Tests of gravity at the solar system scale

M.-T. Jaekel and S. Reynaud

LPT, ENS, Paris LKB, UPMC, Paris

Fundamental Physics Laws:
Gravity, Lorentz Symmetry and Quantum Gravity
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Marc.Jaekel@lpt.ens.fr
Serge.Reynaud@spectro.jussieu.fr

Outline

- Gravitation theory: beyond General Relativity; scale dependence; metric extensions.
- Experimental gravitation: classical tests; anomalies.
- Phenomenology in the solar system: ranging on probes, ephemerides, light deflection.
- Conclusions.

Gravitation theory
Geometry and fields

According to General Relativity (GR), gravitation is described by the geometry of a Riemannian space-time

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu$$

- the geometrical distance is measured as the proper time delivered by ideal clocks along their trajectories: $\tau \equiv \int ds$
- freely falling probes (masses and light) follow geodesics: $\delta(\int ds) = 0$

Coupling to a same metric field leads to the universality of free fall:
The **equivalence principle** is the best tested property of nature.

As a fundamental interaction, gravitation is carried by fields:
curvature couples to the energy-momentum tensor of gravity sources

- one curvature tensor $E_{\mu\nu}$ has a null divergence (Bianchi identities) like the energy-momentum tensor $T_{\mu\nu}$ (conservation laws)

$$E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \qquad \nabla^\nu E_{\mu\nu} = 0, \qquad \nabla^\nu T_{\mu\nu} = 0$$

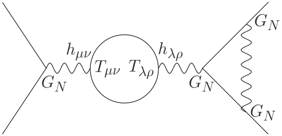
- in GR, the two tensors are simply proportional to each other

$$E_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} \qquad \text{Einstein} - \text{Hilbert} \quad \text{equations}$$

Newton gravitation constant $\mathbf{G_N}$ is the less well known fundamental constant.

Beyond GR

Gravitation is one of the four fundamental interactions.
The electromagnetic, weak and strong interactions share a same property: radiative corrections entail scale dependent couplings.
Fluctuations of metric fields and stress tensors modify the graviton propagator *i.e.* the effective coupling between metric fields and sources



Radiative corrections introduce a coupling to squares of curvatures:

- GR is embedded in renormalizable theories.
- G_N becomes scale dependent (a running coupling constant).
- gravitation involves additional couplings.

GR extends to a theory which preserves its geometric basis:

- gravitation is described by a metric theory.
- it may remain close to GR within a large range of scales.
- the corrections to GR introduce two gravitational sectors.

Metric extensions of GR

The general gravitation equations may be written as response equations

$$E_{\mu\nu} = \chi_{\mu\nu}(T) = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \delta\chi_{\mu\nu}(T)$$

M.-T. Jaekel, S. Reynaud, Ann. Physik (1995) 68

The graviton couples differently to massive and massless fields (trace and traceless energy-momentum tensors): the effective couplings in the two sectors of Weyl (traceless) and scalar (trace) curvatures differ.

In the linearized limit, the two sectors can be separated with (non local) linear projectors. For a stationary pointlike source

$$T_{\mu\nu} = \delta_{\mu 0}\delta_{\nu 0} T_{00}, \qquad T_{00} = Mc^2 \delta(k_0)$$

$$E_{\mu\nu} = E^{(0)}_{\mu\nu} + E^{(1)}_{\mu\nu}, \qquad \pi_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

$$E^{(0)}_{\mu\nu} = \{\pi^0_\mu \pi^0_\nu - \frac{\pi_{\mu\nu}\pi^{00}}{3}\} \frac{8\pi G^{(0)}}{c^4} T_{00}, \qquad E^{(1)}_{\mu\nu} = \frac{\pi_{\mu\nu}\pi^{00}}{3} \frac{8\pi G^{(1)}}{c^4} T_{00}$$

$$G^{(0)} = G_N + \delta G^{(0)}, \qquad G^{(1)} = G_N + \delta G^{(1)}$$

The two couplings are equivalent to two gravitation potentials replacing Newton potential Φ_N

$$g_{00} = 1 + 2(\Phi_N + \delta\Phi_N), \qquad g_{ij} = -(1 - 2(\Phi_N + \delta\Phi_N - \delta\Phi_P))\delta_{ij}$$

Parametrized vicinity of GR

In general, metric solutions appear as perturbations of Einstein curvature

$$E_\nu^\mu \equiv [E_\nu^\mu]_{\mathrm{st}} + \delta E_\nu^\mu \,, \qquad \delta E_\nu^\mu \equiv \delta\chi_{\mu\nu}(T)$$

In the vicinity of GR, the two gravitation running couplings are equivalent to two independent components of Einstein curvature.

