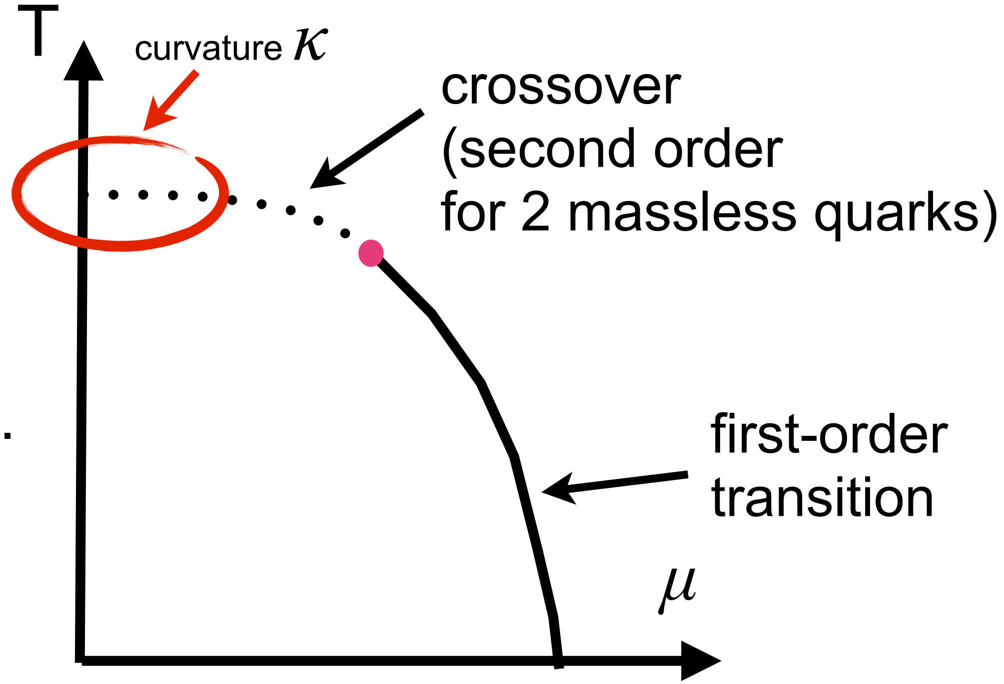
a = 0.079 fm

0.9 fm < L < 2.5 fm

- Wilson fermions (**un**quenched) with **periodic** boundary conditions [B. Orth, T. Lippert and K. Schilling, Phys. Rev. D 72 (2005) 014503.]

QCD phase diagram

at finite chemical potential
...very schematically...



- second-order phase transition for two flavors in the chiral limit
  [R. D. Pisarski and F. Wilczek, Phys. Rev. D 29 (1984) 338]
- crossover at finite quark masses for finite temperature at $\mu = 0$
- conventional expectation:
  first-order phase transition with critical end point

Curvature of the transition line

- at small baryon chemical potential μ, the phase transition line is characterized by the curvature κ

$$\frac{T_\chi(L, m_\pi, \mu)}{T_\chi(L, m_\pi, \mu = 0)} = 1 - \kappa \left(\frac{\mu}{(\pi T_\chi(L, m_\pi, 0))} \right)^2 + \dots$$

- "sign problem" in lattice QCD: simulations are difficult at finite μ

RG methods:
[J. Braun, Eur. Phys. J. C64, 459 (2009)]
→ talk of Jens Braun

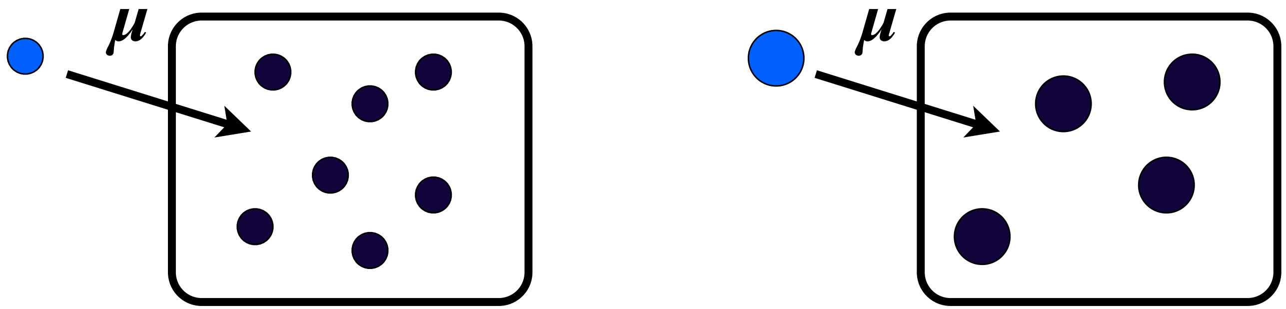
- curvature can be calculated in lattice QCD (imaginary chemical potential, Taylor expansion) [P. de Forcrand and O. Philipsen, Nucl. Phys. B 642 (2002) 290, JHEP 01 (2007) 077; F. Karsch et al., Nucl. Phys. Proc. Suppl. 129, 614 (2004).]

