High-energy physics with particles carrying non-zero orbital angular momentum

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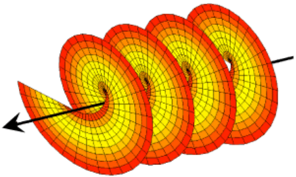
Twisted states: scalar case

Solving free wave equation: monochromatic plane waves.
A plane wave is invariant under azimuthal rotations $\rightarrow$ eigenfunction of the orbital angular momentum (OAM) operator $L_z = -i\partial/\partial\phi$ with zero OAM.

A non-plane wave configuration with ϕ-dependence $\exp(im\phi)$ carries OAM $=$ m. This twisted state has helical wave fronts (=surfaces of equal phase).



Note: at $m \neq 0$, the phase is undefined along the z-axis, so the intensity must be zero.

Coordinate space

Plane wave solutions of the wave equation with definite ω, k_z and $\mathbf{k}$:

$$|PW(\mathbf{k})\rangle = e^{-i\omega t + ik_z z} \cdot e^{i\mathbf{k}\mathbf{r}}\,.$$

Another type of solution in cylindrical coordinates: twisted state $|\kappa, m\rangle$:

$$|\kappa, m\rangle = e^{-i\omega t + ik_z z} \cdot \psi_{\kappa m}(\mathbf{r})\,,$$

where

$$\psi_{\kappa m}(\mathbf{r}) = \frac{e^{im\phi_r}}{\sqrt{2\pi}}\sqrt{\kappa}J_m(\kappa r)\,.$$

Here $r = |\mathbf{r}|$ and κ is the conical momentum spread.

Momentum space

The twisted state $|\kappa, m\rangle$ is superposition of various plane waves $|PW(\mathbf{k})\rangle$ with fixed $|\mathbf{k}| = \kappa$ and all ϕ_k:

$$|\kappa, m\rangle = e^{-i\omega t + ik_z z} \int \frac{d^2\mathbf{k}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) e^{i\mathbf{kr}} \,,$$

where

$$a_{\kappa m}(\mathbf{k}) = (-i)^m e^{im\phi_k} \sqrt{2\pi} \frac{\delta(|\mathbf{k}| - \kappa)}{\sqrt{\kappa}} \,.$$

Twisted photons

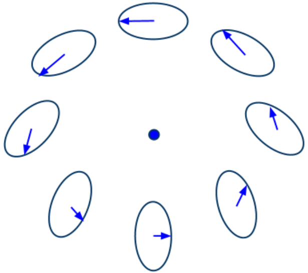
For the photons consider a similar non-plane wave state:

$$A^{\mu}_{\kappa m \Lambda}(\mathbf{r}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) e^{\mu}_{\Lambda}(\mathbf{k}) |PW(\mathbf{k})\rangle \,,$$

- The polarization vector does not factorize because it depends on **k**.
- $e^{\mu}_{\Lambda}(\mathbf{k})$ corresponding to different PWs inside a twisted state don't even lie in the same plane.
- There exists ambiguity to be resolved of how to define relative phase between $e^{\mu}_{\Lambda}(\mathbf{k})$'s with different **k**.

Exotic polarizations

Polarization states of a twisted photon can be very rich, including the exotic cases with polarization singularities.



In paraxial approximation can be described as a superposition of states with different m and λ's.

this example: $|\lambda = +; m = 0\rangle - |\lambda = -; m = +1\rangle$.

Experimental situation: photons

Laser beams with non-zero orbital angular momentum are well-known in the optics and are used in condensed matter physics, atomic physics, quantum information science.

Recently, it was suggested to use Compton backscattering to generate high-energy photons carrying OAM [*Jentschura, Serbo, PRL 106, 013001 (2011)*]:

$$e + \gamma^{optical}_{twisted} \to e + \gamma^{high-energy}_{twisted} .$$

This upconvertion of optical twisted photons to multi-GeV energies seems feasible with today's technology.

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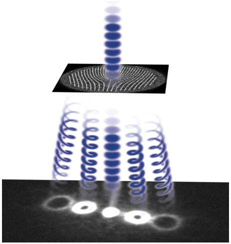
Experimental situation: electrons

In the last months: impressive progress in creating twisted electron states [*Science, 331, 192 (2011)*].



Electrons with $E = 300$ keV and $m \sim 100$ were observed.

Twisted particles enter high-energy physics.