The two independent curvatures are equivalent to two gravitation potentials $\delta\Phi_N$ and $\delta\Phi_P$

$$\delta E_0^0 \equiv 2u^4(\delta\Phi_N - \delta\Phi_P)'', \quad \delta E_r^r \equiv 2u^3\delta\Phi_P' \qquad u \equiv \frac{1}{r}, \quad ()' \equiv \partial_u$$

In the static isotropic case, the two potentials describe anomalous parts of the metric

$$\delta g_{rr} = \frac{2u}{(1-2\kappa u)^2}(\delta\Phi_N - \delta\Phi_P)', \qquad \kappa \equiv \frac{G_N M}{c^2}, \quad \Phi_N = -\kappa u$$

$$\delta g_{00} = 2\delta\Phi_N + 4\kappa(1-2\kappa u)\int \frac{u(\delta\Phi_N - \delta\Phi_P)' - \delta\Phi_N}{(1-2\kappa u)^2} du$$

The two potentials provide a gauge-independent parametrization of metric theories in the vicinity of GR.

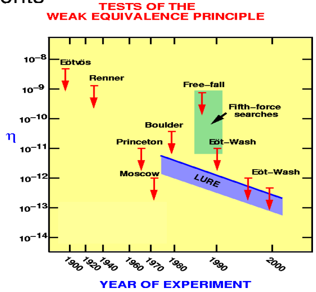
M.-T. Jaekel, S. Reynaud, Class. Quantum Grav. 23 (2006) 777

Experimental gravitation
Tests of the equivalence principle

- Eötvös type experiments
- Tests of the universality of free fall
- Earth-Moon distance measurements

Relative acceleration
between test bodies
of different compositions

$$\eta \equiv 2\frac{a_1 - a_2}{a_1 + a_2}$$



The equivalence principle
is presently tested at 10^{-13}

C.F. Will Living Reviews in Relativity, 9 (2006) 3

to be improved to 10^{-15} (Microscope)

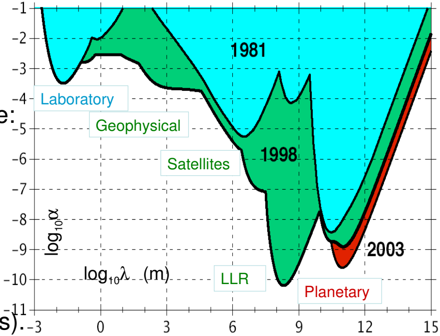
Tests of Newton potential

The Newtonian dependence of the potential in the first sector is well tested within a large range of scales.

Search for a Yukawa correction
$$\delta\Phi_N(r) = \alpha e^{-\frac{r}{\lambda}}\phi(r)$$

In the submillimeter range: fifth force tests.

At planetary range: best fits performed with observations on artificial probes and planets (ephemerides).



J. Coy, E. Fischbach, R. Hellings, C. Talmadge, E.M. Standish (2003)

Significant deviations remain possible at very short and very long ranges

PPN metrics

Tests of GR are performed by comparing observations with predictions obtained from a parametrized family of (PPN) metrics.

In the approximation of a pointlike gravitational source (and ignoring effects due to rotation) PPN metrics may be written (in isotropic coordinates)

$$g_{00} = 1 + 2\Phi_N + 2\beta\Phi_N^2 + \ldots, \qquad \Phi_N = -\frac{G_N M}{c^2 r}$$

$$g_{rr} = -1 + 2\gamma\Phi_N + \ldots$$

Eddington parameters γ and β parametrize the effects of gravitation on light propagation and on the trajectories of massive bodies.