→ differences partially due to finite-volume effects?

Why Finite-volume effects?

- curvature depends on the sensitivity of the system on the chemical potential

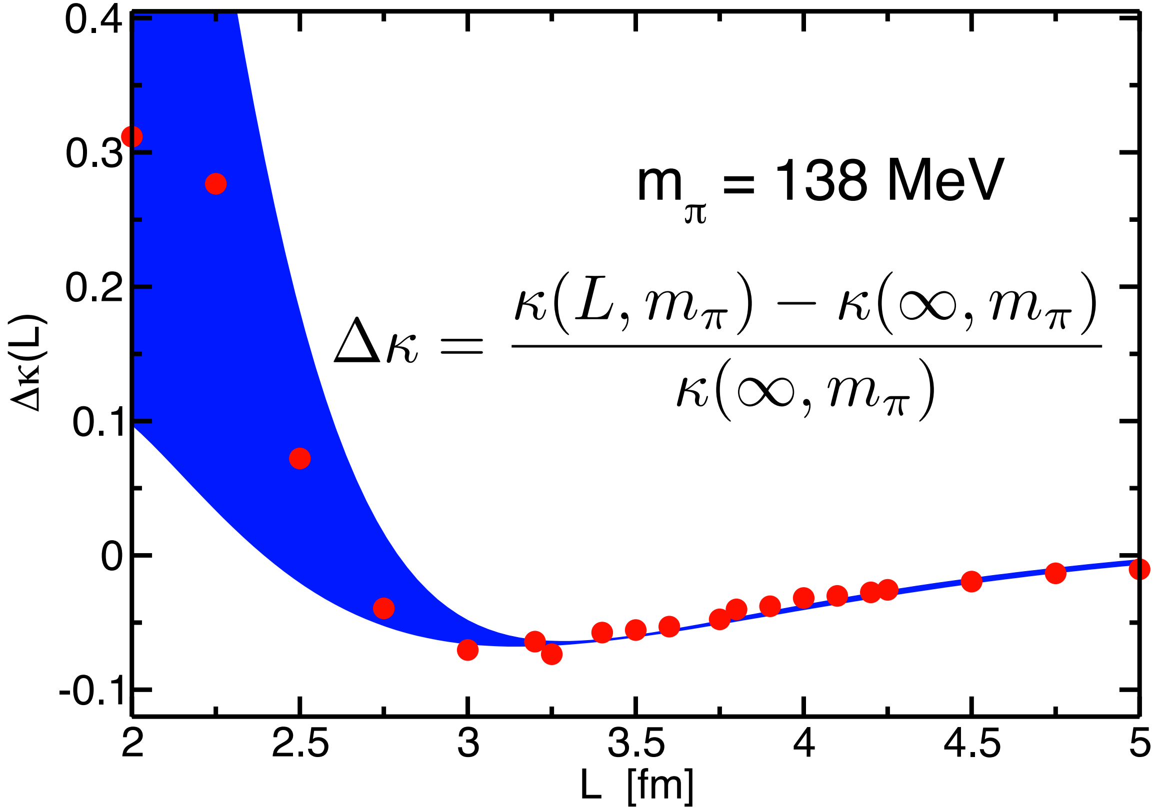
$$\mu = \frac{\partial F}{\partial N_q}\bigg|_{T,V}$$



- sensitivity in turn depends on the "constituent quark mass"
- constituent quark mass affected by volume!

Change of curvature in finite volume

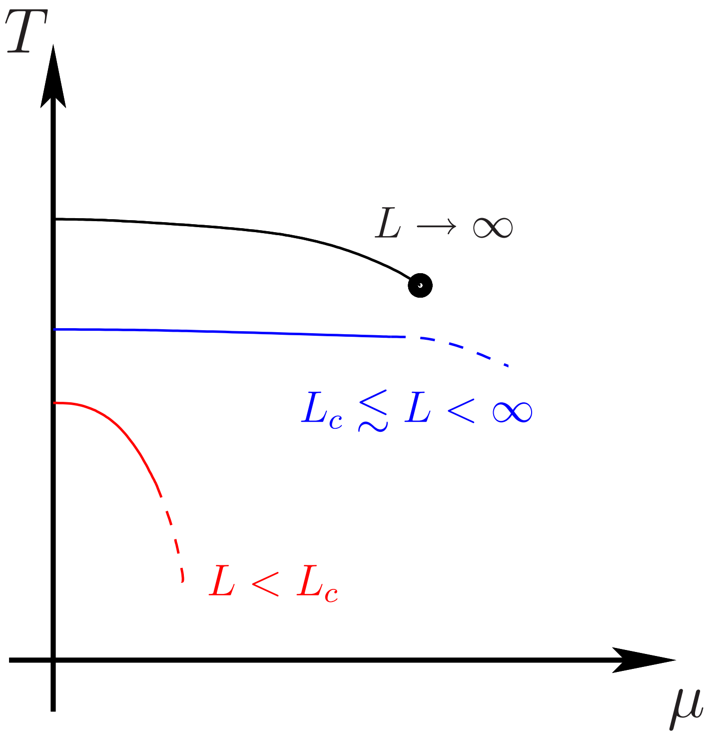
- **periodic** boundary conditions for quarks
- decreasing curvature in intermediate volume
- corresponds to decreasing pion mass/increasing constituent quark mass
- decreased sensitivity to chemical potential