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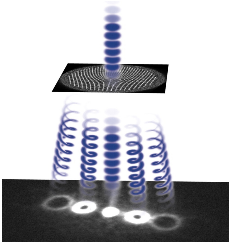

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Single twisted scattering

Now consider the same process but with one twisted particle:

$$|\kappa, m\rangle + |PW(\mathbf{p})\rangle \to X\,.$$

The scattering matrix is

$$S_{tw} = \int \frac{d^2\mathbf{k}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) S_{PW}(\mathbf{k},\mathbf{p})\,.$$

Its square is

$$\begin{aligned} |S_{tw}|^2 &= \int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{d^2\mathbf{k}'}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) a^*_{\kappa m}(\mathbf{k}') S_{PW}(\mathbf{k},\mathbf{p}) S^*_{PW}(\mathbf{k}',\mathbf{p}) \\ &\propto \int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{d^2\mathbf{k}'}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) a^*_{\kappa m}(\mathbf{k}') \delta^{(2)}(\mathbf{k}+\mathbf{p}-\mathbf{p}_X)\delta^{(2)}(\mathbf{k}'+\mathbf{p}-\mathbf{p}_X)\mathcal{M}(\mathbf{k},\mathbf{p})\mathcal{M}^*(\mathbf{k}',\mathbf{p}) \\ &= \int \frac{d^2\mathbf{k}}{(2\pi)^4} a_{\kappa m}(\mathbf{k}) a^*_{\kappa m}(\mathbf{k}) \delta^{(2)}(\mathbf{k}+\mathbf{p}-\mathbf{p}_X)|\mathcal{M}(\mathbf{k},\mathbf{p})|^2\,. \end{aligned}$$

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The cross section can then be written as

$$d\sigma_{tw} = \int \frac{d\phi_k}{2\pi}\, d\sigma_{PW}(\mathbf{k}) \cdot \frac{j_{PW}(\mathbf{k})}{j_{tw}}\,.$$

For paraxial scattering the ratio of fluxes is very close to 1.

- An unusual quantity: $d\sigma_{PW}(\mathbf{k})$ averaged over initial angle and fixed final momenta.
- The single-twisted cross section is m-independent: no smallness associated with OAM.
- No smallness associated with small κ: can be studied experimentally with today's technology.
- $d\sigma_{tw}$ is an incoherent sum of $d\sigma_{PW}(\mathbf{k})$. Initial coherence is lost via non-interfering final states.

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Twisted state in superposition of different m

Example: $a|\kappa, m\rangle + a'|\kappa, m'\rangle$, with $|a|^2 + |a'|^2 = 1$.

$$d\sigma = d\sigma_{tw} + 2|aa'|d\sigma_{tw}^{\Delta m}\,.$$

where

$$d\sigma_{tw}^{\Delta m} = \int \frac{d\phi_k}{2\pi} \cos(\Delta m\,\phi_k + \alpha)\, d\sigma_{PW}(\mathbf{k}) \cdot \frac{j_{PW}(\mathbf{k})}{j_{tw}}\,,$$

with $\Delta m = m - m'$ and α is the relative phase between the two complex coefficients a and a'.

One gets a new tool: Fourier-analyzer of the plane wave cross section w.r.t. initial azimuthal angle ϕ_k.

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A novel way to probe composite particles?

Generic example: elastic scattering of a probe particle by a target particle. $k + p \to k' + p'$; transverse momentum transfer $\mathbf{q} = \mathbf{k}' - \mathbf{k}$.

The target particle has an internal structure ($\to$ formfactor) and can have transverse polarization ($\to$ preferred direction $\mathbf{n}$).

A toy model for the cross-section dependence on $\mathbf{q}$:

$$d\sigma(\mathbf{k},\mathbf{k}') \propto A + B \cdot \mathbf{q}^2 + C \cdot (\mathbf{qn})^2\,,$$

$$\mathbf{q}^2 = \mathbf{k}^2 + \mathbf{k}'^2 - 2kk'\cos(\phi_k - \phi'_k)\,,$$
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Usual way: partial wave analysis requires measuring x-section at different $\mathbf{k}'$, fitting the angular distribution and extracting different spherical harmonics.

Using twisted states with adjustable Δm: Fourier analysis w.r.t. ϕ_k can be performed at fixed $\mathbf{k}'$ by comparing several initial states.

Different requirements for the detectors $\rightarrow$ different systematics $\rightarrow$ complementary tools.