PPN metrics are particular cases of metric extensions of GR

$$\delta\Phi_N = (\beta - 1)\Phi_N^2 + O(\Phi_N^3), \qquad \delta\Phi_P = -(\gamma - 1)\Phi_N + O(\Phi_N^2)$$

$$\delta E_0^0 = \frac{1}{r^2} O(\Phi_N^2), \qquad \delta E_r^r = \frac{1}{r^2}\left(2(\gamma - 1)\Phi_N + O(\Phi_N^2)\right) \qquad \text{[PPN]}$$

Classical tests

Observations are compared with PPN predictions depending on (β, γ)

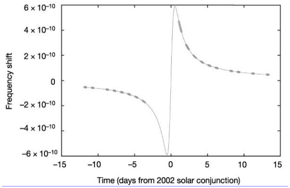
$$\delta\Phi_N = (\beta - 1)\Phi_N^2, \qquad \delta\Phi_P = -(\gamma - 1)\Phi_N$$

- Ranging on planets
- Astrometry and VLBI
- Lunar laser ranging
- Doppler velocimetry on probes
- Light deflection

Tests of β, γ are consistent with GR and bound allowed deviations

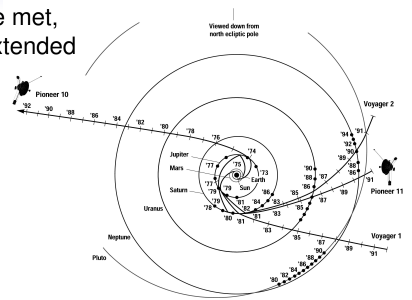
$$|\gamma - 1| < 3 \times 10^{-5}, \qquad |\beta - 1| < 1 \times 10^{-4}$$

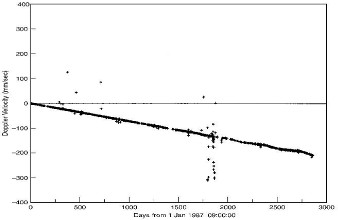
Measurement of the two-way relativistic frequency shift due to the Sun gravitation (Cassini)
B. Bertotti, L. Iess and P. Tortora, Nature 425 (2003) 374



Tests in the outer solar system

After their planetary objectives were met, the Pioneer 10/11 missions were extended by NASA, providing the best long-range test of gravity to date.
Anomalies have been observed after Pioneer 10/11 last flybies.





The radio signals used for navigation showed regular deviations from the values predicted by GR.
J. Anderson et al.,
Phys. Rev. D 65 (2002) 082004

Doppler residuals exhibited a nearly linear dependence in time

$$v_{obs} - v_{model} \simeq -a_P(t - t_{in}), \qquad a_P \simeq 0.9\ \mathrm{nm\ s}^{-2}$$

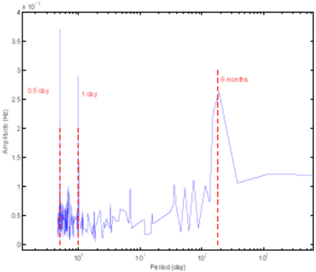
No conventional explanation has been able to totally account for the anomaly.

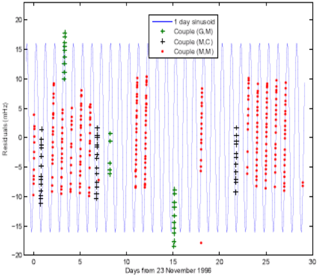
Pioneer 10/11 anomalies

Independent analyses have confirmed the Pioneer anomaly (J. Anderson et al., C Markwardt, O Olsen) and recently recovered data (navigation and on board sensors) have been added to the analysis (Pioneer Anomaly Investigation Team).

ODF for Pioneer 10 has been reanalysed with a dedicated software

A spectral analysis details the periodic modulations present in Doppler residuals





A. Levy et al.,
Advances in Space Research,
43 (2009) 1538

- modulated anomalies may signal a mismodeling of trajectories, ...
- or may provide additional hints on gravitation anomalies

Phenomenology in the solar system
Ranging on probes

Metric extensions of GR provide a framework which is well suited for analysing gravitation tests performed in the solar system. Light-like propagation is characterized by a time delay function

$$c\mathcal{T}(r_1, r_2, \phi) \equiv \int_{r_1}^{r_2} \frac{-\frac{g_{rr}}{g_{00}}(r)dr}{\sqrt{-\frac{g_{rr}}{g_{00}}(r) - \frac{\rho^2}{r^2}}} \quad , \quad \phi = \int_{r_1}^{r_2} \frac{\rho dr/r^2}{\sqrt{-\frac{g_{rr}}{g_{00}}(r) - \frac{\rho^2}{r^2}}}$$