[B.-J. Schaefer, J.Braun, B. Klein, in preparation]

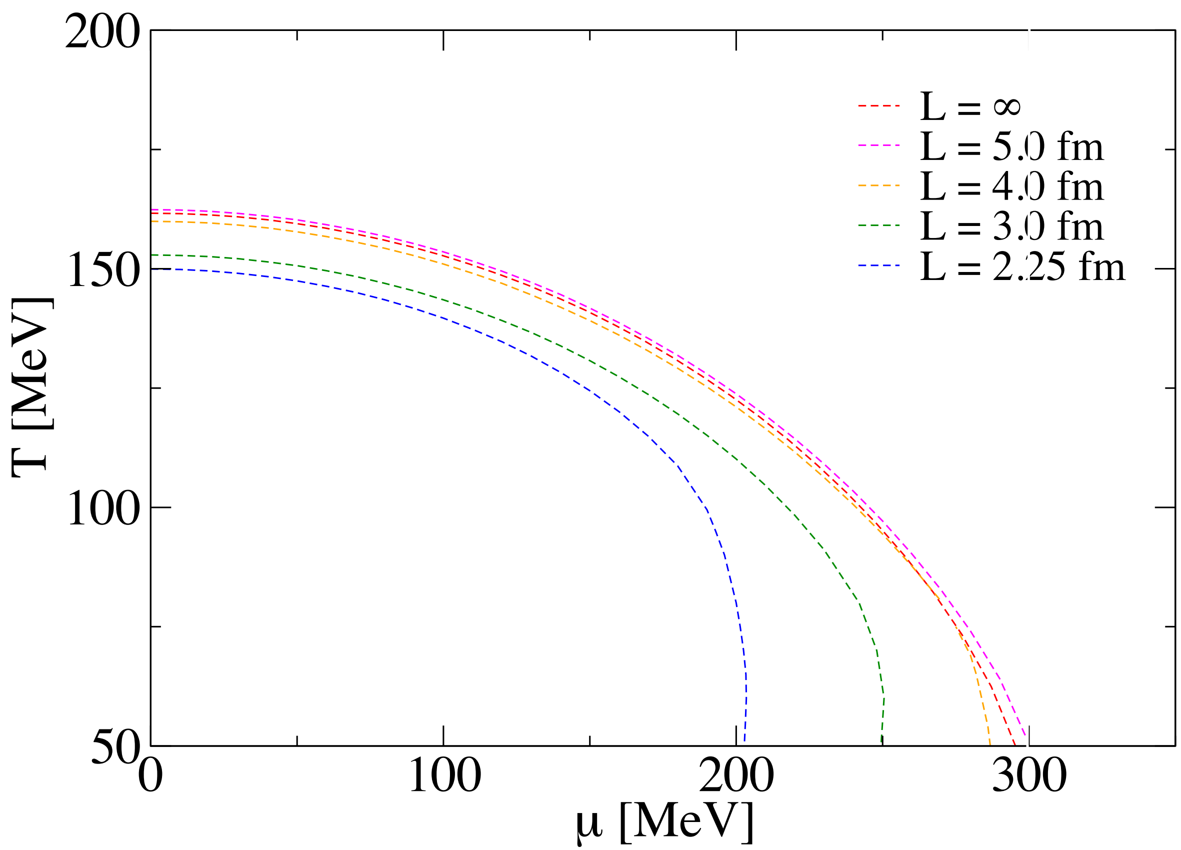
Phase diagram for QCD models in finite volume - qualitative results

- qualitatively clear effects of finite volume on curvature
- phase transition line tends to *flatten* in an intermediate volume range
- curvature increases dramatically for very small volumes



Phase diagram for QCD models in finite volume - first results

- potential discretized on a mesh grid
- first-order phase transition can be determined
- effects on critical point can be determined



[A. Tripolt, B.-J. Schaefer, J.Braun, B. Klein, in preparation]

Summary

- fertile idea: use a finite volume for the analysis of chiral symmetry breaking effects in QCD and combine this with RG methods

- dependence of finite-volume pion mass shift on quark boundary conditions

- qualitative effects on the QCD phase diagram

- finite size scaling analysis [J. Braun and B. Klein, AIP Conf.Proc. 964 (2007) 330, LAT2007 (2007) 198, Eur.Phys.J. C63 (2009) 443; P. Piasecki, J. Braun, and B. Klein, Eur.Phys.J. C71 (2011) 1576.]

Thanks to...

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