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Colliding two twisted particles

$$|\kappa, m\rangle + |\eta, n\rangle \to X\,.$$

The scattering matrix is

$$S_{2tw} = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{d^2\mathbf{p}}{(2\pi)^2} a_{\kappa m}(\mathbf{k}) a_{\eta n}(\mathbf{p}) S_{PW}(\mathbf{k},\mathbf{p})\,.$$

The square $|S_{2tw}|^2$ contains

$$\int \frac{d^2\mathbf{k}\, d^2\mathbf{p}\, d^2\mathbf{k}'\, d^2\mathbf{p}'}{(2\pi)^8} a_{\kappa m}(\mathbf{k}) a_{\eta n}(\mathbf{p}) a^*_{\kappa m}(\mathbf{k}') a^*_{\eta n}(\mathbf{p}') \times \delta^{(2)}(\mathbf{k}+\mathbf{p}-\mathbf{p}_X)\delta^{(2)}(\mathbf{k}'+\mathbf{p}'-\mathbf{p}_X)\mathcal{M}(\mathbf{k},\mathbf{p})\mathcal{M}^*(\mathbf{k}',\mathbf{p}')\,.$$

Kinematical restrictions: $|\mathbf{k}| = \kappa = |\mathbf{k}'|$ and $|\mathbf{p}| = \eta = |\mathbf{p}'|$; as well as $\mathbf{k}+\mathbf{p} = \mathbf{p}_X = \mathbf{k}'+\mathbf{p}'$.

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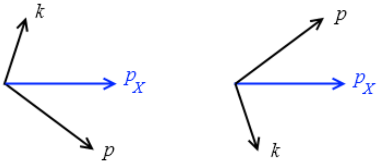

In the product $\mathcal{M}(\mathbf{k},\mathbf{p})\mathcal{M}^*(\mathbf{k}',\mathbf{p}')$ one can have either $\mathbf{k}' = \mathbf{k}$, $\mathbf{p}' = \mathbf{p}$, or

$$\mathbf{k}' = \mathbf{k}^* \equiv -\mathbf{k} + 2(\mathbf{k}\mathbf{n}_X)\mathbf{n}_X\,, \quad \mathbf{p}' = \mathbf{p}^* \equiv -\mathbf{p} + 2(\mathbf{p}\mathbf{n}_X)\mathbf{n}_X\,,$$

where $\mathbf{n}_X \equiv \mathbf{p}_X/|\mathbf{p}_X|$.

These two possibilities interfere. Therefore, the cross section will depend not only on $|\mathcal{M}(\mathbf{k},\mathbf{p})|^2$ but also on $\mathcal{M}(\mathbf{k},\mathbf{p})\mathcal{M}^*(\mathbf{k}^*,\mathbf{p}^*)$.

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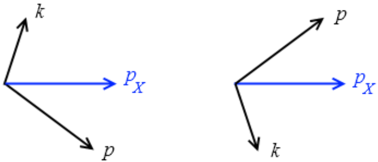

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The cross section is

$$d\sigma_{2tw} = \frac{1}{8\pi \sin(\delta_k + \delta_p)} \int d\phi_k d\phi_p \frac{j_{PW}(\mathbf{k},\mathbf{p})}{j_{2tw}} \left[d\sigma_{PW}(\mathbf{k},\mathbf{p}) + d\sigma'_{PW}(\mathbf{k},\mathbf{p})\right] ,$$

where

$$\begin{aligned} d\sigma'_{PW}(\mathbf{k},\mathbf{p}) &= \frac{(2\pi)^4 \delta(E_i - E_f)\delta(p_{zi} - p_{zf})\delta^{(2)}(\mathbf{k}+\mathbf{p}-\mathbf{p}_X)}{4E_p \omega j_{PW}} \\ &\times \mathrm{Re}\left[e^{2im(\phi_k-\phi_X)+2in(\phi_p-\phi_X)}\mathcal{M}(\mathbf{k},\mathbf{p})\mathcal{M}^*(\mathbf{k}^*,\mathbf{p}^*)\right] d\Gamma_X . \end{aligned}$$

and

$$\delta_k = \arccos\left(\frac{\mathbf{p}_X^2 + \kappa^2 - \eta^2}{2|\mathbf{p}_X|\kappa}\right) , \quad \delta_p = \arccos\left(\frac{\mathbf{p}_X^2 - \kappa^2 + \eta^2}{2|\mathbf{p}_X|\eta}\right) .$$

The double-twisted cross section is m, n-dependent and stays finite at small κ, η.

Conclusions

- Many effects described are observable with today's technology.
- Fourier transform w.r.t. initial azimuthal angle might emerge as a new tool, complementary to the standard PWA.
- Double-twisted x-section is sensitive to the autocorrelation function, which is inaccessible in plane wave collisions.
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