The time delay function is parametrized by the potentials in the two sectors. The second time derivative (or time derivative of the Doppler signal) gives a difference with GR which appears as an anomalous acceleration

$$\delta a \simeq \delta a_{\mathrm{sec}} + \delta a_{\mathrm{mod}}$$

$$\delta a_{\mathrm{sec}} \simeq -\frac{c^2}{2}\partial_r(\delta g_{00}) + [r_2]_{\mathrm{st}} \left\{ \frac{\delta(g_{00}g_{rr})}{2} - \delta g_{00} \right\} - \frac{c^2}{2}\partial_r^2 [g_{00}]_{\mathrm{st}}\, \delta r_2$$

$$\delta a_{\mathrm{mod}} \simeq \frac{d}{dt}\left\{ \left[\dot{\phi}\right]_{\mathrm{st}} \delta\rho \right\}$$

The Pioneer-like anomaly has a secular part δa_{sec} and a modulated part δa_{mod}. The secular and modulated anomalies are correlated.

M.-T. Jaekel, S. Reynaud, Class. Quantum Grav. 23 (2006) 7561

Planet ephemerides

Metric extensions of GR provide equations for light-cones and geodesics which are parametrized by two functions (the potentials Φ_N and Φ_P).

Expressions for the perihelion precessions of planets generalize those obtained from PPN metrics

$$\begin{aligned} \frac{\delta \Delta \varpi}{2\pi} \quad \simeq \quad & u\left(u\delta\Phi_P\right)'' - \frac{c^2 u}{2G_N M}\delta\Phi_N'', \qquad\qquad (u = \frac{1}{r}) \\ & + \frac{e^2 u^2}{8}\left(\left(u^2\delta\Phi_P'' + u\delta\Phi_P'\right)'' - \frac{c^2 u}{2G_N M}\delta\Phi_N''''\right) + \ldots \end{aligned}$$

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Simple realistic models for the two potentials may be designed to provide small sets of parameters to be tested in place of the usual PPN parameters. The same parametrized functions can be used

- when analysing ranging and Doppler data obtained from probe tracking
- when fitting the parameters determining planet ephemerides.

Probe ranging and planet ephemerides provide sensitive probes for a scale dependence of gravitation above the A.U scale.

Light deflection

Light deflection depends on a (conformal) combination of the two potentials

$$2\delta\Phi_N(r) - \delta\Phi_P(r) \equiv -\frac{G_0 M}{c^2 r} + \frac{M}{c^2} r \zeta_0(r)$$

Comparing with the usual PPN framework, the second sector results in an Eddington parameter γ which depends on the impact parameter ρ

$$\delta\gamma(\rho) = \frac{2(G_0 - G_N)}{G_N} - \frac{\zeta_0(\rho)\rho^2}{G_N}$$

The deflection angle exhibits an anomalous dependence with respect to GR

$$\delta\Delta\theta \simeq -\frac{G_N M}{c^2} \frac{\partial}{\partial\rho} \left(\delta\gamma(\rho) \ln \frac{4 r_1 r_2}{\rho^2} \right)$$

GR deflection angles increase with smaller impact parameters
but anomalies may increase with larger impact parameters.

Precise light deflection tests at large angles (GAIA) provide sensitive probes for a scale dependence of gravitation below the A.U. scale.

M.-T. Jaekel, S. Reynaud, Class. Quantum Grav. 22 (2005) 2135

Conclusions

- General Relativity is well confirmed by present tests of gravity, except possible anomalies in the outer part of the solar system.
  Observations at larger scales however point at the necessity to test gravitation more precisely and to look for potential scale dependences.
  This is conforted by a theoretical analysis, which suggests a parametrization of the vicinity of GR appropriate for analysing tests.
- Metric extensions of GR are parametrized by two gravitation potentials which generalize the PPN parameters β, γ.
  They provide the necessary parameters for studying scale dependences.
  They can account for Pioneer-like anomalies and predict correlated anomalies. These could be exhibited by further analyses of available data or by experiments in future space missions.
- The usefulness of the extended famework is not limited to the solar system. Although bound to remain small at this scale and mainly significant in the second sector, modifications of GR may become more important at larger scales and also affect the first sector.
  As suggested by observations at galactic and cosmological scales.